

Low-threshold Raman Silicon Lasers Using Photonic Crystal High-Q Nanocavities

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Abstract: By utilizing stimulated Raman scattering, it is possible to generate continuous-wave laser light in silicon, an indirect bandgap semiconductor. The first part of this chapter explains the mechanism of the Raman laser using a silicon resonator with a high-quality factor (Q). In the second part, the mechanism of the ultra-low threshold Raman silicon laser using a photonic crystal high- Q nanocavity is summarized, and recent advancements are explained.

1. Introduction

Integrated electronic circuits (ICs) and semiconductor lasers are indispensable devices for the current smart society. These technologies are essential for driving the innovations and infrastructures that sustain modern-day life. The core material of ICs is silicon, a Group IV element. On the other hand, semiconductor lasers, which are indispensable for high-speed optical communication, are mainly made from Group III-V compound semiconductors. These semiconductors are composed of Group III elements (e.g., aluminum, gallium, indium) and Group V elements (e.g., nitrogen, phosphorus, arsenic) [1]. If a compact silicon laser compatible with complementary metal-oxide-semiconductor (CMOS) fabrication procedures is realized, silicon photonic circuits are expected to be integrated into IC chips. Such optoelectronic integrated silicon chips are widely anticipated to drive significant advancements in both the electronics and photonics industries [2–6].

Silicon and germanium, IV group semiconductors, are indirect bandgap semiconductors. As shown in the band diagram in Fig. 1a, the momentum vector of the conduction band minimum differs from that of the valence band maximum. When an electron transits between different two energy states, both the energy conservation law and the momentum conservation law must be satisfied. Since the momentum vector of light is too small, it is difficult for an electron at the conduction band minimum of silicon to release its energy as light via interband transition while fulfilling the momentum conservation law. To satisfy the momentum conservation law, it is more likely that the energy will be released as multiple phonons, which have a larger momentum compared to light. Consequently, in indirect bandgap semiconductors, the most of energy for the electron in conduction band is emitted as heat rather than light.

On the other hand, III-V compound semiconductors such as GaAs and InGaN are direct bandgap semiconductors. As shown in Fig. 1b, the momentum vector at the conduction band minimum is identical to that at the valence band maximum. Therefore, electrons in the conduction band easily release their energy as light. While phonon emission also occurs in direct bandgap semiconductors, the maximum energy of a single phonon is only about 5% of

the bandgap energy. The quantum efficiency (η) of the radiative process is determined by the competition between the radiative recombination rate (k_R), where all energy is released through a single photon, and the non-radiative recombination rate (k_{NR}), where energy is lost through multiple phonon emissions.

$$\eta = \frac{k_R}{k_R + k_{NR}} \quad (1)$$

In the high-quality semiconductors with few defects, the radiative process can occur at room temperature with a probability comparable to that of the non-radiative process.

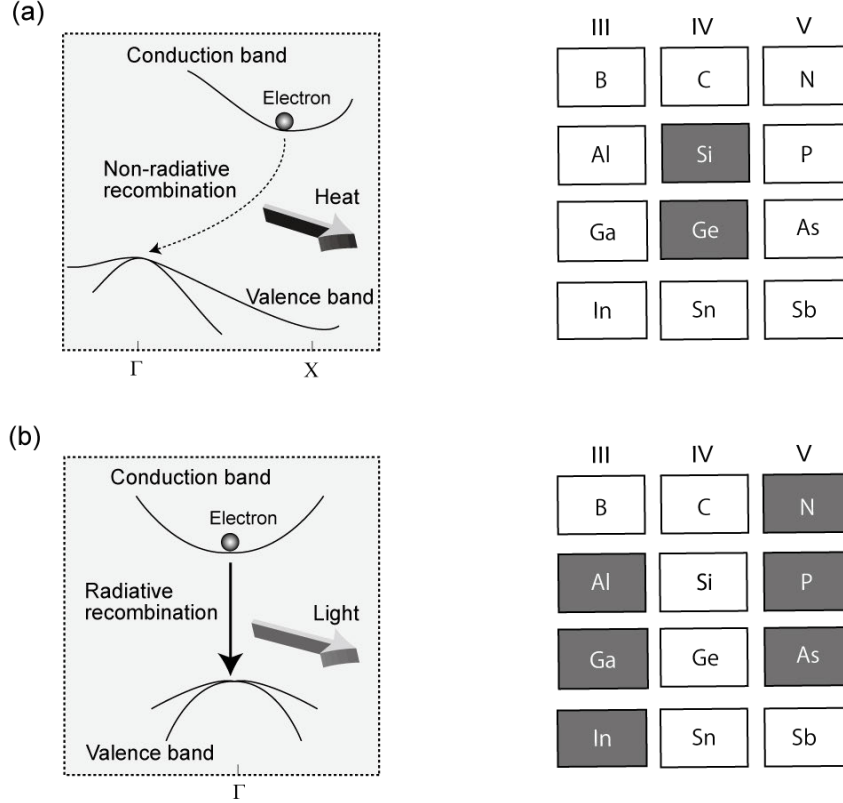


Figure 1. (a) Band diagram with non-radiative recombination process in silicon. (b) Band diagram and radiative process in group III-V compound semiconductor (e.g., GaAs).

It is possible for an electron in the conduction band of silicon to release its energy as light through the emission of a transverse-optical (TO) phonon, which helps satisfy the momentum conservation law (Fig. 2a) [7]. At low temperatures around 10 K, this emission peak can be clearly measured in the spectrum [8]. In addition, when silicon is strongly excited at low temperatures, an emission peak from the electron-hole droplet state is also observed [9–11]. However, for silicon at room temperature, k_{NR} is generally much larger than k_R . Thus, the η is much smaller than 10^{-4} . Various engineering approaches have been attempted to enhance η for the interband transition by increasing the k_R and decreasing the k_{NR} . Several nanostructured silicon systems and silicon engineered materials have shown promising properties such as nanocrystals [12–14], nanolayers [15,16], nanowires [17–19], nanocavities [20–22], and heterostructures with SiO_2 and Ge [23–26]. However, there is still no faultless solution for practical silicon lasers in order to achieve inexpensive, monolithic optoelectronic chips with CMOS compatibility.

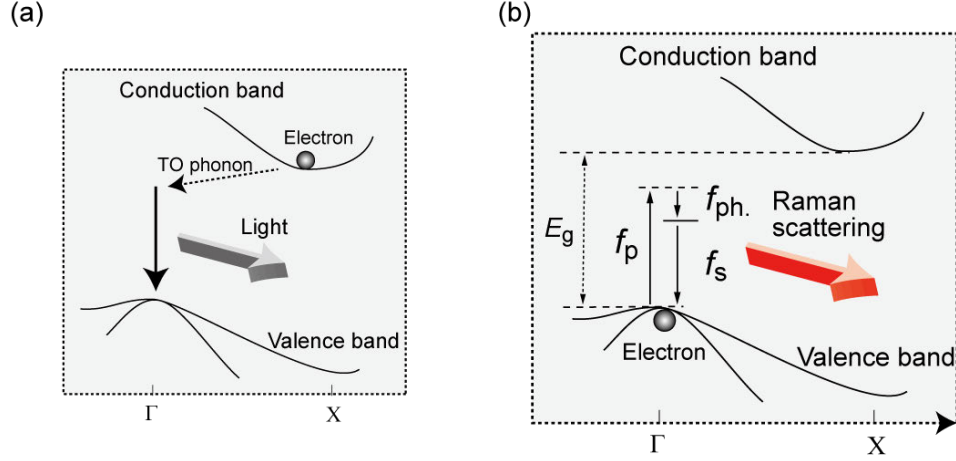


Figure 2. (a) Radiative process involving TO-phonon emission in silicon. (b) The Raman scattering process, specifically spontaneous Stokes Raman scattering, in silicon. f_p : frequency of the excitation light, f_s : frequency of the Stokes Raman scattered light, f_{ph} : Raman frequency of silicon.

In 2005, the first room-temperature continuous-wave silicon laser was reported [27]. Unlike the interband transition shown in Fig. 2a, it was an optically pumped laser based on Raman scattering, as illustrated in Fig. 2b. In the following section 2, the mechanisms of the Raman silicon laser are explained, and an overview of Raman silicon lasers employing rib waveguides is provided. Section 3 offers a detailed description of Raman silicon lasers using photonic crystal high- Q nanocavities with an ultra-low threshold of approximately 1 μ W [28]. Section 4 presents recent advancements in nanocavity-based Raman silicon lasers toward applications in integrated optoelectronic chips.

2. Raman Silicon Laser

Raman silicon laser stably oscillates in continuous-wave mode at room temperature. In 2002, the development of a Raman silicon laser utilizing a silicon rib-waveguide was proposed [29]. The magnitude of the Raman gain in silicon was estimated through the measurement of the spontaneous Raman scattering spectrum, paving the way for the development. The subsequent reports immediately presented optical amplification by stimulated Raman scattering in silicon waveguide [30–33]. Subsequently, it was revealed that free-carrier absorption caused by the carriers generated via two-photon absorption interferes with laser oscillation [34–37]. Finally, room-temperature pulsed oscillation was reported in 2004 [38], followed by continuous-wave operation at room temperature in 2005 [27,39]. This section describes the development of the Raman silicon laser.

2.1 Spontaneous Raman Scattering

Spontaneous Raman scattering was discovered in 1928 by C. V. Raman and K. S. Krishnan [40]. This groundbreaking work was the first to demonstrate that light, when interacting with matter, can be scattered at frequencies different from the incident light. This discovery marked a major breakthrough in the study of light-matter interactions, providing new insights into the internal structures of materials as well as the vibrational and rotational states of molecules. In 1930, C. V. Raman was awarded the Nobel Prize in Physics.

Raman scattering is a phenomenon in which light with a frequency f_p irradiates a material, and light with a frequency f_s , shifted by the phonon vibrational frequency f_{phonon} , is emitted. When red-shifted scattered light is observed, the relationship $f_s = f_p - f_{\text{phonon}}$ holds for the frequencies. This is called Stokes Raman scattering. Blue-shifted scattered light can also be

obtained, where the relationship $f_s = f_p + f_{\text{phonon}}$ holds for the frequencies, called anti-Stokes Raman scattering. Since Stokes Raman scattering is used in Raman silicon lasers, henceforth, when referring to Raman scattering, it implies Stokes Raman scattering.

The frequency f_{phonon} is unique to the material and can also vary depending on strain and crystal structure. Therefore, by precisely measuring the spectra of f_p and f_s , one can obtain information of the crystal structure and strain of the irradiated material, without causing damage to the material. Raman spectral analysis has become an indispensable tool in various research fields, including chemistry, physics, materials science, and biology. Furthermore, portable Raman spectrometers are used in non-research applications, such as the detection of hazardous substances, gemstone authentication, and cultural heritage investigations.

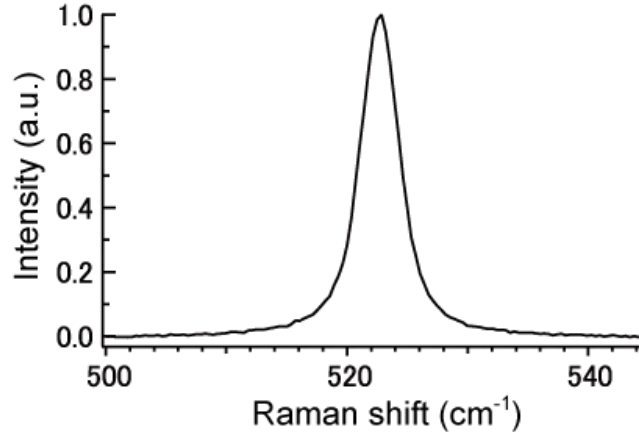


Figure 3. Raman scattering spectrum for silicon.

Figure 3 shows the spontaneous Raman spectrum of silicon measured by typical Raman spectrometer. The horizontal axis is in wavenumbers (cm^{-1}), a unit commonly used in materials science research. In contrast, terahertz (THz) is often used as the unit for Raman shift in the field of photonics. A strong peak is observed around 520 cm^{-1} , which corresponds to 15.6 THz.

In Raman scattering process, the momentum conservation law must also be satisfied. As a result, the Raman peak originating from phonons with zero momentum is strongly observed in Raman spectrum as shown in Fig. 3. Figure 4 shows the phonon dispersion diagram of silicon, calculated using first-principles methods [41]. The phonons with zero momentum correspond to the Γ -point in the diagram. In silicon, the frequency of these phonons is known to be around 520 cm^{-1} (15.6 THz). This value is commonly referred to as the Raman shift or Raman frequency for silicon.

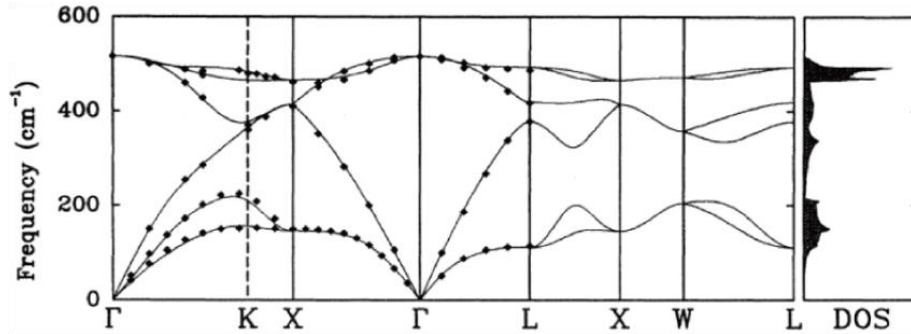


Figure 4. Phonon dispersion and density of state for silicon. Reprinted with permission from [41]. Copyright 1991 American Physical Society

It is technically challenging to accurately determine the absolute value of the Raman shift with a precision of 0.1 cm^{-1} . Such high-precision measurement is essential when investigating the crystal quality and strain of semiconductor substrates. When the Raman spectrum of undoped bulk silicon is measured at room temperature under weak excitation power of 0.5 mW using a high-wavelength-resolution spectrometer (e.g., a double monochromator with a focal length of 1 m), a value of 520.5 cm^{-1} is typically obtained. The full width at half maximum (FWHM) of the Raman peak in this measurement is 2.75 cm^{-1} . It should be noted that the FWHM of the Raman peak varies significantly across different reports due to variations in measurement equipment and techniques. For example, a report in 1967 shows Raman shift value of $520.2 \pm 0.5 \text{ cm}^{-1}$ and the FWHM of 4.6 cm^{-1} [42]. While the Raman shift value is almost the same as the currently well-known value, there is a large difference in the FWHM. Additionally, both the Raman shift and FWHM are slightly affected by the strain applied to the silicon. Silicon-on-insulator (SOI) substrates are commonly used for silicon photonic devices, and the top silicon layer typically exhibits tensile strain [43]. Generally, the changes in Raman shift and FWHM due to this strain are about 0.1 cm^{-1} .

2.2 Stimulated Raman Scattering

In a conventional semiconductor laser, the radiative recombination process between the conduction band and the valence band, shown in Fig. 1b, are utilized for laser oscillation. There are two types of radiative processes: spontaneous emission and stimulated emission. The source of laser oscillation is the stimulated emission. Similarly, there are two types of processes in Raman scattering: spontaneous Raman scattering and stimulated Raman scattering [44]. The source of a Raman laser is stimulated Raman scattering, which is a third-order nonlinear optical effect. Therefore, a Raman laser is always optically pumped and requires an excitation light source. Since a third-order nonlinear effect requires a strong electric field, the excitation light for a Raman laser is usually provided by laser source.

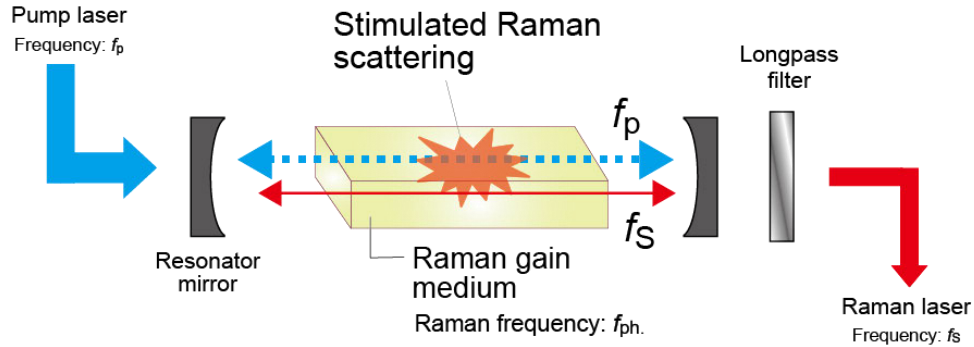


Figure 5. Schematic of conventional Raman laser.

The first Raman laser oscillation was reported in 1962 for the nitrobenzene using a ruby laser as the excitation light source [45]. Thus far, Raman laser oscillation has been demonstrated in a variety of materials. Figure 5 illustrates the basic configuration of a Raman laser. The system consists of a Raman gain medium placed inside a resonator, which is excited by an external laser source with a frequency f_p . The excitation laser light and the Raman scattered light with the frequency f_s are confined within the resonator, efficiently inducing stimulated Raman scattering, resulting in the emission of Raman laser light. When a semiconductor is used as a Raman medium, laser oscillation can occur if the energy of f_s is lower than the bandgap energy of E_g (Fig. 2b).

Since a Raman laser requires an external pump laser, its applications are somewhat restricted compared to those of electrically injected semiconductor lasers. However, Raman silicon lasers have several attractive advantages. The first advantage is the capability for

oscillation over a wide wavelength range. In fact, Raman silicon lasers have been reported to operate for the pump wavelengths ranging from 1.20 μm and 1.68 μm [27,28,46–48]. A single Raman silicon laser chip has also been reported to enable tunable oscillation wavelengths from 1.33 μm to 1.84 μm [49]. The second advantage is a large SMSR (side mode suppression ratio) compared to conventional semiconductor lasers. This is because silicon's Raman shift is as large as 15.6 THz, which prevents the excitation light from contributing to background noise, and the efficiency of spontaneous Raman scattering, which could contribute to background noise, is more than five orders of magnitude smaller than that of stimulated Raman scattering. In a Raman silicon lasers, an SMSR of larger than 80 dB was reported [50]. This value is 30 dB higher than that of DFB lasers for optical communications. The third advantage is the capability of cascade laser oscillation, which enables the wavelength of near-infrared pump laser light to be converted to mid-infrared wavelengths [51]. Common drawbacks of Raman lasers are their large device sizes and high oscillation thresholds. For example, the first continuous-wave fiber Raman laser required more than 10 meters of silica fiber and had an oscillation threshold of approximately 3 watts [52].

2.3 Raman Silicon Lasers Based on Rib-Waveguide Resonators

Stimulated Raman scattering in silicon was first reported in 1970 [53]. The reported Raman gain coefficient for silicon is 10 – 70 cm/GW, which is 3 to 4 orders of magnitude higher than that of silica fiber (0.01 cm/GW). In the 1990s, advances in silicon microfabrication technology led to the development of low-loss silicon wire waveguides [54–56]. Silicon's high refractive index allows for reducing the cross-sectional area of waveguides, drawing attention due to the enhancement of nonlinear optical effects [57]. The development of Raman silicon lasers was proposed in 2002 [29]. Then, in 2005, continuous-wave Raman laser oscillation at room temperature was demonstrated using a rib-type silicon waveguide in a Fabry–Pérot cavity structure [27].

Figure 6 illustrates a Raman silicon laser utilizing a silicon rib-waveguide [50]. As shown in Fig. 6a, a racetrack-shaped ring resonator is employed. This resonator configuration allows the quality (Q) factor to exceed one million and enables a longer waveguide length within a small footprint. The length of this ring resonator is about 3 cm. A directional coupler couples the pump laser light from the pump waveguide into the ring resonator. Figure 6b shows the cross-sectional view of the rib waveguide. A rib waveguide is formed on the top silicon layer of the SOI substrate. The waveguide has p- and n-doped regions on either side, forming a p-i-n diode structure [39]. This diode structure is essential for extracting free carriers generated within the rib-waveguide due to two-photon absorption (TPA) [34–37].

Figure 6(c) shows the input-output characteristics of this Raman silicon laser at room temperature. The pump laser is a continuous-wave laser with a wavelength of 1550 nm. Accordingly, the Raman laser wavelength is about 1686 nm, which is redshifted by 15.6 THz from the pump wavelength. Laser oscillation occurs at a threshold of 20 mW. When the reverse bias applied to the p-i-n silicon waveguide is 0 V, saturation of the Raman laser output is clearly observed. This is attributed to free carriers generated by TPA, which reduce the Raman signal due to free carrier absorption (FCA). As the reverse bias on the diode waveguide is increased, the lifetime of the free carriers shortens, thereby reducing the saturation of the Raman laser output. When a reverse bias of 25 V is applied, the carrier lifetime is estimated to be 0.32 ns [50]. Due to this short lifetime, a laser output of 50 mW and an energy efficiency of approximately 17% are achieved.

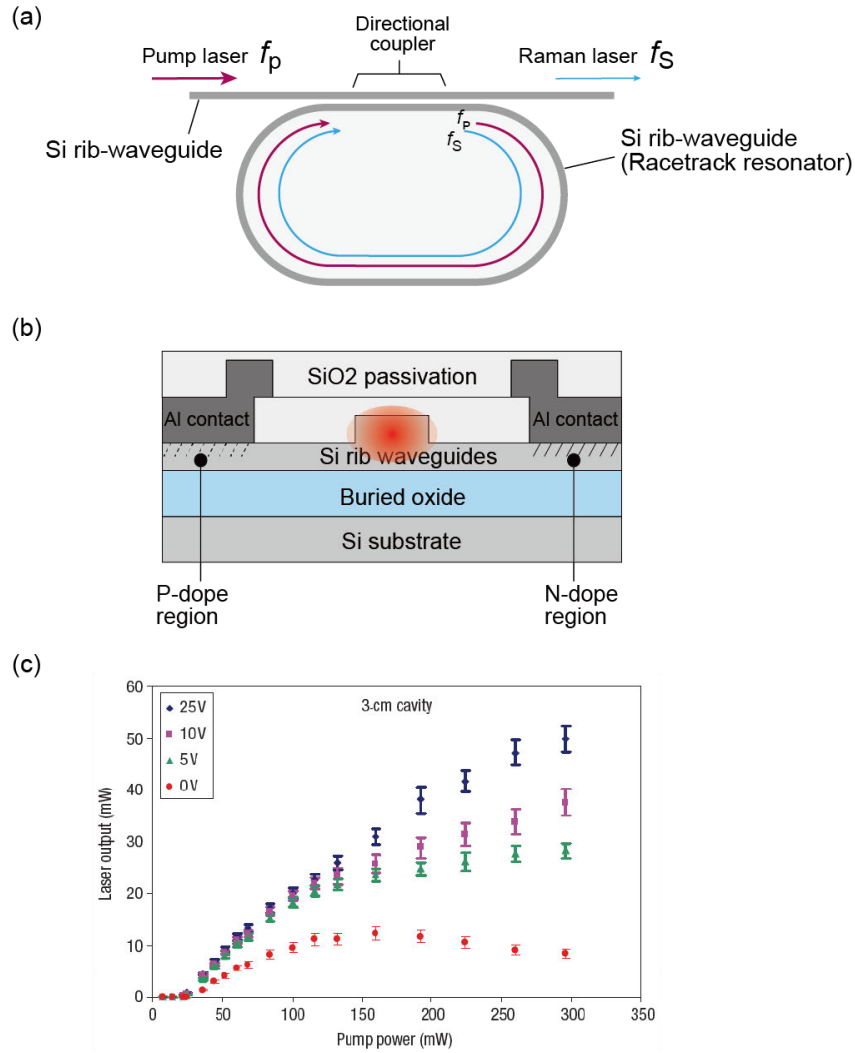


Figure 6. (a) Raman silicon laser with a racetrack resonator. (b) Cross-sectional structure of a silicon rib waveguide. (c) Laser output power against coupled input pump power at different reverse biases. Adapted with permission from [50]. Copyright 2007, Springer Nature

In realizing optoelectronic integrated silicon chips to significantly advance both optical and electronic technologies, it is essential to reduce the size of silicon lasers. Compact silicon lasers offer advantages, including enabling high integration density, reducing power consumption, facilitating easier circuit design, and enhancing compatibility with existing silicon-based electronic circuits. A centimeter-sized resonator, as shown in Fig. 6, is considered too large for optoelectronic integration. Ideally, a resonator in the micrometer size range, or at least in the millimeter range, is required. Recently, significant advancements have been made in the miniaturization and threshold reduction of racetrack-shaped Raman silicon lasers [48,49,58]. As shown in Fig. 7, the lowest threshold of 0.2 mW with a resonator length of 3.0 mm has been reported [59]. Continuous-wave lasing with a widely tunable wavelength range of 157 nm has been demonstrated. Such compact Raman silicon lasers are expected to find applications in optoelectronic integrated silicon chips. Achieving laser oscillation without the reverse-biased p-i-n structure will be important.

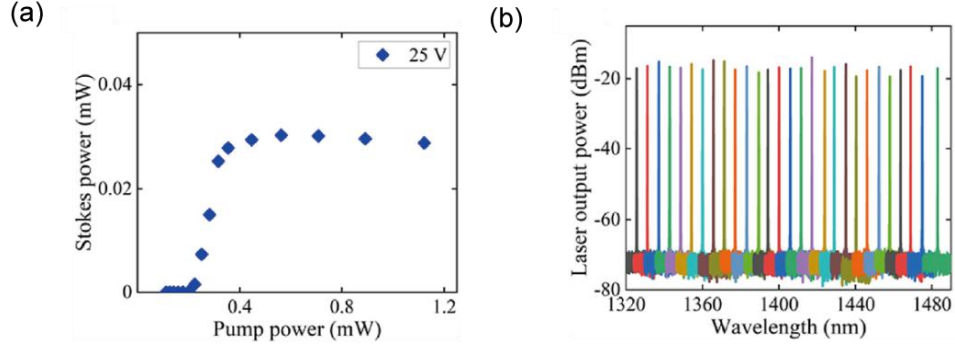


Figure 7. Lasing characteristics of Raman silicon laser with a 3.0 mm-long rib-waveguide. (a) Measured Stokes output power as a function of the pump power at a pump wavelength of 1330 nm. (b) Measured Raman lasing output spectra with pump wavelengths from 1240 to 1380 nm. Adopted with permission from [59]. Copyright 2023 American Institute of Physics

2.4 Key factors essential for the oscillation mechanism of a Raman silicon laser

In a typical laser, oscillation occurs when the optical gain from stimulated emission exceeds the total losses within the resonator. This principle also applies to Raman silicon lasers. In a Raman laser, oscillation is achieved when the Raman gain from stimulated Raman scattering surpasses the total losses occurring in the resonator (Fig. 5). This subsection explains this relationship and the strategies for reducing the laser threshold.

To maximize the Raman gain, it is essential to understand the Raman selection rules of silicon. The polarization of Raman-scattered light often differs from that of the pump light. Furthermore, the Raman selection rules that determine the polarization of the Raman-scattered light depend on the crystal orientation. Therefore, it is important to determine the crystal orientation of silicon for fabricating the resonator so that the Raman gain is maximized, taking into account the electromagnetic field distribution of the resonator modes that confine the pump light f_p and the Raman light f_s , as well as the polarization characteristics of these resonant modes. The racetrack-type Raman silicon lasers shown in Figs. 6 and 7 are fabricated along the [110] crystal orientation of silicon [29,30].

There are primarily two types of losses. The first is the loss due to the leakage of Raman light from the resonator, and the second is the loss due to the absorption of Raman light in the resonator. Resonator loss is determined by the Q -factor (Q_s) of the resonant mode that confines the Stokes Raman-scattered light. The higher the Q_s of the Stokes mode, the lower the resonator loss. Ring resonators with Q -factors exceeding one million have been used in rib-waveguide Raman lasers shown in Fig. 6. (The Q -factor, Q_p , of the resonant mode that confines the pump light is also important for decreasing the threshold, but it does not affect the oscillation condition.) In terms of absorption loss, FCA induced by TPA is a problem for Raman silicon lasers [34–37]. As illustrated in Fig. 8a, even when the pump light energy is less than the silicon bandgap of 1.12 eV, free carriers are generated in the conduction band due to TPA. These carriers in the conduction band can absorb the Raman-scattered light through free carrier absorption. Since TPA is a second-order nonlinear effect, this loss becomes significant as the pump intensity increases. To reduce this loss, reverse-biased p-i-n waveguides have been employed in Raman silicon lasers with rib-type waveguides.

Figure 8b illustrates the relationship between Raman gain, resonator loss, FCA loss, and the photon density of the pump light in the resonator. The Raman gain is linearly proportional to the photon density. The resonator loss, determined by the Q_s , is independent of the pump photon density and remains constant. The FCA loss increases nonlinearly since it originates from two-photon absorption. Under the conditions shown by the thin lines in Fig. 8b (low Raman gain, small Q_s , and long carrier lifetime), there is no excitation range in which the Raman gain exceeds the total loss. Therefore, under these conditions, laser oscillation does not occur.

In the excitation range where the Raman gain exceeds all losses, laser oscillation can be achieved. Since FCA loss increases nonlinearly, an increase in pump intensity will inevitably result in total losses surpassing the gain. However, it is possible to achieve a regime where the Raman gain exceeds the total losses in the low photon density region. To realize this, the following three strategies, illustrated by the bold line in Fig. 8b, are essential:

- 1) Increasing the Raman gain.
- 2) Enhancing the Q_S -factor to reduce resonator loss.
- 3) Reducing the free carrier lifetime in order to decrease the FCA loss.

These strategies are crucial for achieving lasing in a Raman silicon laser. (A higher Q_p of the pump resonant mode results in an increased photon density; therefore, a high Q_p is preferable for reducing the threshold power.)

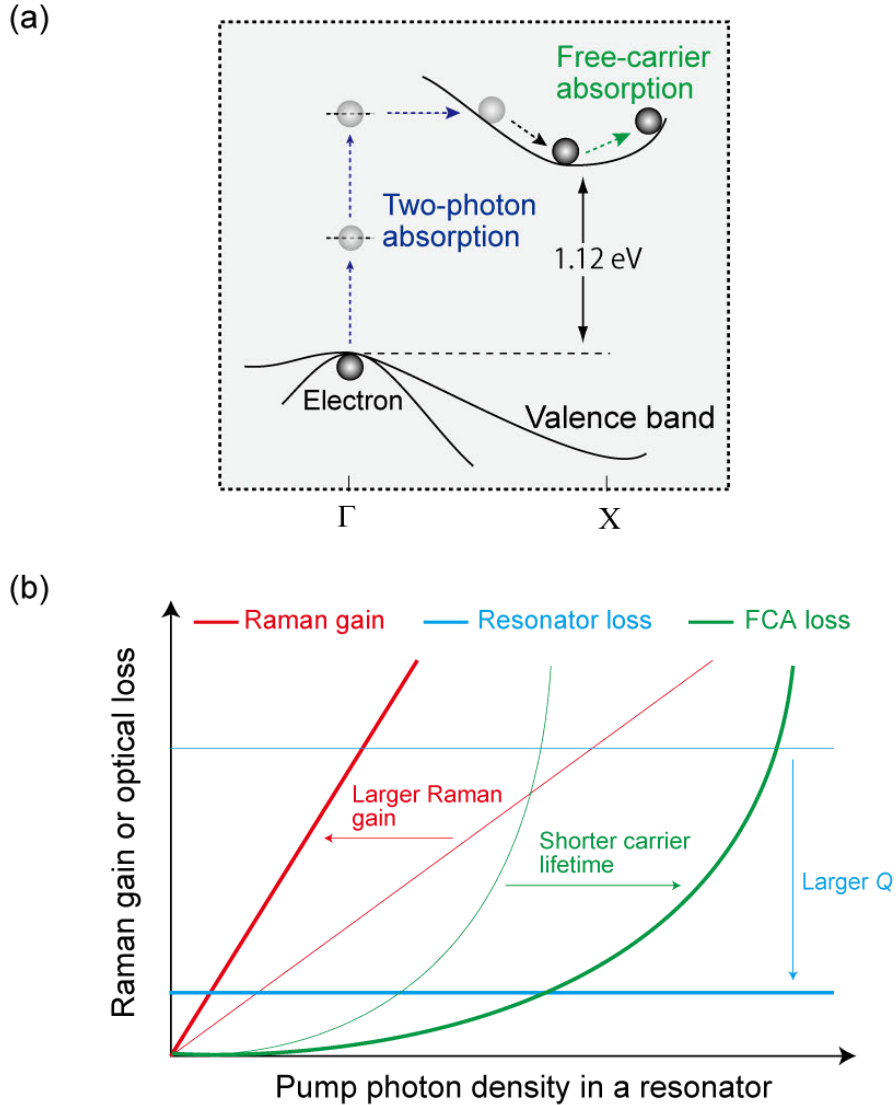


Figure 8. (a) Schematic illustration of TPA and FCA in silicon. (b) Relationship diagram of the three key factors essential for Raman laser oscillation. Thin lines indicate a state where the Raman gain does not exceed the sum of losses by cavity and FCA. Thick lines indicate a state where oscillation is possible since the Raman gain exceeds the sum of losses.

3. Raman Silicon Lasers Based on Photonic Crystal High- Q Nanocavities

The structure consisting of nanoscale periodic air holes, as shown in Fig. 9, is called a photonic crystal. Optical waveguides and optical resonators using photonic crystals have been actively studied since around 2000 [60–68]. In 2013, we developed a Raman silicon laser using a high- Q photonic crystal nanocavity [28]. This cavity had a length of 10 μm and an oscillation threshold of 1 μW , both values are more than two orders of magnitude lower than those of Raman silicon rib-waveguide lasers. Furthermore, the nanocavity does not employ a p-i-n structure to reduce the FCA loss. These remarkable performance can be attributed to the unique properties of the high- Q nanocavity used in the Raman laser:

- 1) A cavity structure that enhances Raman gain.
- 2) High Q_p and Q_s factors comparable to those of a racetrack resonator.
- 3) A short carrier lifetime inherent to photonic crystals.

This section describes the Raman silicon lasers based on high- Q nanocavities.

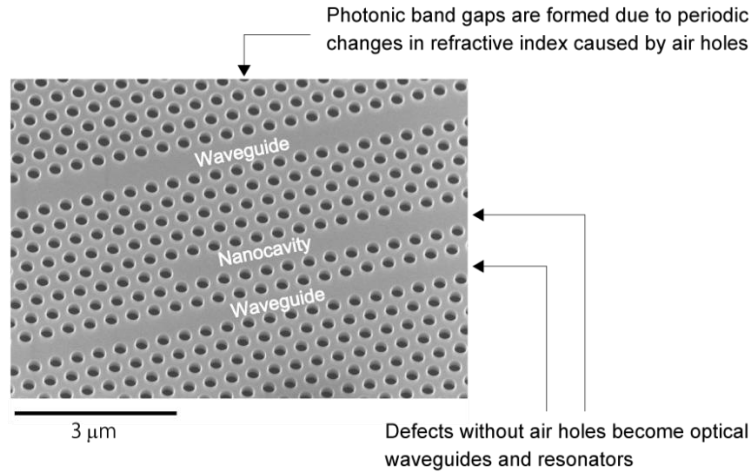


Figure 9. Electron microscope image of a high- Q photonic crystal nanocavity.

3.1 Towards Threshold Reduction and Miniaturization of Raman Silicon Lasers

In 2005, the first demonstration of continuous-wave operation of a Raman silicon laser at room temperature was reported [27]. Lowering the threshold and reducing the size were essential for applications in optoelectronic silicon chips. To realize these aims, the use of resonators with high Q -factors and small modal volumes is effective. The average time a photon remain within a resonator before it escapes or is absorbed (photon lifetime) is proportional to the Q -factor, and the photon density within the resonator increases as its volume decreases. Therefore, in high- Q small resonators, photons can interact efficiently with the material. This indicates that the strength of the light-matter interaction is proportional to the Q -factor of the resonator and inversely proportional to its volume [69].

According to the general principles of ray optics, reducing the volume of a resonator results in a lower Q -factor. This decrease occurs because the rapid change in the electromagnetic field at the resonator edges increases the components of light that fail to meet the total reflection conditions at the interface between the resonator material and the cladding. However, in photonic crystal nanocavities, the electromagnetic field distribution at the edges can be gradually adjusted by finely tuning the position of air holes [61,70,71]. As a result of various improvements, nanocavities with volumes on the order of the wavelength and theoretical Q -factors exceeding 100 million have been achieved [67–71]. Experimentally, ultra-high Q -

factors about 10 million have been demonstrated [72–74]. In nanocavities fabricated using CMOS-compatible processes, average Q -factors above three million have been reported [75].

Shortly after the demonstration of a Raman silicon laser using a rib waveguide, the development of a Raman silicon laser utilizing a photonic crystal nanocavity was proposed [76–78]. However, a cavity structure suitable for the Raman silicon laser was not identified. This is because the pump mode and Stokes mode, required to confine both the pump and Stokes Raman light, must satisfy the following conditions:

1. They have high Q_p and Q_s factors comparable to those of a racetrack resonator.
2. Their frequency difference match the Raman shift of silicon.
3. Their electromagnetic field distributions enhance the Raman gain.

Finding a cavity structure that meets these requirements proved challenging. The first observation of spontaneous Raman scattering from a high- Q nanocavities was reported in 2011, demonstrating that the efficiency of Raman scattering is enhanced by the high- Q nanocavity [79].

3.2 Raman Silicon Lasers based on High- Q Nanocavities: Fundamental Principles and Performance Analysis

Figures 10a and 10b illustrate the fundamental characteristics of the pump nanocavity mode and Stokes nanocavity mode utilized in a Raman silicon nanocavity laser. The photonic crystal comprises a well-known triangular lattice structure, created by circular air holes within a suspended membrane. The heterostructure nanocavity is constructed from a line-defect waveguide (Fig. 10c), which supports two types of propagation modes within the photonic bandgap: the 2nd-propagation mode and the 1st-propagation mode. The corresponding band structure in wavenumber space is shown in Fig. 10d, where the blue and red curves represent the dispersion relations for the 2nd- and 1st-propagation modes, respectively. The dashed line represents the light line, below which low-loss propagation is possible. The nanocavity is generated by a slight increase (Δa) in the lattice constant over a short waveguide section, shifting the dispersion relation curves to lower frequencies. As a result, the frequencies of the 1st- and 2nd-propagation bands decrease in this region, as shown in Fig. 10b [62], resulting in pump- and Stokes-nanocavity modes formed through optical confinement by mode-gap differences. The 1st-mode gap and 2nd-mode gap correspond to frequency ranges lacking low-loss propagation modes. These nanocavity modes confine the pump light and Stokes-Raman scattered light, respectively.

Figure 11a plots the Raman laser output power as a function of the pump power coupled into the nanocavity for a sample with $Q_p = 140,000$, $Q_R = 1,500,000$, $\lambda_p = 1428.38$ nm, and $\lambda_R = 1543.02$ nm [28]. It shows lasing oscillation above a threshold of 1 μ W. Figure 11b indicates that the efficiency for Raman light generation above threshold is more than 1,000 times higher than that below threshold. Figure 11c the laser efficiency of 4 % is obtained without any p-i-n diodes. This low-threshold lasing result is a consequence of the fact that nanocavity design in Fig. 10 produces net Raman gain in the low excitation range before TPA-induced FCA becomes dominant as explained in Fig. 8. Because Raman gain has a linear dependence on pump power while TPA-induced FCA has a super-linear dependence, the Raman gain can exceed losses in the low pump region in this design. The lasing output power gradually saturates with increasing pump power due to TPA-induced FCA loss. Stimulated-Raman-scattering excitation spectroscopy reveals that this nonlinear loss increases significantly once a certain threshold is exceeded [80]. Time-domain measurements also reveal that free-carriers generated by TAP significantly influence the oscillation dynamics via carrier-plasma effect and thermo-optic effect [81].

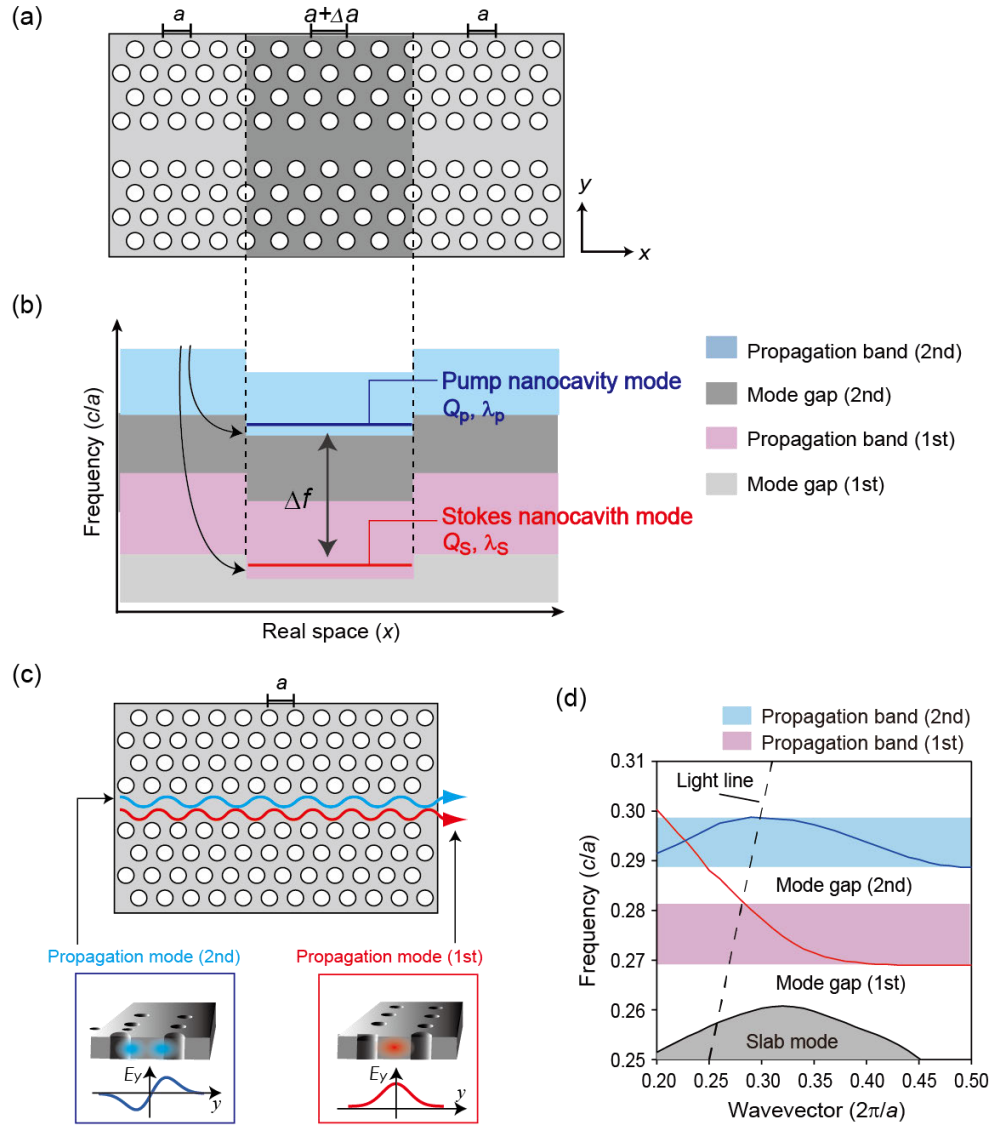


Figure 10. Two nanocavity modes formed in a heterostructure nanocavity applicable to Raman silicon laser. (a) Schematic of the heterostructure nanocavity. (b) Band diagram of the nanocavity along the x -direction, showing that the Pump and Stokes nanocavity modes originate from the 2nd- and 1st-propagation modes, respectively. (c) Line-defect waveguide created by a missing row of air holes, in which 2nd- and 1st-propagating modes are formed. The insets illustrate the profiles of the propagating modes. (d) Calculated band structure for the line-defect waveguide. Adapted with permission from [28]. Copyright 2013, Springer Nature

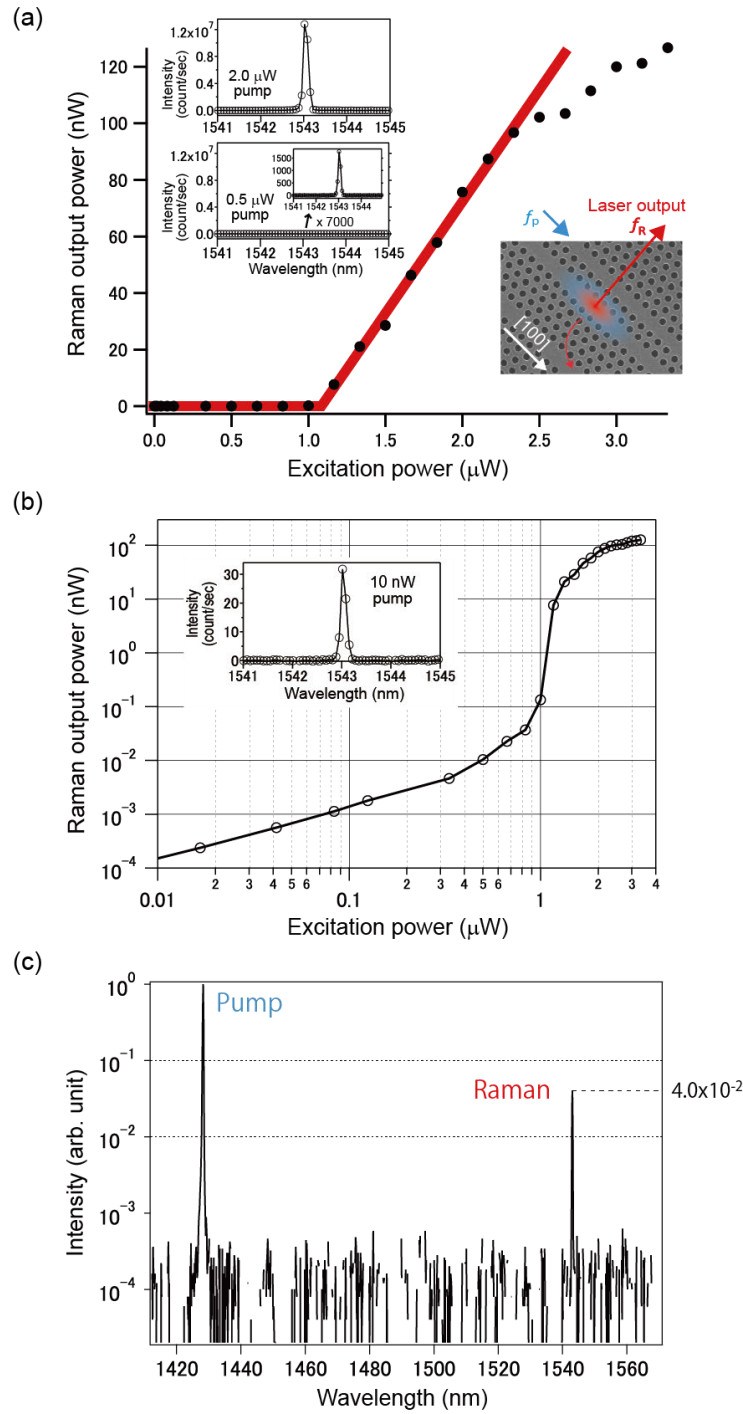


Figure 11. Lasing properties for a conventional Raman silicon nanocavity laser. (a) Laser output power as a function of pump power coupled into the nanocavity. The insets illustrate Raman spectra at power levels below and above the threshold on a consistent scale, reflecting device behavior. (b) Logarithmic plot corresponding to the graph a, with the inset displaying the Raman scattering spectrum 10 nW input. (c) Complete laser spectrum showing total output, including the pump wavelength, at an input of 3.0 μW . Adapted with permission from [28]. Copyright 2013, Springer Nature

As shown in Fig. 11, room-temperature continuous-wave operation of a Raman silicon laser with an ultralow threshold is achieved for a nanocavity-based Raman silicon laser. This is because the mode pair of pump and Stokes mode in Fig. 10 has several advantages for realizing a Raman silicon laser.

The first advantage of this design is that the pump mode and Stokes mode achieve high Q_p and Q_s values. When TPA-induced FCA loss in the nanocavity is neglected, the laser threshold (I_{th}) is inversely proportional to the product $Q_p \cdot Q_s$ and the Raman gain coefficient of the nanocavity, g_R^{cav} [77,81]:

$$I_{th} \propto (g_R^{cav} Q_p Q_s)^{-1}. \quad (2)$$

Therefore, a lower I_{th} can be obtained by increasing Q_p and Q_s . The value of $Q_p \cdot Q_s$ in the laser cavity shown in Fig. 11 is 2.1×10^{11} , which is slightly smaller than those for racetrack-based Raman silicon lasers due to the smaller Q_p value [48–51,59].

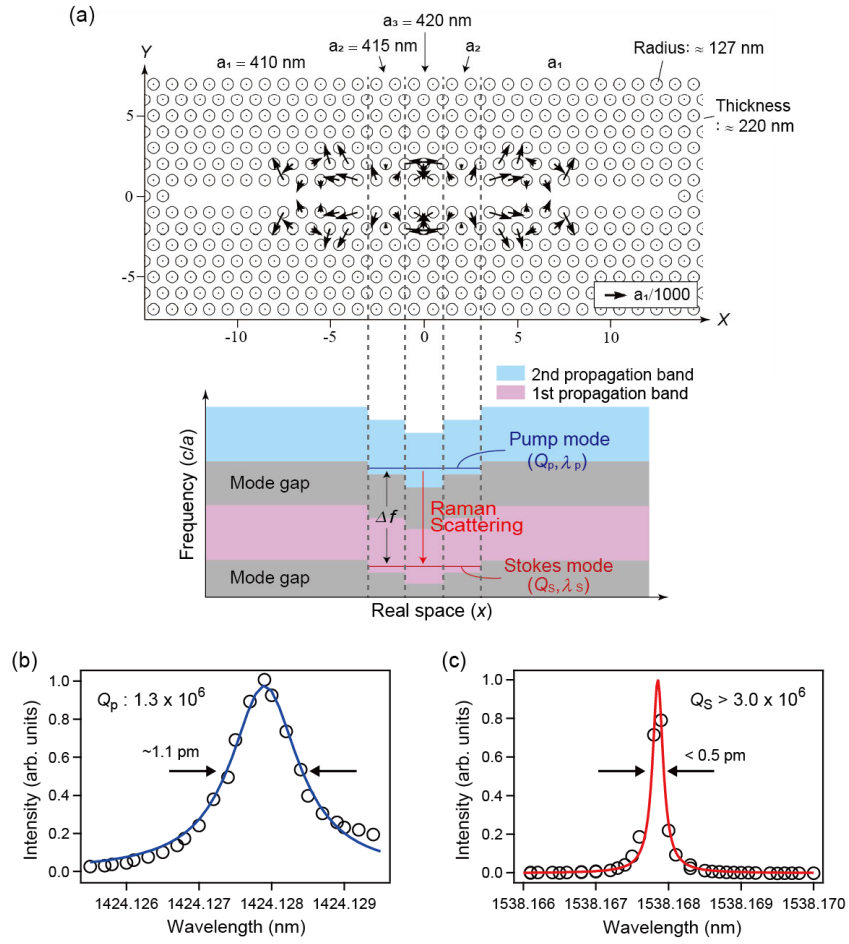


Figure 12. (a) Schematic of the heterostructure nanocavity designed by a method based on machine learning. (b) Resonance spectrum of the pump nanocavity mode. (c) Resonance spectrum of the Stokes nanocavity mode. Adapted from [82] under the terms of the Creative Commons Attribution 4.0 International license

Figure 12 shows the results for the heterostructure nanocavity with the air hole positions optimized with machine learning techniques [82]. The experimental value of $Q_p \cdot Q_s$ exceeding 4×10^{12} is obtained, which is comparable to those of racetrack-based Raman silicon lasers [48–

51,59]. It is noted that the theoretical $Q_p \cdot Q_s$ value calculated by three-dimensional (3D) finite-difference time-domain (FDTD) method is in fact more than one order of magnitude larger than the $Q_{p_exp} \cdot Q_{s_exp}$. In heterostructure nanocavities, the experimental Q values are expected to be lower than the theoretical Q values because the radius, position, and sidewall angle of the air holes deviate from the ideal values, which causes scattering losses in the nanocavity [83,84]. In addition, impurities and defects on the surface and inside of the silicon resonator absorb light. This also reduces the experimental Q value since it causes absorption losses in the resonator [72,73]. In order to reduce the I_{th} , improvements in the fabrication process is significant.

A second advantage of the design in Fig. 10 is that the cavity volumes for both modes (V_p and V_s) can be reduced to approximately a cubic wavelength. Consequently, this design enables an extraordinarily high $Q_p Q_s / V$, resulting in a laser threshold of sub-100 nanowatt [82].

Third, the frequency spacing ($\Delta f = f_p - f_s = c/\lambda_p - c/\lambda_s$) between these nanocavity modes can be tuned to the Raman shift of silicon nanocavity, which is 15.606 THz [43], by adjusting the air hole radius as shown in Fig. 13a [85]. This tunability arises from the inherent difference in parity, causing the electric field densities of the two modes within the air holes to be fundamentally unequal. Conveniently, the Δf in Figs. 10 and 12 aligns closely with 15.606 THz when the lattice constant a is set for operation within telecommunication bands between 1.30 and 1.60 μm . Precise tuning of the frequency spacing to 15.606 THz is crucial for the efficient generation of Raman scattered light in the Stokes nanocavity mode, as the g_R^{cav} in equation (2) follows the relation below [81]:

$$g_R^{cav} \propto \frac{g_{R_Si}}{V_R} \frac{\left(\frac{\Delta}{2}\right)^2}{\Delta f_{det.}^2 + \left(\frac{\Delta}{2}\right)^2} . \quad (3)$$

The variables g_{R_Si} and Δ represent the Raman gain coefficient of bulk silicon and the Raman gain bandwidth, respectively, both of which depend on silicon's crystallinity. V_R denotes the effective modal volume for Raman scattering, as shown in equation (9). The variable $\Delta f_{det.}$ indicates the detuning of the actual Δf from 15.606 THz. The Δf values in Fig. 11 and Fig. 12 are 15.589 THz and 15.603 THz, respectively. Since increasing $\Delta f_{det.}$ decreases the Raman gain of g_R^{cav} , and thus the I_{th} increases as shown in Fig. 13b [86]. The ability to control $\Delta f_{det.}$ by adjusting the air hole radius is a significant advantage.

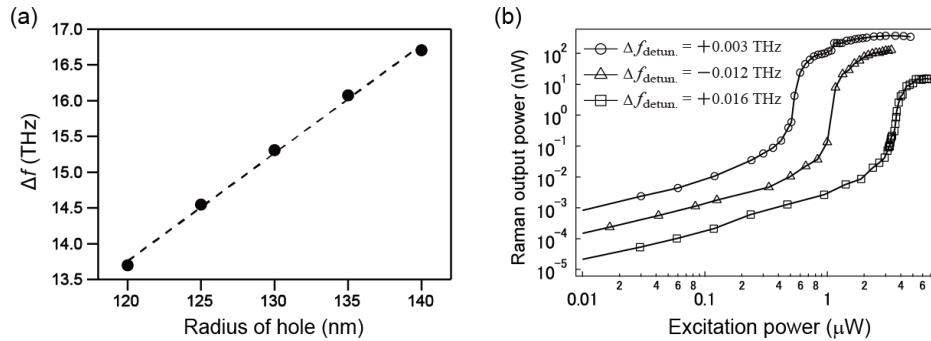


Figure 13. (a) Relation between Δf and the air hole radius determined by FDTD calculations for the structure presented in Fig. 12a. A change in the air hole radius leads to a shift of Δf with a rate of about 0.15 THz/nm. (b) The log-scale diagram of Raman laser output power as a function of power coupled into pump nanocavity mode for three laser cavities with various $\Delta f_{det.}$. Adapted with permission from [85] and [86]. Copyright 2020, IEEE and 2015, Optica

Lastly, it should be noted that the heterostructure nanocavity achieves a high Raman gain when fabricated along a specific crystallographic direction of the silicon-on-insulator (SOI) substrate. Figure 14a displays the calculated x - and y -components of the electric field distribution for the pump nanocavity mode, labeled as E_{x_pump} and E_{y_pump} , respectively. Similarly, Figure 14(b) shows the corresponding electric field distributions for the Stokes nanocavity mode, labeled E_{x_Stokes} and E_{y_Stokes} . These field distributions were derived using the FDTD method. Due to their origin in different propagation modes, E_{x_pump} and E_{x_Stokes} exhibit distinct parities. This is also the case for E_{y_pump} and E_{y_Stokes} . However, the crossed field components, such as E_{x_pump} and E_{y_Stokes} , possess the same line symmetry and show similar distributions. Consequently, if the polarization of the light confined in the pump mode undergoes a 90-degree rotation in the x - y plane through the Raman scattering process, the Raman scattered light can be effectively confined in the Stokes mode. This finding is crucial, as it demonstrates that this unique pair of modes can significantly enhance the Raman gain by carefully choosing the crystallographic orientation in which the nanocavity is fabricated.

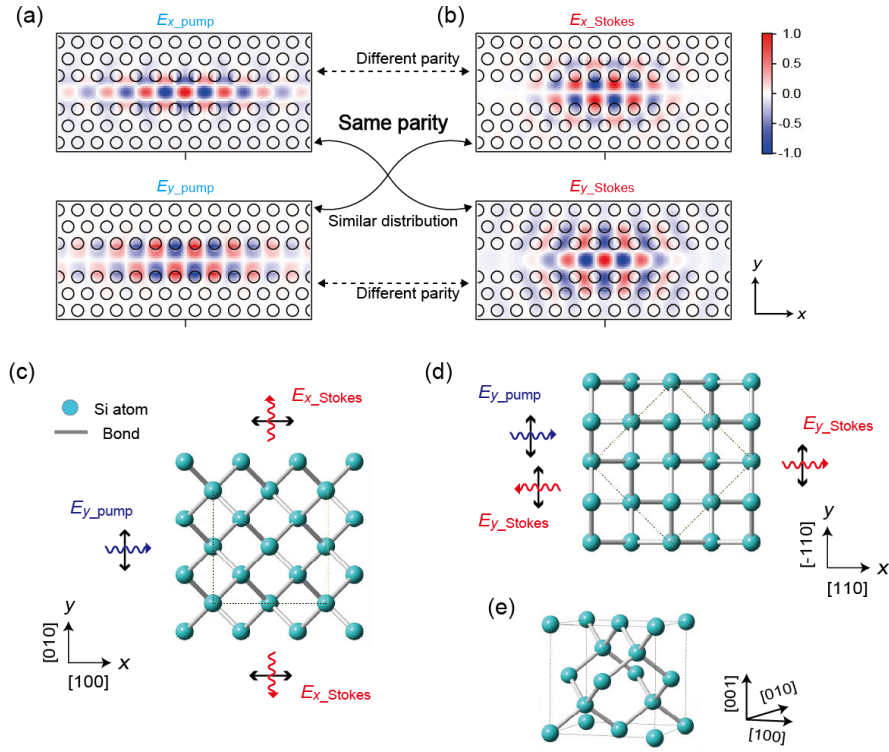


Figure 14. (a),(b) Calculated electric field distributions for the pump nanocavity mode and the Stokes nanocavity mode, respectively. The color scale represents the intensity of the electric field. (c) Schematic of in-plane Raman scattering in a (100) silicon in which the x -axis aligned to [100] crystal direction. Dotted lines show the outline of a unit cell. (d) Raman scattering for the x -axis aligned to [110] direction. (e) Unit cell of silicon. Adapted with permission from [28]. Copyright 2013, Springer Nature

If the device is built on a standard (100) silicon-on-insulator substrate, the x -axis of the nanocavity can align with either the [100] or [110] crystallographic direction. Figure 14c illustrates a microscopic model of in-plane Raman scattering when the x -axis of the cavity aligns with the [100] direction in silicon. Only transverse-electric (TE) polarized modes are considered here, as shown in Figures 14(a) and 14(b), while transverse-magnetic (TM) polarized waves (E_z) are excluded from analysis [87]. In this model, blue spheres indicate silicon atoms, with grey lines representing covalent bond projections on the x - y plane. The blue

and red arrows depict the directions of pump light and Raman scattered light, respectively. As shown by the wavy arrows, the pump and Raman scattered light polarizations are orthogonal to each other within the x - y plane for this setup. For the cavity oriented along $[100]$, the Raman gain is proportional to the following integral [28]:

$$\iiint |E_{x_Stokes}^* E_{y_pump} + E_{y_Stokes}^* E_{x_pump}|^2 dx dy dz \quad (4)$$

where an asterisk denotes complex conjugate. It is noted in this equation that the overlap involving the cross components is significant. Therefore, the pump and Stokes nanocavity modes shown in Fig. 10 have parities and distributions that are desirable for stimulated Raman scattering.

The theoretical derivation of equation (4) can be explained as follows. By setting the three principal axes of the coordinate system (x, y, z) along the $[100]$, $[010]$, and $[001]$ directions, respectively, the Raman tensors corresponding to the Γ -point phonons in crystalline silicon can be expressed by the following matrices [88–90],

$$R_x = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad R_y = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad R_z = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (5)$$

where the subscript denotes the direction of phonon oscillation. Using these Raman tensors, the third-order nonlinear susceptibility, which characterizes the stimulated Raman scattering, can be formulated as shown below.

$$\chi_{ijmn}^R \sim \sum_{k=x,y,z} (R_k)_{ij} (R_k)_{mn} \quad (6)$$

In this context, the indices i, j, m, n take the on the values x, y , and z . The induced nonlinear polarization through χ_{ijmn}^R is described as

$$P_i^{NL} = \epsilon_0 \sum_{j,m,n} \chi_{ijmn}^R (E_{pump})_j (E_{pump}^*)_m (E_{Stokes})_n \quad (7)$$

Accordingly, the Raman gain due to stimulated Raman scattering is given by

$$g_R^{cav} = \left(\frac{\hbar \omega_p c^2}{2 n_p n_S V_R} \right) g_{R_Si} \quad (8)$$

where n_p and n_S are the refractive indices for the pump mode and the Stokes mode, respectively. Additionally, g_{R_Si} refers to the bulk Raman gain coefficient for silicon, which is intrinsic to bulk silicon (with reported values around 10–70 cm/GW) and remains unaffected by the nanocavity structure [29,30,50,89].

$$V_R^{-1} \sim \sum_{i,j,m,n} \chi_{ijmn}^R \frac{\epsilon^2 \iiint_{Si} (E_{Stokes}^*)_i (E_{pump})_j (E_{pump}^*)_m (E_{Stokes})_n dx dy dz}{\iiint \epsilon |E_{Stokes}|^2 dx dy dz \cdot \iiint \epsilon |E_{pump}|^2 dx dy dz} \quad (9)$$

Here, the subscript "Si" specifies that the integral calculation excludes air holes. The parameter V_R^{-1} indicates the effective modal volume for Raman scattering, where achieving spatial overlap between the pump and Stokes modes via χ_{ijmn}^R is essential. Consequently, while equation (6) defines a total of 81 components, only 12 unique nonzero components remain when employing the Raman tensors from equation (5).

$$\begin{aligned} \chi_{xyyx}^R &= \chi_{xyxy}^R = \chi_{yxxy}^R = \chi_{yxyx}^R = \chi_{zzxx}^R = \chi_{xxzz}^R = \chi_{zxzx}^R = \chi_{zxxz}^R \\ &= \chi_{yzzy}^R = \chi_{yzyz}^R = \chi_{zyyz}^R = \chi_{zyzy}^R \end{aligned} \quad (10)$$

Additionally, the contributions from the z -components of the electric field in the cavity, specifically E_{z_pump} and E_{z_Stokes} , can be disregarded since both modes exhibit TE polarization. This leaves us with four key components, $\chi_{xyyx}^R = \chi_{xyxy}^R = \chi_{yxxy}^R = \chi_{yyxx}^R$, which stem from the R_z in equation (5) and determine the Raman gain. Substituting these four components in equation (9), the equation (4) is derived.

It should be noted that χ_{xyyx}^R and χ_{yxxy}^R primarily contribute to the Raman gain due to the in-plane symmetries of the pump and Stokes nanocavity modes, as shown in Figs. 14(a) and 14(b). Additionally, the distributions of E_{x_pump} and E_{y_Stokes} exhibit almost identical oscillation periods along the x -axis, as each nanocavity mode represents the ground state within its respective propagation mode. This relationship also holds for E_{y_pump} and E_{x_Stokes} . Consequently, the nearly optimal overlap for achieving Raman gain is expected when the heterostructure nanocavity is oriented along the [100] direction.

Typically, SOI-based photonic devices are fabricated along the [110] direction or its equivalent, as wafers are readily cleaved in this orientation [91]. Previous designs of rib-waveguide lasers also followed this [110] orientation [27,50,51], and prior theoretical studies were based on the same alignment [76,77].

Figure 14(d) shows a microscopic model of in-plane Raman scattering when the cavity's x -axis is aligned with the [110] direction. As indicated by the wavy arrows, the polarizations of the pump light and Raman scattered light are aligned. For a cavity oriented along [110], the Raman gain is proportional to the integral shown below [28].

$$\iiint \left| E_{x_Stokes}^* E_{x_pump} - E_{y_Stokes}^* E_{y_pump} \right|^2 dx dy dz \quad (11)$$

In this case, the Raman gain is proportional to the overlap integrals between corresponding components, specifically E_{x_pump} and E_{x_Stokes} , as well as. This combination of pump and Stokes mode pairs is unfavorable. The magnitude of the integral in Eq. (9) is 20 times smaller than equation (4) [92]. Additionally, the presence of negative terms in equation (11) further limits the effectiveness of any mode pair. The Raman gain cannot be significantly large even if two nanocavity modes with appropriate line-symmetries could be prepared.

The theoretical derivation of Equation (11) is outlined as follows. By defining the three axes as [110], [-110], and [001], the Raman tensors take the following forms [90]

$$R_x = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \quad R_y = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix} \quad R_z = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \quad (12)$$

In this case, using the Raman tensors in Equation (12), there are also 12 non-zero components of χ_{ijmn}^R . However, two components of χ_{xxyy}^R and χ_{yyxx}^R have negative values:

$$\begin{aligned} \chi_{xxxx}^R &= \chi_{yyyy}^R = -\chi_{xxyy}^R = -\chi_{yyxx}^R = \chi_{zzzz}^R = \chi_{zzxz}^R = \chi_{xxzz}^R = \chi_{zxzx}^R \\ &= \chi_{yzzz}^R = \chi_{zyyz}^R = \chi_{zyyz}^R = \chi_{zyzy}^R \end{aligned} \quad (13)$$

The contributions from the z -components can be once again disregard and therefore, the four components $\chi_{xxxx}^R = \chi_{yyyy}^R = -\chi_{xxyy}^R = -\chi_{yyxx}^R$, which originate from the R_z , remain for the nanocavity fabricated along [110]. By substituting these four values into equation (9), the equation (11) is obtained.

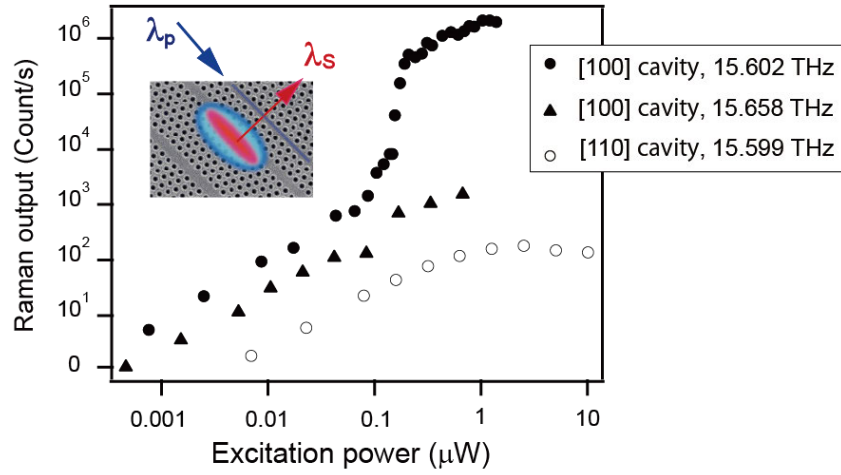


Figure 15. Intensity of Stokes Raman scattering for three nanocavities. Only the nanocavity fabricated along [100] with the small Δf_{det} exhibited lasing. The vertical axis provides the photon counts per second that were measured by the InGaAs detector. Adapted with permission from [85]. Copyright 2020, IEEE

Figure 15 displays the experimental results for Stokes Raman scattering for three nanocavities [85]. The inset shows the setup during laser operation, where the pump nanocavity mode is strongly excited by an external continuous laser, and the Stokes output is measured perpendicularly to the slab. Details of the experimental setup are provided in previous studies [28,46]. The black circles in Fig. 15 represent results for the [100]-oriented cavity with minimal detuning from the Raman shift and high Q_{exp} factors ($\Delta f = 15.602$ THz, $Q_{\text{p_exp}} = 4.02 \times 10^5$, and $Q_{\text{S_exp}} = 1.65 \times 10^6$). The black triangles correspond to the [100]-aligned cavity with a larger detuning ($\Delta f = 15.658$ THz) and $Q_{\text{p}} = 2.61 \times 10^5$ and $Q_{\text{S}} = 1.25 \times 10^6$. The open circles show results for the [110]-aligned cavity with small Δf_{det} and high Q_{exp} ($\Delta f = 15.599$ THz, $Q_{\text{p}} = 2.10 \times 10^5$, and $Q_{\text{S}} = 2.23 \times 10^6$). In the weak excitation region below $0.1 \mu\text{W}$, the Stokes intensities for all cavities increase linearly, as spontaneous Raman scattering is predominant. Notably, the efficiency of spontaneous Raman scattering in the [100]-aligned cavity is over ten times greater than in the [110]-aligned cavity, corresponding well with the overlap integral differences described in equations (4) and (11). Above $0.1 \mu\text{W}$, the [100]-aligned cavity with minimal Δf_{det} exhibits clear laser emission. Conversely, the Stokes intensity for the other two cavities begins to saturate near $0.1 \mu\text{W}$ due to increased optical losses from FCA induced by TPA. The details of how Raman lasing is inhibited by FCA loss as the pump intensity increases, have been investigated by numerical calculations [80].

4. Advancements Toward Applications of Nanocavity-Based Raman Silicon Lasers

This section describes recent advancements in applying Raman silicon nanocavity lasers to optoelectronic integrated silicon chips. Topics discussed include the mass fabrication of Raman lasers using CMOS-compatible processes, the expansion of the operating wavelength range, and laser structures with strong edge emission from the waveguide.

4-1 Raman silicon nanocavity laser by using a (100) SOI wafer with a 45-degree-rotated top silicon layer

In this subsection, we describe the fabrication of a Raman silicon nanocavity laser on a 45°-rotated SOI substrate, which is necessary for integrating the Raman laser onto an optoelectronic chip. Details are seen in Ref. [91].

Figure 16 illustrates the core design of a typical Raman silicon nanocavity laser, optimized so that the Stokes mode wavelength (λ_s) aligns with the C-band for optical communication. The primary structure consists of a triangular lattice with $a = 410$ nm in the light-gray region. A multi-heterostructure nanocavity is created by a central line-defect composed of 27 missing air holes. In the dark-gray regions, the air-hole spacing in the x -direction is incrementally adjusted by 5 nm. An air-hole radius of approximately 128 nm is selected to achieve a Δf closely to the Raman shift of silicon nanocavity, 15.606 THz. The two line-defects positioned above and below the nanocavity are utilized as waveguides for exciting the pump mode and the Stokes mode, respectively. The width of the pump excitation waveguide, defined by the distance between the air holes along the y -direction in the upper line defect, is $0.88\sqrt{3}a$. Similarly, the width of the Stokes waveguide is $1.10\sqrt{3}a$.

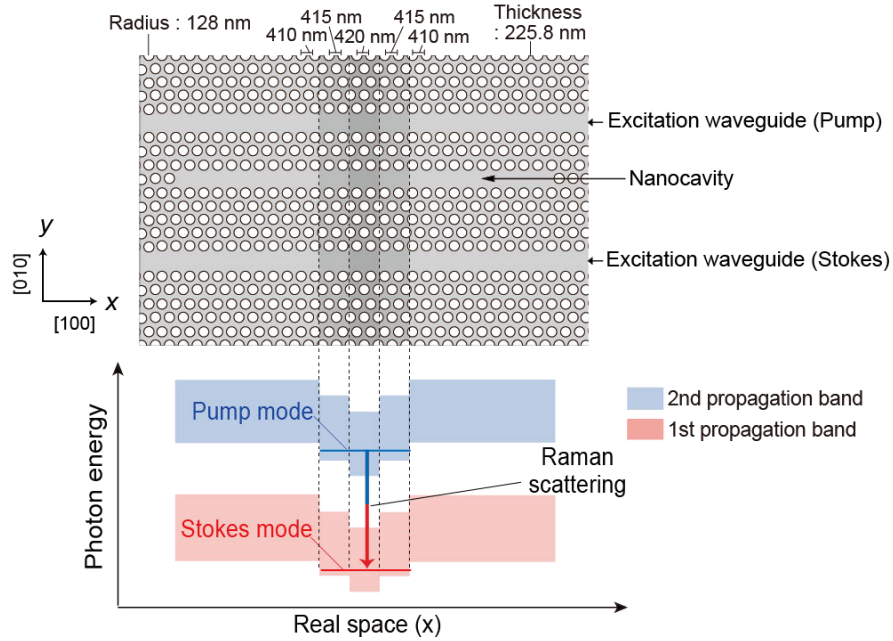


Figure 16. Diagram of the photonic crystal (PC) heterostructure nanocavity and the two excitation waveguides utilized in Raman silicon laser. Band structure of the nanocavity, showing the pump and Stokes modes emerging from the two propagation bands. The white region between the first and second propagation bands indicates the mode gap area. Adapted from [93] under the terms of the Creative Commons Attribution 4.0 International license

To enhance the Raman gain, the heterostructure nanocavity should be fabricated along the [100] crystal orientation of silicon, as explained by equation (4). However, silicon photonic devices are usually produced along the [110] direction since crystalline silicon cleaves along [110] (or an equivalent direction). Fabricating along [110] is essential for increasing the integration density of opto-electronic components. The 45-degree misalignment between this cleavable direction and the longitudinal orientation of the Raman silicon nanocavity laser will be problematic for various applications.

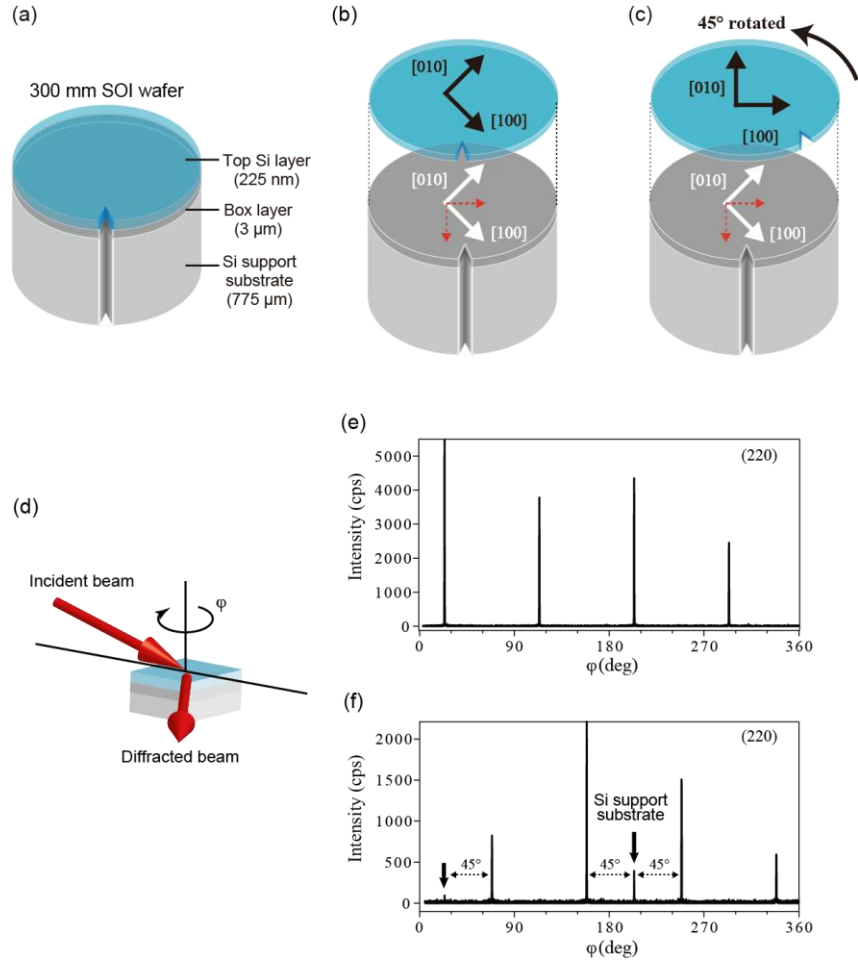


Figure 17. (a) Typical structure of a (100)-oriented SOI wafer. The wafer notch in the figure indicates the cleavable direction of silicon. (b) Illustration of a conventional (100) SOI wafer with red arrows marking the cleavable [110] and [1-10] directions. (c) Illustration of the (100) SOI wafer rotated by 45° used in this study. (d) Schematic setup of the ϕ -scan for in-plane XRD measurement. (e) XRD ϕ -scan spectrum of Si (220) for the standard SOI wafer and (f) that for the 45°-rotated SOI wafer. Adapted from [91] under the terms of the Creative Commons Attribution 4.0 International license

Figure 17 presents the SOI substrate employed to address this issue. Currently, (100) SOI wafers are widely used in the optoelectronics industry and thus constitute an important platform for silicon photonics research. Cleaving along the [110] direction is essential in photonics for creating waveguide facets and segmenting small chips from larger substrates. Figure 17a is a schematic of a conventional (100) SOI wafer. It consists of three layers: a thin silicon layer (about 225 nm in this study) at the top, a buried-oxide (BOX) layer (3 μm thick), and a silicon support substrate (775 μm). The Raman silicon nanocavity lasers and the other silicon photonic devices are fabricated in the top silicon layer. This conventional SOI wafer is in fact utilized in the lasing results shown in Figs. 11, 13b, and 15 [28,85,86].

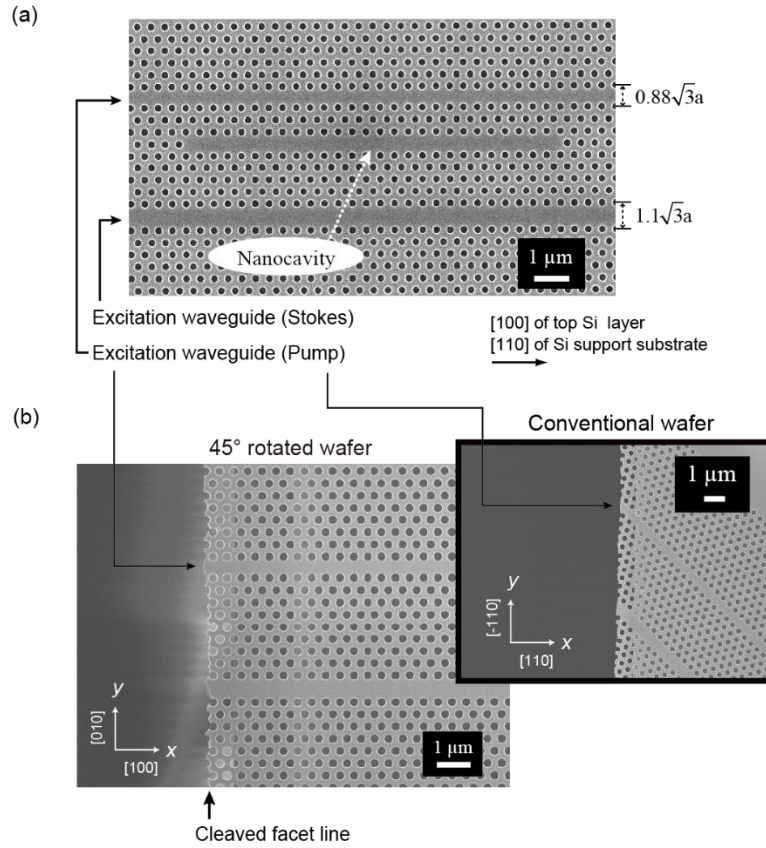


Figure 18. (a) SEM image of the fabricated laser sample on the 45°-rotated SOI wafer. (b) SEM image at the cleaved facet. The inset is the corresponding image for the Raman silicon laser on the conventional SOI wafer. Adapted from [91] under the terms of the Creative Commons Attribution 4.0 International license

SOI wafers are produced using the thermal fusion bonding of two wafers [94,95]. Typically, this bonding aligns the crystal orientations of both wafers, as illustrated in Fig. 17b. Figure 17c presents a schematic of the SOI wafer with a 45° rotation. In this case, fusion bonding is conducted with the top silicon layer rotated 45 degrees relative to the silicon support substrate, so that the [100] direction of the top layer aligns with the [110] orientation of the support substrate. It is noted that cleaving of this SOI wafer is limited to the [110] or [1-10] directions of the Si support substrate, because the top Si layer is much thinner than the support substrate. Therefore, this adjustment allows cleaving along the [100] or [010] directions of the top layer [96]. Since most silicon photonic devices do not rely on optical effects that are sensitive to crystal orientation, their performance should remain consistent even when they are fabricated along the [100] direction. A comprehensive comparison of high- Q heterostructure nanocavities fabricated along the [100] and [110] directions was documented in another study [85].

The crystal orientations of both SOI wafers were investigated using in-plane XRD. Figure 17d illustrates a schematic of the measurement setup. Figures 17e and 17f display ϕ -scan spectra of Si (220) for both the conventional and 45°-rotated SOI wafers, respectively. In Fig. 17e, four prominent peaks appear, spaced 90 degrees apart, reflecting the 4-fold symmetry of the silicon wafer. In Fig. 17f, the positions of these four peaks are shifted by 45 degrees compared to the conventional wafer. The two smaller peaks, marked by solid arrows, originate from the Si support substrate and clearly show that the crystal orientation of the top Si layer in the modified SOI wafer is rotated by 45 degrees relative to the support substrate.

Figure 18a presents a scanning electron microscope (SEM) image of the core section of the fabricated Raman laser device on the 45°-rotated SOI wafer. Figure 18b shows the image of the waveguide edge, with the inset on the right displaying a sample fabricated on a conventional SOI wafer. Both samples were cleaved using the same method. It should be emphasized that the waveguide on the 45°-rotated SOI wafer is oriented perpendicular to the cleaved facet. This characteristic is critical for ensuring integration and achieving low-loss connections with other components.

Figures 19a and 19b depict the resonance spectra for the pump nanocavity mode and the Stokes nanocavity mode, respectively. The experimental data is represented by black circles, while the solid curves show the fitting results based on Lorentzian functions, from which the values for λ and the full width at half-maximum ($\Delta\lambda$) are extracted. Using the relation $Q = \lambda / \Delta\lambda$ to calculate the quality factor, it is estimated to be $Q_p = 126,000$ for the pump mode and $Q_s = 1,860,000$ for the Stokes mode. These Q values are comparable to those measured for laser samples fabricated on conventional SOI wafers [28,80,81,85]. The Q_{exp} factor of photonic crystal (PC) cavities tends to decrease as imperfections in the air holes' radii and positions increase [83–85]. Therefore, the high Q_{exp} values obtained here indicate that the accuracy of fabricating PC structures is maintained even with the use of the 45°-rotated SOI wafer. Δf is 15.619 THz, resulting in a $\Delta f_{\text{det.}}$ of 0.013 THz.

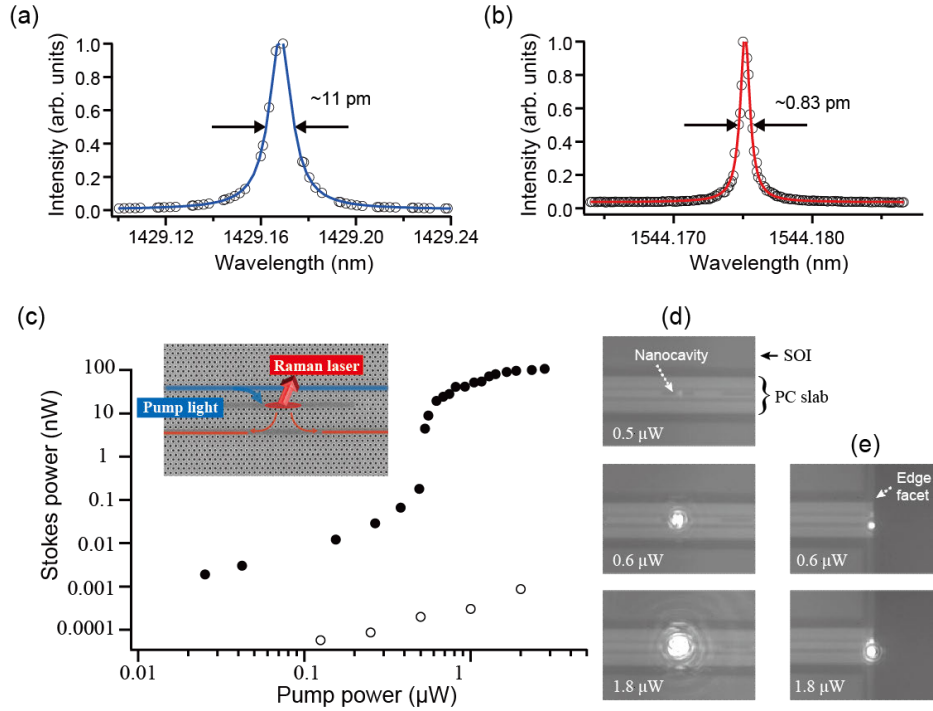


Figure 19. Optical properties for a laser sample fabricated on the 45°-rotated SOI wafer. (a) Resonance spectrum of the pump nanocavity mode. (b) Resonance spectrum of the Stokes nanocavity mode. (c) Raman laser output power versus excitation power coupled into a nanocavity on the 45°-rotated SOI wafer (filled circles). For comparison, open circles show results from a [110]-aligned cavity on a standard SOI wafer. The inset depicts the excitation setup. (d) Camera images of the nanocavity under three different excitation powers, with the pump laser light removed by a long-pass filter (1500 nm cutoff) placed before the camera. (e) Camera image showing Stokes light emission from the facet (transmission side) of the lower waveguide in the inset of (c). All images share the same intensity scale. Adapted from [91] under the terms of the Creative Commons Attribution 4.0 International license

The filled circles in Fig. 19c represent the input/output characteristics of the Raman laser. The Stokes Raman scattered light, emitted perpendicular to the slab (as shown in the inset of Fig. 19c), is measured as a function of the excitation power. The Stokes intensity shows a rapid increase of over two orders of magnitude around the threshold power of 0.53 μW . The highest energy efficiency achieved with this sample is 5.6%, observed at 1.8 μW . An energy efficiency greater than 20% is possible in this type of laser if the detuning of Δf is small [81,86]. The saturation of the laser output at pump powers above 2.0 μW is due to FCA [34–37,80,81]. Figure 19d displays near-infrared (NIR) camera images of the nanocavity for excitation below the threshold (0.5 μW), just above the threshold (0.6 μW), and at optimum performance (1.8 μW).

The open circles in Fig. 19c represent the results for the [110]-aligned cavity fabricated on the conventional SOI wafer. For this sample, $\Delta f = 15.585$ THz, $Q_p = 137,000$, and $Q_s = 1,480,000$ are obtained. Although these values are comparable to those of the laser device on the 45°-rotated SOI wafer, the efficiency of spontaneous Raman scattering in the low excitation range (below the threshold) is about two orders of magnitude lower. This discrepancy in emission efficiencies arises from the differences between equations (4) and (11).

Figure 19e shows the emission of Stokes light from the Stokes waveguide facet below the nanocavity (as depicted in the inset of Fig. 19c). These camera images illustrate that a portion of the Raman laser light is simultaneously extracted through Stokes waveguide, even as the laser properties are examined via the vertical emission. From the intensity observed in Fig. 19e, it is roughly estimated that several tens of percent of the Stokes power in the current cavity are extracted by the waveguide. For the realization of dense optical circuits with multiple functions, efficiently collecting Stokes light in Stokes waveguides will be crucial [97].

The laser performance observed for the device fabricated along the [100] direction of the top Si layer on the 45°-rotated SOI wafer is comparable to that of Raman lasers fabricated along [100] on conventional SOI wafers, as shown in Fig. 11. Therefore, it can be concluded that nanocavity-based Raman silicon lasers can be fabricated on SOI wafers with a 45°-rotated top Si layer without any performance degradation.

4-2 CMOS-Compatible Mass Fabrication of Raman Silicon Nanocavity Lasers

Fabrication of silicon photonic devices using CMOS-compatible processes on large-diameter silicon wafers is crucial for integrated optoelectronic chip applications. In this subsection, we present the large-scale production of a Raman silicon nanocavity laser, with further details available in Refs. [75,93,98].

Recently, significant advancements have been achieved in fabricating silicon photonic devices on SOI wafers with large diameters using CMOS-compatible methods [99–101]. Devices such as high-speed optical modulators [102], wavelength demultiplexers [103], and beam steering components [104] based on photonic crystals have been realized. Of these, compact silicon-based lasers are especially promising for a range of applications, including opto-electronic integrated circuits, short-range optical communication, and low-cost environmental sensing solutions.

Figure 20 illustrates the process flow for fabricating a CMOS-based Raman silicon laser. A 300-mm-diameter SOI wafer is utilized, featuring a 225-nm-thick silicon top layer, a 3- μm -thick BOX layer, and a 775- μm -thick silicon substrate. Notably, the SOI wafer rotated by 45° as described in section 4.1 is employed. The photonic crystal pattern is created using an immersion scanner (Nikon NSR-S610C, 193-nm ArF excimer laser) [105]. A binary mask with a chromium film, which has 0% transmittance, is prepared to define the air-hole patterns. The photomask pattern size projected onto the SOI wafer was 26 mm $\times$ 33 mm, allowing nearly 60 identical chips to be fabricated. The dry-etching process used to form the air holes was refined to minimize random structural variations and achieve more vertical air-hole sidewalls. These process steps were carried out in the 300-mm pilot line at AIST.

After the silicon etching process, the wafer was divided into smaller chips ($800\text{ }\mu\text{m} \times 2000\text{ }\mu\text{m}$) using stealth dicing. The surfaces of chips underwent cleaning following a standard procedure. Then a thermal oxidation process at about $500\text{ }^{\circ}\text{C}$ and removal of the surface oxide film by 1.0% dilute hydrofluoric acid were added several times. It was observed that, without this thermal treatment, the Q_p value remained below 20,000, while the Q_s exceeded 1,000,000. This repeated thermal process is believed to reduce defects and impurities near the surface of the silicon nanocavity. Lastly, the BOX layer was selectively etched away using 48% hydrofluoric acid (HF) to create an air-bridge structure. After rinsing with water, the chip surface was dried with a nitrogen stream.

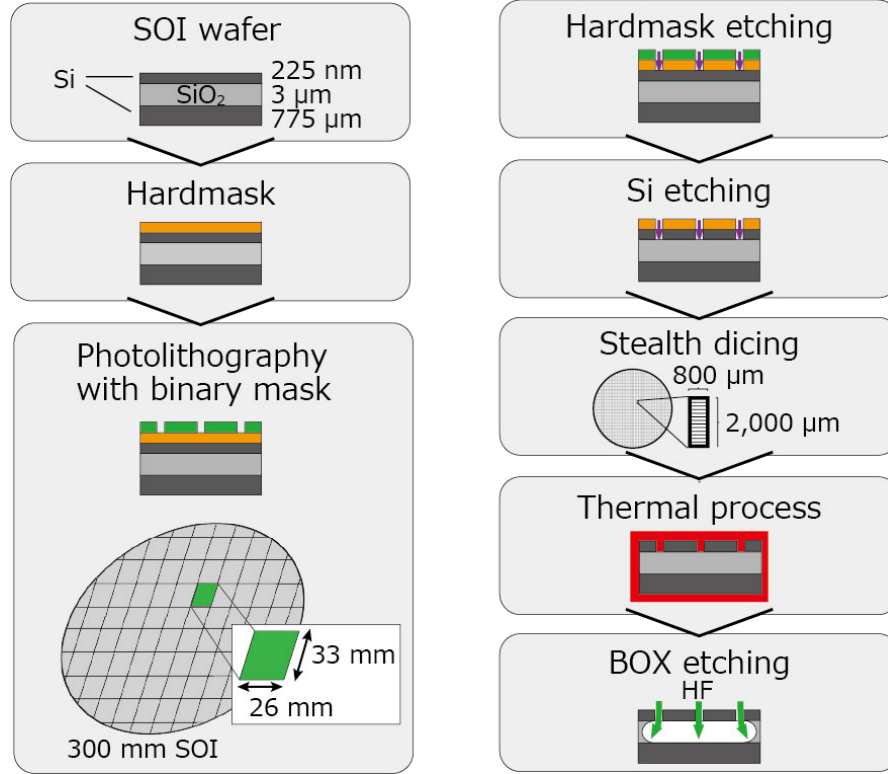


Figure 20. Flow chart illustrating the steps involved in sample fabrication utilizing CMOS-compatible technologies. Adapted from [75] under the terms of the Creative Commons Attribution 4.0 International license

The cross-sectional SEM image of one device prior to HF treatment is shown in Fig. 21a. The sidewalls of the fabricated air holes show a slight curvature, which contributes to a reduction in Q_{exp} values [75]. Figure 21b presents a cross-sectional SEM image after the air-bridge structure was formed. Although there is minor deformation in the photonic crystal slab caused by compressive stress, it has little effect on the Q_{exp} of the two modes [43]. Figure 21c shows a top view of the laser device's core region, confirming that no visible hardmask or photoresist residues remain on the silicon surface. Since the rate of oxide film formation in silicon varies depending on crystal orientation [106], the hole shape appears hexagonal. Repeated thermal processing can enhance Q_p , but this may also lead to a reduction in Q_s due to this deformation effect. It is believed that this repeated thermal process helps to decrease defects and impurities in the silicon nanocavity. The SEM image of the waveguide edge in Fig. 21d demonstrates that the stealth dicing process produced a clean, well-cleaved facet.

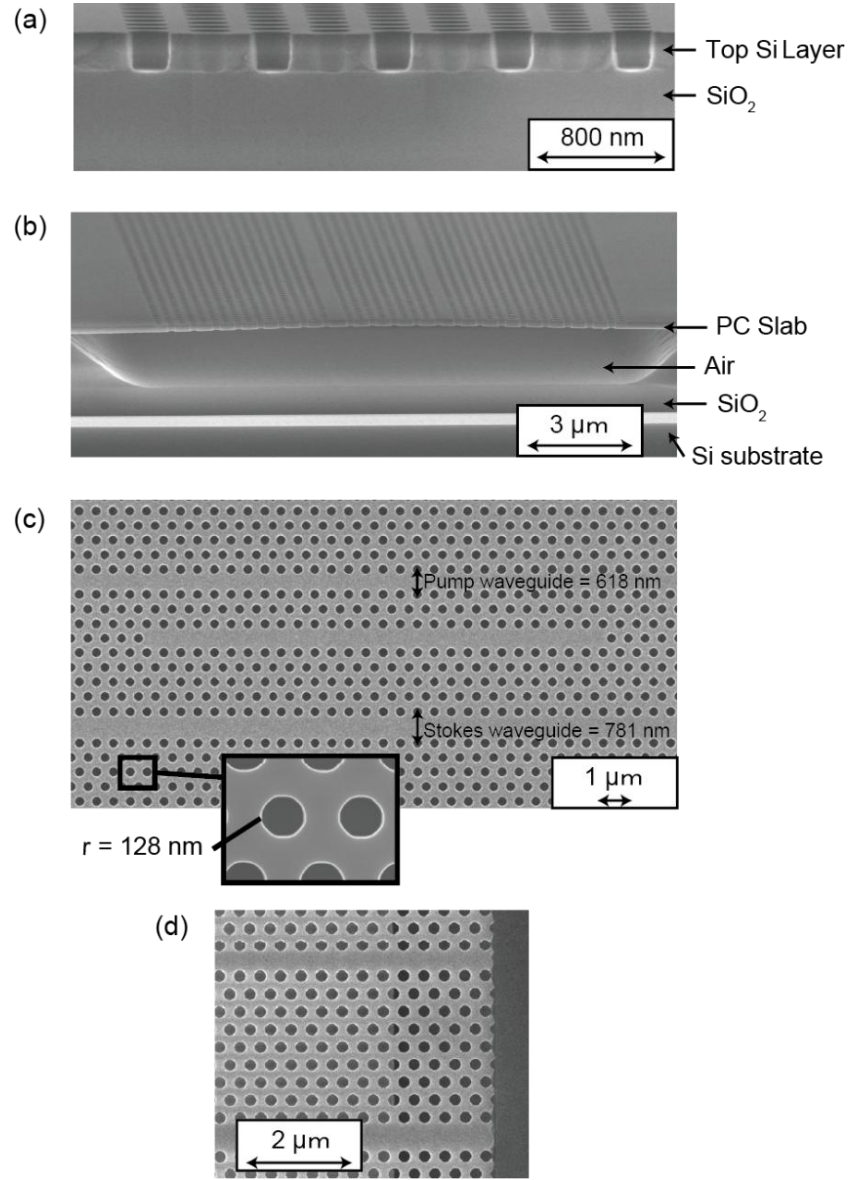


Figure 21. (a) SEM image of a fabricated Raman silicon laser prior to the etching of the SiO_2 layer. (b) SEM image of the sample after the SiO_2 layer has been etched. (c) A top-down view of the device's core region. (d) A top view of the cleaved facet of the device. Adapted from [93] under the terms of the Creative Commons Attribution 4.0 International license

Figure 22a shows the results for the sample with the highest energy efficiency. The estimated values from the resonance spectra for Δf , Q_p , and Q_s are 15.584 THz, 260,000, and 1,190,000, respectively. The input-output relationship reveals a threshold of 1.1 μW and an energy efficiency of 3.0%. These laser properties are comparable to those of Raman lasers with similar Q_p and Q_s values fabricated by electron beam lithography, as shown in Fig. 19. Figure 22b presents the characteristics of 10 samples which is fabricated on the same chip. Six out of the ten cavities emitted laser light, achieving a fabrication yield of 60%. These graphs on Q_p , Q_s , and Δf suggest that the $\Delta f_{\text{det.}}$ is the most detrimental fluctuation for the fabrication yield, originating to equation (3) [85].

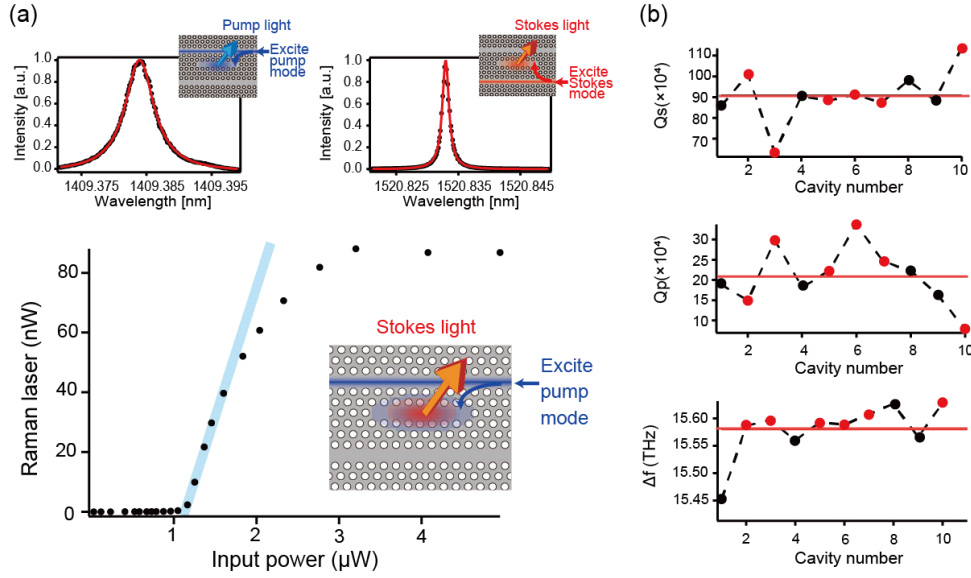


Figure 22. (a) Resonance spectrum of the pump mode (upper left panel), the resonance spectrum of the Stokes mode (upper right panel), and the input-output characteristics of the Raman laser emission (lower panel). (b) The values of Q_p , Q_s , and Δf for the ten fabricated laser samples, with red circles marking the cavities that exhibited laser emission. The red lines indicate the average values. Adapted with permission from [98]. Copyright 2022, Optica

4-3 Short-Wavelength Operation of Raman Silicon Nanocavity Lasers

To advance the intelligent society, it is essential to shift from copper wiring to fiber communication for short-distance data transmission. In the near future, wavelength-division multiplexing (WDM) should be adopted even for short-range optical communication. Silicon is transparent across all optical communication wavelength bands, from 1.26 μm to 1.63 μm , and thus optoelectronic chips incorporating WDM systems hold significant potential. In this subsection, we explain the development of Raman silicon nanocavity lasers with the Stokes wavelength in the O-band, with further details available in Refs. [46,47].

As explained in section 2.4 using Fig. 8, the condition for laser oscillation is that the Raman gain exceeds the total cavity losses. The Raman gain of the nanocavity, g_R^{cav} , is given by equation (8). The g_R^{cav} should be roughly proportional to λ_p^{-4} [47]. Therefore, the performance of Raman silicon nanocavity laser is improved by reducing the operation wavelengths if nonlinear optical losses related TPA are ignored.

An ideal method to change the operating wavelength of a photonic structure without degrading its performance is to resize all dimensions of the device geometry by the same factor. For instance, reducing the dimensions of a silicon photonic device operating at 1.6 μm to 75% shifts the operating wavelength to 1.2 μm . Since the effect of refractive index dispersion on the operating wavelength is relatively small, it can be neglected. In this scenario, both devices can be treated using a single photonic band diagram defined by normalized frequency [107–109]. In other words, silicon photonic devices for different wavelength bands can be fabricated using the same device design.

Figure 23a shows a SEM image of a Raman silicon nanocavity laser with a Stokes wavelength λ_s of 1278 nm, in the O-band, fabricated using this method. The lattice constant a is 328 nm, the radius of the air holes r is 92 nm, and the thickness of the silicon slab t , which corresponds to the thickness of the top Si layer in Fig. 17c, is 177 nm. The value of a is precisely controlled by electron beam lithography. The value of r is also controlled by electron beam lithography, accounting for the increase during the fabrication process. The value of t is thinned

to the desired value by chemical etching [46]. The ratio of a to that in Fig. 16 is 0.80 ($= 328/410$), and the ratio of t is 0.78 ($= 177/225.8$). These values are similar to the ratio of λ_s , which is 0.83 ($= 1278/1544$). The ratio of r is 0.72 ($= 92/128$), which differs from the other ratios. This difference arises because the radius is chosen to ensure that the frequency shift, Δf , matches the Raman shift of silicon, 15.606 THz, as explained in Fig. 13a.

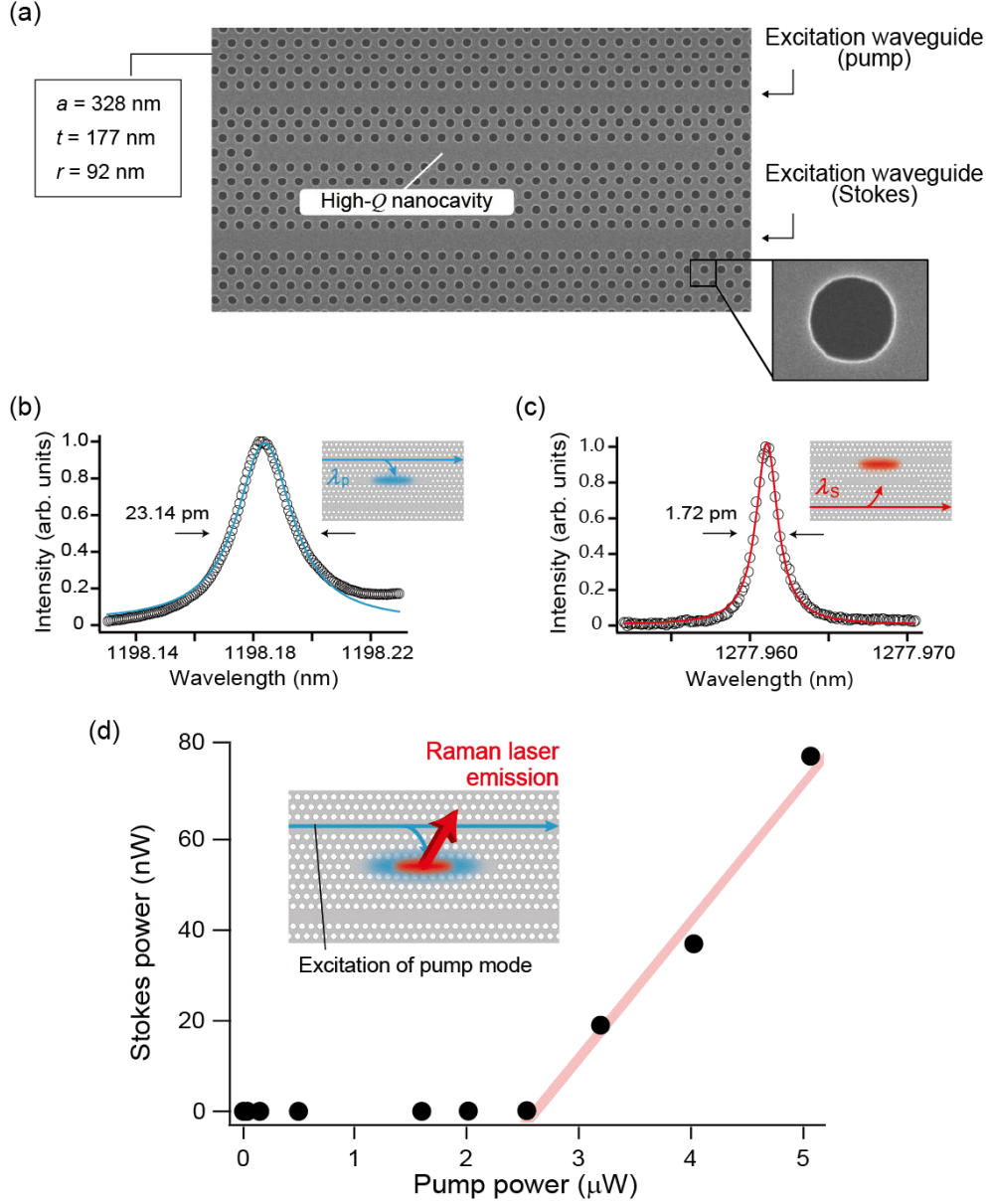


Figure 23. (a) SEM image of the Raman silicon nanocavity laser operating in the O-band. (b) Resonance spectrum of the pump mode, and (c) resonance spectrum of the Stokes mode. (d) Output power of the laser as a function of the pump power coupled into the nanocavity. The insets in (b) to (d) depict the excitation setup used during the measurements. Adapted from [47] under the terms of the Creative Commons Attribution 4.0 International license

Figures 23b and 23c display the resonance spectra for the pump and Stokes modes, respectively. The resonance wavelengths for the pump mode (λ_p) and that of the Stokes mode (λ_s) are 1198.18 nm and 1277.96 nm, respectively, resulting in a Δf of 15.615 THz. The experimentally measured Q factors of the pump and Stokes modes are $Q_{p_exp} = 5.18 \times 10^4$ and $Q_{s_exp} = 7.43 \times 10^5$, respectively. These values are lower than those of the C-band Raman lasers presented in Figs. 11, 19, and 22, where most of these C-band lasers had a $Q_{p_exp} > 1.00 \times 10^5$ and $Q_{s_exp} > 1.00 \times 10^6$. The threshold I_{th} depends on the value of $(Q_p \cdot Q_s)^{-1}$ and accordingly, the nanocavity shown in Fig. 23a–c might not oscillate at all.

Figure 23d presents the input–output characteristic of 1.2 μ m-pumped Raman silicon nanocavity laser. The x-axis represents the excitation power coupled into the nanocavity, while the y-axis indicates the total Stokes Raman power generated within the nanocavity. As shown by the red solid line, the Stokes power begins to rise significantly above the threshold of approximately 2.5 μ W. A distinct input-output relationship with a clear laser threshold is observed, despite the relatively low values of Q_{p_exp} and Q_{s_exp} . Since a variable ND filter, adjustable in 0.1 optical density increments, was used to control the excitation power, detailed input-output data near the threshold could not be recorded. It is worth noting that saturation of the Raman laser output, caused by TPA-induced FCA, was not observed in the excitation power range up to 2.0 times the threshold. The reason such good laser performance was achieved despite the low Q_{exp} factors is likely due to the increased g_R^{cav} , resulting from the shorter operating wavelength.

It is known that the effective mode volume for TPA is proportional to λ^{-3} [81] and the TPA coefficient β is inversely proportional to λ [110]. However, the influence of TPA on a Raman silicon nanocavity laser at threshold is relatively small (for example, the carrier density have been estimated to be $2.36 \times 10^{15} \text{ cm}^{-3}$ [81], which is relatively small). Additionally, the small volume of the nanocavity may shorten the carrier lifetime due to a higher surface-to-volume ratio. It is also known that magnitude of FCA is approximately proportional to λ^2 [111]. As a result, the increase in g_R^{cav} can be greater than the increase in nonlinear losses.

The Q_{p_exp} and Q_{s_exp} of the cavities designed for the 1.2- μ m band can be enhanced through improvements in the fabrication process. Compared to the C-band Raman silicon nanocavity lasers, these optimized 1.2- μ m-band cavities could deliver better laser performance in terms of threshold, energy efficiency, and output power. Additionally, achieving laser oscillation with a lower Q_p will be crucial for developing Raman lasers that utilize broadband light as the excitation source [112–114]. In this context, the use of superluminescent diodes based on InAs quantum dots holds significant promise [115,116]. The results presented in Fig. 23 demonstrate the potential of Raman silicon nanocavity lasers at shorter wavelengths.

Figure 24a shows the 3D image for dual band Raman silicon nanocavity lasers operating at the O-band and C-band on a single chip. An SOI substrate with two different thicknesses is used. A Raman silicon laser operating in the C band is fabricated in the thicker region, while a Raman silicon laser operating in the O band is fabricated in the thinner region. Figures 24b and 24c present SEM images of the O-band and C-band nanocavities, respectively. The O-band cavity is characterized by the following parameters: $a_1 = 350 \text{ nm}$, $\Delta a_1 = 4 \text{ nm}$, $r_1 = 100 \text{ nm}$, $t_1 = 182.5 \text{ nm}$ with a cavity length of 9.83 μ m. The C-band cavity is described by $a_2 = 410 \text{ nm}$, $\Delta a_2 = 5 \text{ nm}$, $r_2 = 128 \text{ nm}$, and $t_2 = 215.6 \text{ nm}$. Here, the length of the line defect is 11.52 μ m. The parameters for the C-band Raman laser are almost the same as those shown in sections 3.2 and 4.1. The ratios a_1/a_2 and t_1/t_2 are 0.854 and 0.846, respectively, which are very close to 1.31/1.55. However, due to the necessary tuning of Δf the ratio r_1/r_2 is smaller at 0.781.

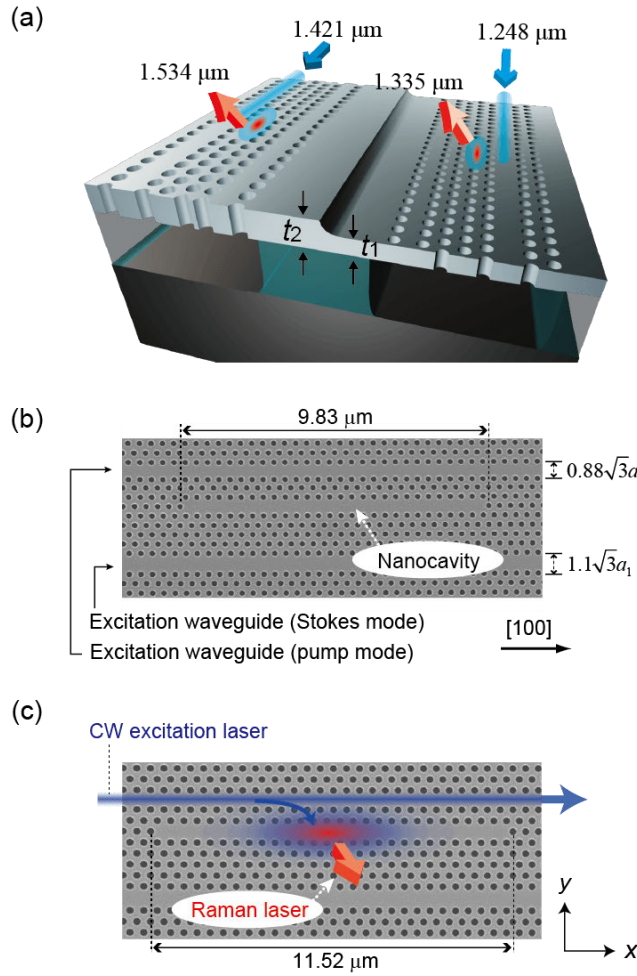


Figure 24. (a) 3D view of the nanocavity-based Raman silicon laser device operating in both the O-band and C-band. (b) SEM image of the core region of the O-band Raman silicon laser, and (c) SEM image of the C-band Raman silicon laser. Both images are shown at the same magnification. The arrows in (c) indicate the generation of the Raman laser emission (red) from the excitation beam (blue). Adapted with permission from [46]. Copyright 2019, Wiley-VCH

Figure 25a illustrates each step of the process for fabricating photonic crystal devices operating in both the O-band and C-band on a dual-thickness substrate. The first four steps focus on creating the dual-thickness substrate. Initially, a 1 cm² (100)-oriented SOI substrate with a thickness of $t_2 \approx 217$ nm is prepared (A). Next, a mask in the form of a protective epoxy resin layer, which is resistant to etching, is partially applied to the surface (B). Inkjet printing is used to apply the coating. The unmasked area is chemically etched using an alkaline-based solution [$\text{H}_2\text{O} : \text{KOH} (4.0 \text{ mol/l}) : \text{H}_2\text{O}_2 (10.1 \text{ mol/l}) = 50 \text{ ml} : 15 \text{ ml} : 0.3 \text{ ml}$] at 297 K, while the thickness reduction is monitored by spectroscopic reflectometry. The etching process is stopped when the ratio $t_1:t_2$ reaches approximately 1.31:1.55 (C). The mask is then chemically removed (D). In process step (E), the dual-thickness substrate is coated with electron beam (EB) resist via spin coating. No significant variation in resist thickness is observed near the step edge. EB lithography is then used to pattern the nanocavity for the C-band on the thicker area and the O-band on the thinner area (F). After immersion in the developer, the PC patterns in the resist are transferred to the top silicon layer using inductively coupled plasma (ICP) etching with SF_6 gas (G). Importantly, the ICP etching is performed in a single step. Despite this, issues

such as over-etching in the thinner area or under-etching in the thicker area are avoided. Step (H) shows the device after the remaining resist is removed. Finally, the BOX layer beneath the patterned region is selectively etched using HF acid to create an air-bridge structure (I). Figures 25b and 25c display SEM images of the air holes formed in the thinner and thicker regions, respectively, both shown at the same magnification. The sidewalls of the PC holes are highly vertical in both cases. Figures 25d and 25e provide corresponding top-view images, confirming the good circularity of the holes.

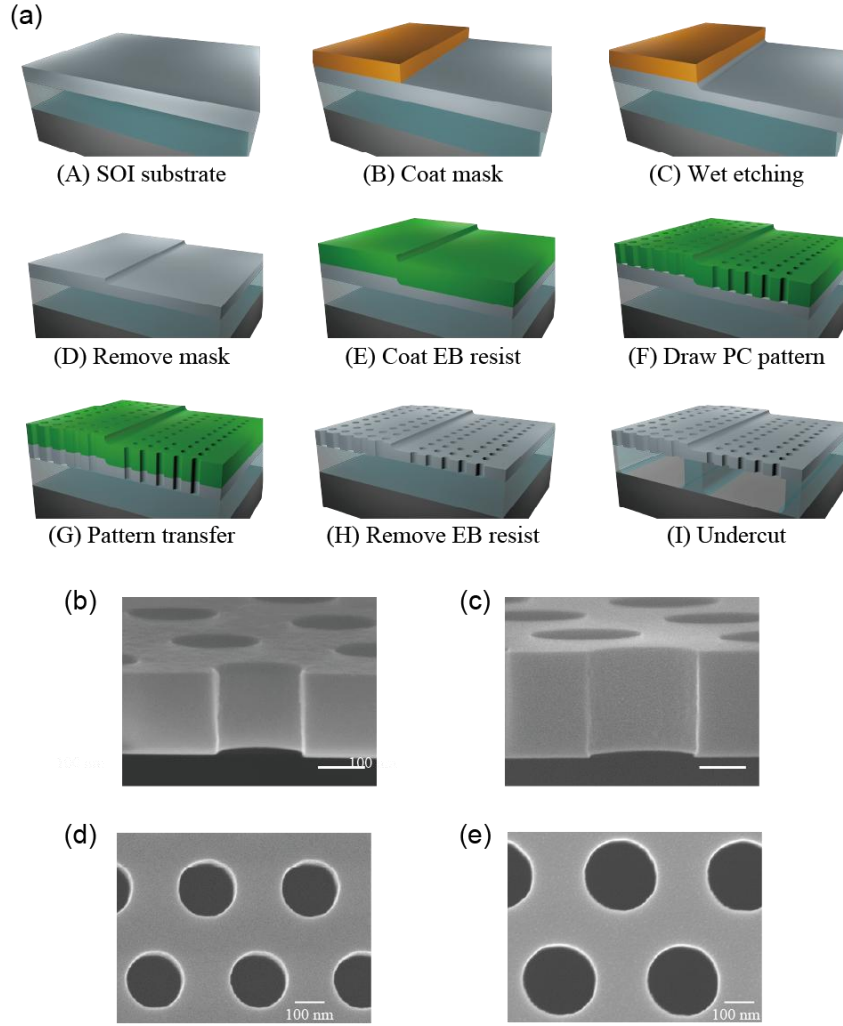


Figure 25. (a) Process steps for fabricating the O-band and C-band photonic crystal cavities on a dual-thickness SOI substrate. (b) Cross-sectional SEM image of the air holes for the O-band, and (c) for the C-band. (d) Top-view SEM image of the air holes for the O-band, and (e) for the C-band. Adapted with permission from [46]. Copyright 2019, Wiley-VCH

Figure 26a shows the resonance spectra for the pump mode (blue) and the Stokes mode (red) of the O-band Raman laser, where λ_s falls within the O-band range. Figure 26b displays the spectra for the C-band laser, with λ_s within the C-band. The quality factors Q_p and Q_s for the O-band cavity are 1.95×10^5 and 1.29×10^6 , respectively. For the C-band cavity, Q_p and Q_s are 1.28×10^5 and 1.42×10^6 , respectively. The high device quality is evident from the Q_s values exceeding one million in both cavities. Figure 26c presents the input/output characteristics for the O-band and C-band Raman silicon lasers. These results confirm that both

lasers possess sufficient quality for Raman lasing. The threshold power for the O-band laser is 220 nW, with a maximum efficiency of 1.0%. The relatively low efficiency could be attributed to the large frequency detuning ($\Delta f_{\text{det.}}$) from the Raman shift of silicon, while the low threshold might be due to the high $Q_{\text{p_exp}}$ or the enhanced $g_{\text{R}}^{\text{cav}}$. For the C-band laser, the threshold power is 340 nW, and the maximum efficiency reaches 12.2%. The successful demonstration of O-band and C-band Raman silicon nanocavity lasers on a single chip, with a minimal number of processing steps, indicates strong potential for the integration of Raman silicon nanocavity lasers into optoelectronic circuits.

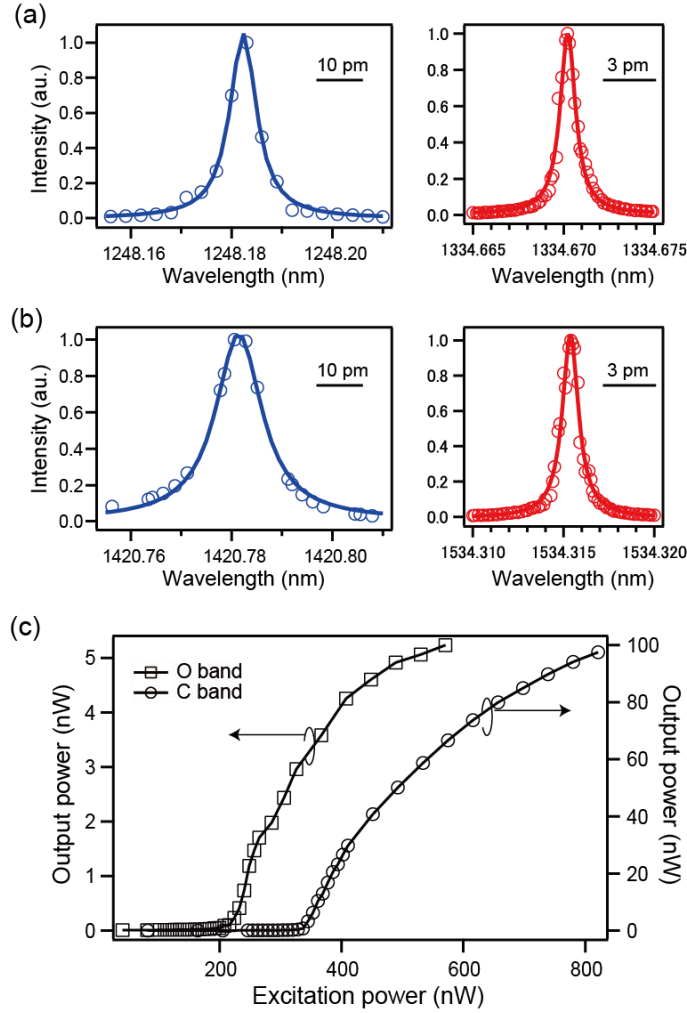


Figure 26. (a) Resonance spectra of the O-band Raman silicon laser and (b) C-band laser. The open circles represent experimental data, while the solid lines show the fitting results based on a Lorentzian line shape. (c) Laser output power for both lasers as a function of the excitation power coupled into the nanocavity. Adapted with permission from [46]. Copyright 2019, Wiley-VCH

4-4 Edge-Emitting Raman Silicon Nanocavity Lasers

A Raman nanocavity laser with a common device design shown in previous sections can strongly emit light into free space from the nanocavity surface, while the emission from the edge of the waveguide adjacent to the cavity is relatively weak as shown in Fig. 19. A Raman silicon nanocavity laser with strong emission from the waveguide edge will be advantageous for integrated optoelectronic chip applications. In this subsection, the device design to increase the edge emission is explained, with further details available in Refs. [97].

The device design used in previous sections has a significant limitation: the power emitted from the edge of an adjacent waveguide (P_{edge}) is several times smaller than the surface emission power from the nanocavity (P_{surface}). A edge-emitting Raman silicon lasers can often be advantageous not only for silicon photonic circuit but also for the sensing applications [117,118]. The increase in P_{edge} of a Raman silicon nanocavity laser can be achieved by introducing heterointerface mirrors (HMs). Hereafter, we call this type of laser a HM Raman laser.

Figure 27 clarifies the HM Raman laser device design. The core-region design of the laser for heterostructure nanocavity and two excitation waveguide are the same as those in previous sections as shown in Figs 27a–c. Figure 27c shows the entire HM-laser device layout. Between the two waveguides, ten nanocavities with the same structure are placed with a spacing of about 20 μm . This sample design enables an efficient measurement of the individual laser samples [84]. The magnitudes of the variation in λ_p and λ_s are usually more than one order of magnitude larger than the full widths at half-maximum of the pump and Stokes modes [85]. Therefore, in a structure that contains only a few nanocavities, it is unlikely that even one of the resonance modes interferes with any of the other resonance modes [119]. In this study, the waveguide edge was formed by manually cleaving the sample.

The positions of the two HMs used to reflect the light propagating in the waveguides are indicated by the blue and red rectangles in Fig. 27c for the pump light and the Stokes light, respectively. The lattice constant of the holes in the colored areas in Figs. 27d and e is 380 nm in the x -direction (that in the y -direction is 710 nm to maintain structural uniformity). The propagation bands in the colored area shift to higher frequencies as shown in the diagrams at the bottom of Figs. 27d and e. The mode-gap differences at the heterointerfaces result in the reflection of light at λ_p and λ_s . For the evaluation of the performance increase that can be achieved by the HMs, ten conventional Raman laser cavities were also fabricated on the same chip. The design of the core region is the same as that shown in Fig. 27a.

Figure 28 shows the experimental results of the cavity which provides the largest P_{edge} . In Figure 28a, the resonance and transmission spectra of the pump mode are shown by the closed and open circles, respectively. To obtain these spectra, the output of the tunable laser was focused at the input port indicated in Fig. 27c to excite the pump mode. As illustrated in the inset of Fig. 28a, the surface emission from the cavity in the vertical direction of the slab ($+z$ direction) was measured, as well as the transmitted intensity at the waveguide edge opposite to the input port. Figure 28b shows the spectra for the Stokes mode. For this measurement, the tunable laser irradiated the output port indicated in Fig. 27c.

Due to the reflection at the heterointerfaces shown in Figs. 27d and 27e, the transmitted intensities shown in Figs. 28a and 28b are very small. The obtained Q_{p_exp} and Q_{s_exp} values are 1.49×10^5 and 7.87×10^5 , respectively. This Q_{s_exp} is smaller than the values reported in previous sections so that the edge emission increases. The measured Δf is 15.602 THz. These optical characteristics allow Raman laser oscillation. Figure 28c shows the input–output characteristics of this HM Raman laser. The orange points represent P_{edge} (the Stokes light emitted via the output port), while the black points represent P_{surface} (the Stokes light emitted from the cavity into free space). The pump power I is the power coupled into the pump mode of the nanocavity.

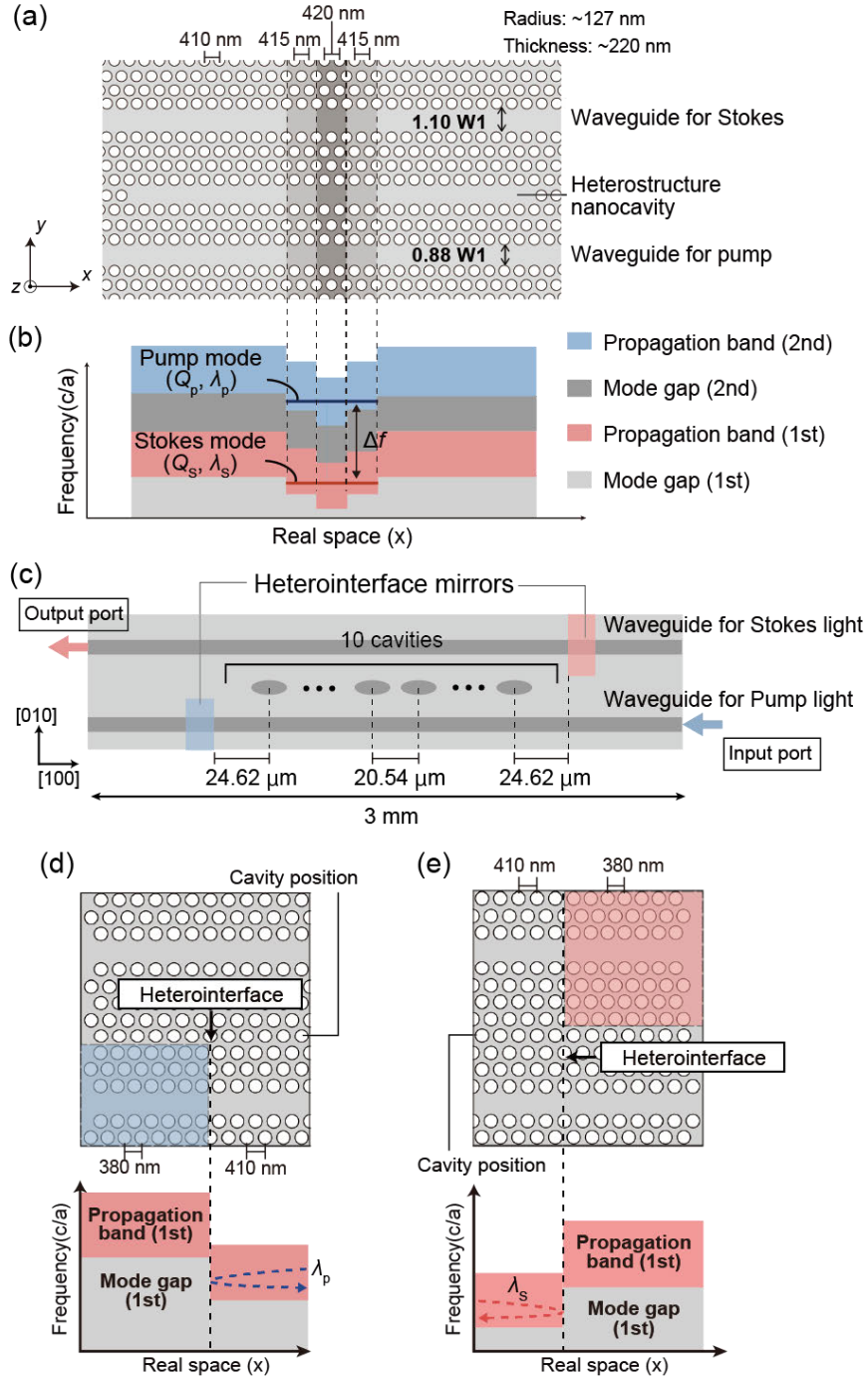


Figure 27. Device structure of the HM Raman laser. (a) Diagram of the heterostructure nanocavity along with the two neighboring waveguides. (b) Energy level diagram of the heterostructure nanocavity. (c) Full layout of the HM-laser device. (d) and (e) illustrate the structures of the photonic mirrors with a heterointerface, which provide reflection for the pump and Stokes waveguides, respectively. Reprinted with permission from [97] under the terms of the Creative Commons Attribution 4.0 International license

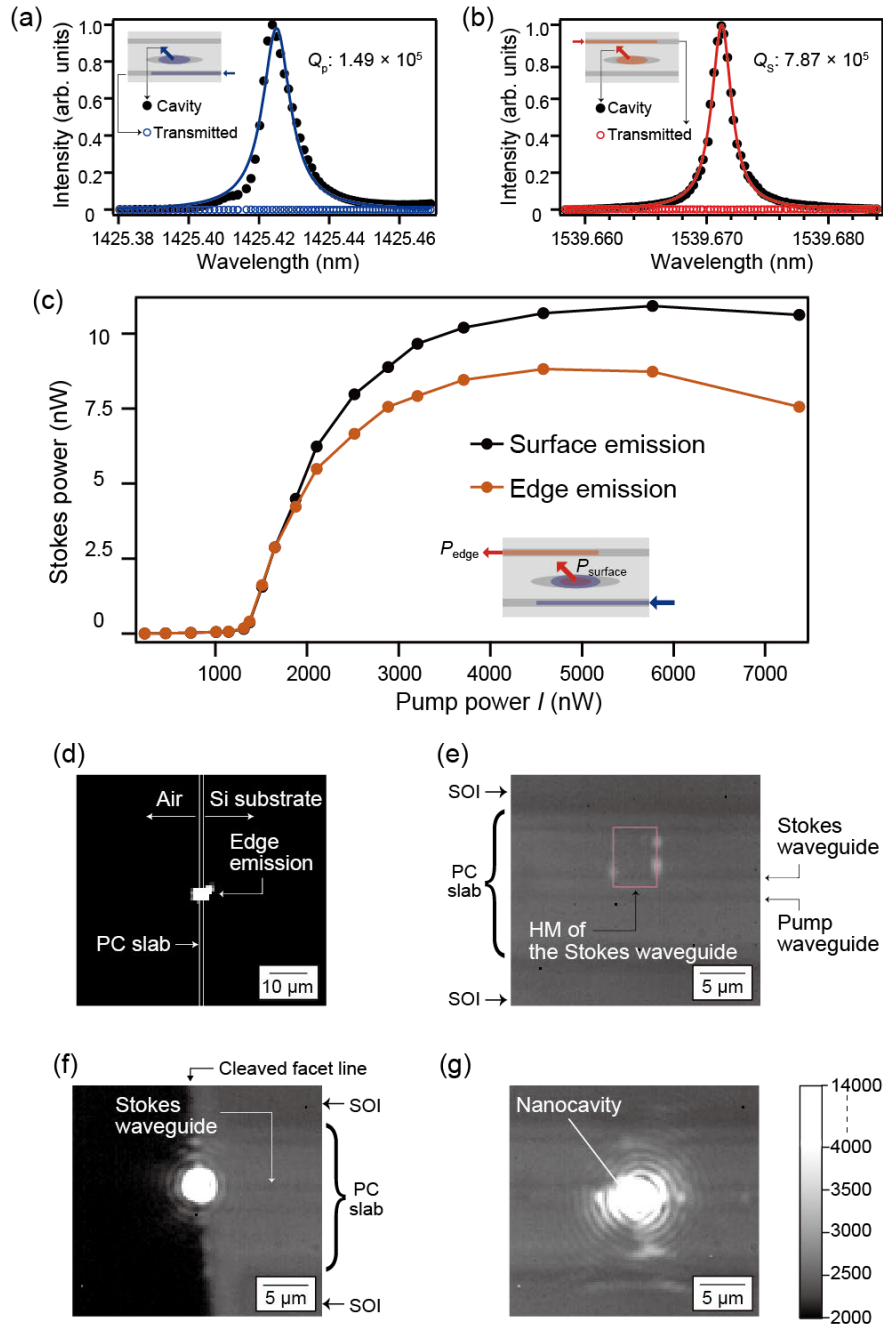


Figure 28. Details of the results for HM Raman laser. (a) Resonance spectrum of the pump mode (solid circles) and the corresponding transmission spectrum (open circles). The solid line represents the fitting result. (b) Resonance and transmission spectra of the Stokes mode. (c) P_{edge} and P_{surface} as a function of the current I . (d)–(g) NIR camera images for $I = 1.2 I_{\text{th}}$ taken using a long-pass filter to block the pump light. The images display (d) edge emission at the output port, (e) scattered light at the heterostructure mirror of the Stokes waveguide, (f) scattered light at the output port, and (g) surface emission from the nanocavity. Reprinted with permission from [97] under the terms of the Creative Commons Attribution 4.0 International license

As shown in Fig. 28c, P_{edge} and P_{surface} increase steeply when I starts to exceed the threshold $I_{\text{th}} = 1200$ nW, but the Raman laser output saturates as I increases further. This is due to FCA [80]. The maximum P_{edge} (8.8 nW) is obtained at $I = 4580$ nW and the experimentally obtained ratio of $P_{\text{edge}}/P_{\text{surface}}$ is 0.83. Note that our experimental setup leads to an underestimation of this ratio, because the numerical aperture (NA) of the objective lens that collects the surface emission is 0.7 and this value is larger than that of the lens that collects the edge emission, $\text{NA} = 0.4$.

Figures 28d–g present different near-infrared (NIR) camera images obtained during laser oscillation at $I = 1.2 I_{\text{th}}$. Figure 28d is an image of the edge emission at the output port (view direction: parallel to the x-axis shown in Fig. 27). A clear emission spot is obtained. Figure 28e is an NIR camera image at the HM of the Stokes waveguide (top view). The scattered light at the heterointerface is very small (this scattered light is probably due to Stokes photons with TM polarization [87]). This small intensity of the scattered light and the small intensity of the transmitted light in Fig. 28b indicates that the reflectivity of this HM is close to 100%.

Figure 28f is the top view of the silicon slab at the position of the output port. A spot with strong scattering to the z -direction is observed. The intensity of this emission spot is $0.80 \times P_{\text{surface}}$ at $I = 4,580$ nW (determined by a photodiode), and thus it is almost the same as $P_{\text{edge_max}}$. A similar amount of scattered light is probably also radiated into the direction of the silicon substrate ($-z$ direction). If a sophisticated structure, for example a spot size converter (SSC), had been added to the output port to reduce the scattering and to control the radiation pattern, the $P_{\text{edge_max}}$ would have exceeded maybe even 30 nW.

The above results are now compared with those of conventional Raman laser5 with the largest P_{edge} . Figures 29a and 29b indicate a $Q_{\text{p_exp}}$ and a $Q_{\text{S_exp}}$ of 1.59×10^5 and 6.89×10^5 , respectively. The Δf is 15.588 THz. These values are equivalent to those of HM laser in Fig. 28. Figure 29c shows the input–output characteristics of this Raman laser and indicates an I_{th} of about 1600 nW. The saturation of the Stokes power due to FCA is also observed. It is found that $P_{\text{edge_max}} = 2.0$ nW at $I = 4520$ nW and $P_{\text{edge}}/P_{\text{surface}} = 0.11$. The latter value is only 13% of the $P_{\text{edge}}/P_{\text{surface}}$ of HM cavity. On the other hand, the sum $P_{\text{edge}} + P_{\text{surface}}$ is almost the same (20 nW for conventional laser and 19.5 nW for HM laser).

Figures 29d–f present the NIR camera images obtained at $I = 1.2 I_{\text{th}}$. The intensities of the emission spots in Figs. 29d and 29e are much smaller than those in Fig. 28f. The power of the light scattered at the output port into the z -direction is also small, $0.08 \times P_{\text{surface}}$ at $I = 4520$ nW. This is 10 times smaller than that of HM laser. These results demonstrate that the conventional Raman laser emits most of the laser light from the cavity into free space, while the HMs lead to an increase in the P_{edge} of a Raman laser. The method to further increase P_{edge} is described in Ref. [97].

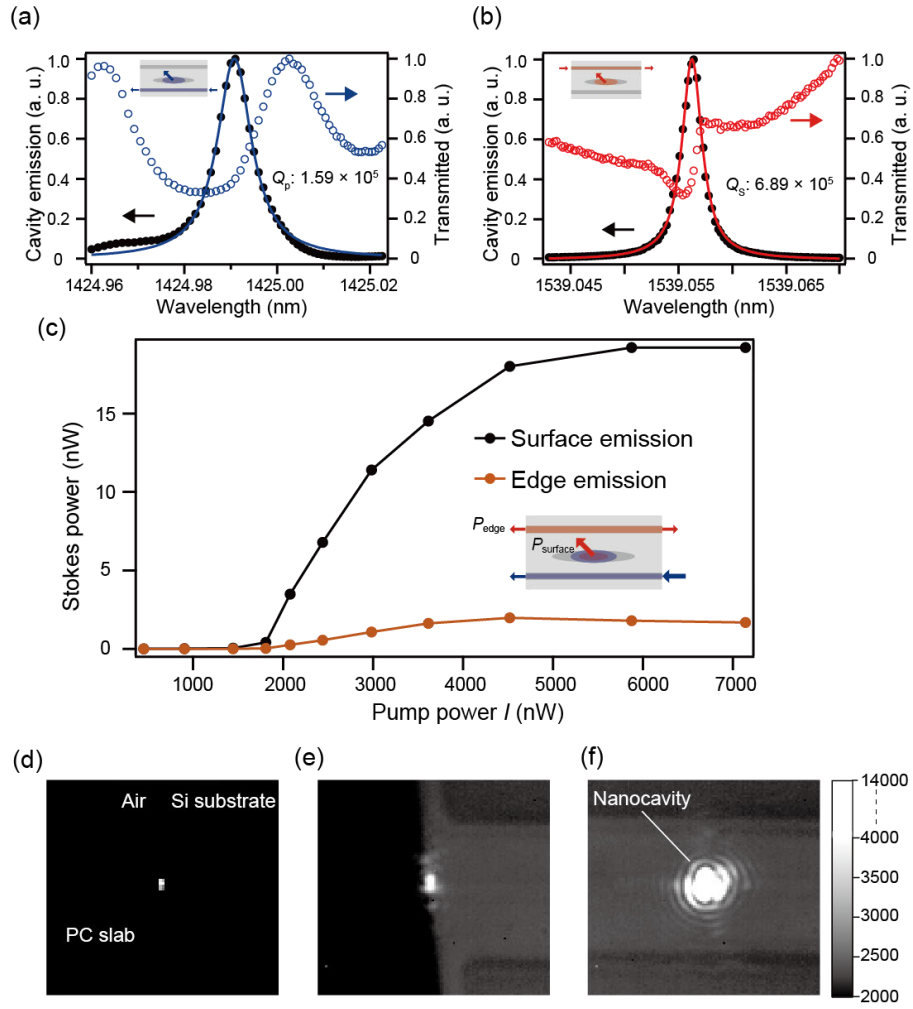


Figure 29. Details of the results for conventional Raman laser. (a) Resonance spectrum of the pump mode (solid circles) and the corresponding transmission spectrum (open circles), with the solid line representing the fitting result. (b) Resonance and transmission spectra of the Stokes mode. (c) Input-output characteristics of the laser. (d)–(f) NIR camera images taken at $I = 1.2 I_{\text{th}}$, showing (d) edge emission at the output port, (e) scattered light at the output port, and (f) surface emission from the nanocavity. Reprinted with permission from [97] under the terms of the Creative Commons Attribution 4.0 International license

6. Future Prospects

In this chapter, we covered the progress in developing Raman silicon lasers using rib-type waveguides [27,30,50], along with our work on nanocavity-based Raman silicon lasers [28,47,80]. The large-scale production of Raman silicon lasers has been realized using CMOS-compatible fabrication techniques [91,93]. Developing a novel excitation approach is essential for the integration of these lasers into optoelectronic chips. Raman lasers generate Raman gain by using laser light to excite the medium. Due to fabrication limitations, each nanocavity exhibits slight variations in resonance wavelength [83,84], and in addition, the resonance wavelength changes by approximately 80 pm/C° as the cavity temperature fluctuates. As a result, continuous-wave laser diodes with tunable wavelengths are needed for excitation. However, such lasers tend to be costly and physically large. Moreover, it is necessary to fine-tune the laser diode's wavelength to align with the nanocavity's resonance wavelength, which can restrict their application scope.

A promising alternative to using a tunable laser and its corresponding tuning process is to employ an excitation source with a broad emission spectrum [112,114]. Research has shown that nanocavities excited by a superluminescent diode maintain nearly consistent resonant emission intensities, even when temperature fluctuations occur [113]. Additionally, a superluminescent diode can excite numerous nanocavities at once, each with considerably different resonance wavelengths. Achieving continuous-wave operation of Raman silicon nanocavity lasers with broadband light could significantly broaden the application possibilities of these lasers. Given that Raman silicon nanocavity lasers have thresholds as low as 1 μ W, this approach could allow multiple lasers to oscillate simultaneously on an integrated optoelectronic chip, all powered by a single broadband light source. This excitation method is also promising for sensing applications [121,122].

Improving the fabrication yield and reducing the performance variation of Raman silicon nanocavity lasers are also important. The variations in Q -factor and in Δf for the fabricated nanocavities have the impact on the performance of Raman lasers [85]. To minimize these variations, it is essential to enhance the fabrication precision of the air holes. Another effective approach would be to design a nanocavity where the Q -factor and Δf are less sensitive to structural fluctuations of the air holes. Such nanocavity can be designed using machine learning [71]. It has been reported that nanocavity with air-hole patterns with broken mirror symmetry can reduce Q -factor variation [123].

Twenty years have passed since the first demonstration of the Raman silicon laser. During these two decades, the industrial applications of silicon photonics have advanced significantly. The demand for optoelectronic silicon chips is higher than ever, driven by the rapid progress in AI. We expect that the industrial application of Raman silicon lasers will soon begin.

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