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Numerical Study of Helical Pipe Flow

Ph.D Thesis

September, 2017

Anup Kumer Datta



**The Graduate School of Natural Science and Technology
(Doctor Course)
OKAYAMA UNIVERSITY, JAPAN**

Numerical Study of Helical Pipe Flow

A thesis

Submitted in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

by

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Under the supervision of

Professor Shinichiro YANASE



The Graduate School of Natural Science and Technology

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Dedication

This dissertation is dedicated to my late father who shaped part of my vision and taught me the good things that really matter in life

Anup Kumer Datta

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Abstract

Flows and heat transfer in helical pipes have attracted considerable attention not only because of their ample applications in chemical, mechanical, civil, nuclear and biomechanical engineering but also their physical interest of secondary flow patterns and soon. In this dissertation, three dimension (3D) direct numerical simulations (DNS) of viscous incompressible isothermal (without heat transfer) and thermal (with heat transfer) laminar and turbulent fluid flows through a helical pipe with circular cross section are presented, where the wall of a helical pipe is heated for thermal case. Note that the case of turbulent flow through a helical pipe, we also used three different turbulent models based on the Reynold averaged Navier-Stokes equation (RANS). Numerical calculation were carried out over a wide range of Dean number, Dn , curvature, δ , torsion parameter, β , the Prandtl number, Pr , and the Reynolds number, Re , for both the laminar and turbulent flow cases.

OpenFOAM was used as a tool for the numerical approach. To generate a suitable mesh in the flow domain, an appropriate mesh system with 3D orthogonal helical coordinates was successfully created to conduct accurate DNSs and turbulence models of helical pipe flow using an FVM-based open-source computational fluid dynamics package (OpenFOAM).

First, the laminar flow in a helical pipe with circular cross section was investigated. The instability of the steady solutions of the helical pipe flow was studied instead of the linear stability analysis. We found the neutral curve of the critical Reynolds numbers of the laminar to turbulent transition by observing unsteady behaviors of the solution. The present results of the critical Reynolds number nearly agree with the two-dimensional (2D) linear stability analysis (Yamamoto et al. (1998)) except for the lowest critical Reynolds number region, where the present study gave the critical Reynolds numbers much less than those by the 2D linear stability analysis and showed explosive 3D instability occurred slightly in the marginal instability state. It is interesting that we found a close relationship between the disturbances of unsteady solutions (such as nearly 2D state, nonlocal 3D modification, highly nonlinear behaviours, continuous nonperiodic oscilation), and the vortical structure of steady solutions in helical pipe flow.

Then, the effect of torsion on the friction factor of helical pipe flow was investigated. Well-developed axially invariant regions were obtained where the friction factors were calculated, and good agreement with the experimental data was obtained. It was found that the friction factor sharply increases as β increases from zero, then decreases after taking a maximum, and finally slowly approaches that of a straight pipe when β tends to infinity. It is interesting that a peak of the friction factor exists in the region $0.2 \leq \beta \leq 0.3$ for all the Reynolds numbers and curvatures studied in the present study, which manifests the importance of the torsion parameter in helical pipe flow.

Next, laminar forced convective heat transfer in a helical pipe with circular cross section subjected to wall heating was investigated comparing with the experimental data. In 3D steady calculations, we found the appearance of fully-developed axially invariant flow regions, where the averaged Nusselt number (averaged over the peripheral of the pipe cross section) were calculated, being in good agreement with the experimental data. Because of the effect of torsion on the heat transfer characteristics, the averaged Nusselt number exhibits repetition of decrease and increases as torsion increases from zero for all Reynolds numbers. It was found that there exists two maximums and two minimums of the averaged Nusselt number. It is interesting that the global minimum of the Nusselt number occurs at $\beta \cong 0.1$ and the global maximum at $\beta \cong 0.55$.

Finally, turbulent flow through helical pipes with circular cross section was numerically investigated by use of three different turbulence models comparing with the experimental data. We numerically obtained the secondary flow, the axial flow and the intensity of the turbulent kinetic energy by use of three turbulence models. We found that the change to fully developed turbulence is identified by comparing experimental data with the results of numerical simulations using turbulence models. We found that RNG $k - \varepsilon$ turbulence model can predict excellently the fully developed turbulent flow with comparison to the experimental data. It was found that the momentum transfer due to turbulence dominates the secondary flow pattern of the turbulent helical pipe flow. It is interesting that torsion effect is more remarkable for turbulent flows than laminar flows.

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Nomenclature

d_p	diameter of the pipe
H	pitch of the helical pipe
$\mathbf{t}$	unit tangential vector
$\mathbf{n}$	unit normal vector
$\mathbf{b}$	unit binormal vector
R	distance of the center line from the axis of the whole system
Pr	Prandtl number
k	Turbulent kinetic energy
T_w	Wall temperature
R_h	Radius of curvature
P	Pressure
T_b	bulk temperature
Dn	Dean Number
Re	Reynolds number
Nu	Nusselt number
q_w	rate of heat flux
u	velocity component in the x -direction
v	velocity component in the y -direction
w	velocity component in the z -direction
$\langle w \rangle$	Ensemble-average axial velocity.
x	horizontal axis
y	vertical axis
z	axis in the direction of the main flow
a	Radius of the pipe

Greek letters

δ	curvature
β	torsion parameter
α	thermal diffusivity
μ	viscosity
τ	torsion
ν	kinematic viscosity
ρ	density

Dissertation contents

This dissertation consists of the following papers and conference proceedings

- **Papers**

1. Anup Kumer Datta, Yasutaka Hayamizu, Toshinori Kouchi, Yasunori Nagata, Kyoji Yamamoto and Shinichiro Yanase, (2017). *Numerical study of turbulent helical pipe flow comparison to the experimental results*, **ASME Journal of Fluid Engineering**, Vol. **139(9)**, pp. 091204(1-13).
2. Anup Kumer Datta, Shinichiro Yanase, Yasutaka Hayamizu, Toshinori Kouchi, Yasunori Nagata and Kyoji Yamamoto, (2017). *Effect of torsion on the friction factor of helical pipe flow*, **Journal of Physical Society of Japan**, Vol. **86**, pp. 064403(1-7).
3. Anup Kumer Datta, Shinichiro Yanase, Toshinori Kouchi, and Mohammed M.E. Shatat, (2017). *Laminar forced convective heat transfer in helical pipe flow*, **International Journal of Thermal sciences**, Vol. **120**, pp. 41-49.

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1. Anup Kumer Datta, Yasutaka Hayamizu, Yasunori Nagata, Toshinori Kouchi, Kyoji Yamamoto and Shinichiro Yanase (2015). Numerical study of transition in helical pipe flow. **Proceedings, branch meeting of the Japan Society of Fluid Mechanics**, November 28-29, 2015, Tottori University, Japan.
2. Anup Kumer Datta, Yasutaka Hayamizu, Toshinori Kouchi, Yasunori Nagata, Kyoji Yamamoto and Shinichiro Yanase (2016). Numerical and experimental study of turbulent helical pipe flow. **Proceedings, International symposium on near-wall flows and turbulence**, June 20-22, 2016, Kyoto University, Japan.
3. Anup Kumer Datta, Yasutaka Hayamizu, Toshinori Kouchi, Yasunori Nagata, Kyoji Yamamoto and Shinichiro Yanase (2016). Numerical and experimental study of

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The present dissertation is designed as follows

In Chapter 1, an introduction regarding curved and helical pipe flow with circular cross section are presented from analytical, numerical and experimental. In chapter 2, the basic governing equation for laminar and turbulent flow, related to the problems considered hereafter, are shown. In chapter 3, the calculation techniques for solving the basic equations, presented in chapter 2, are discussed. In chapter 4, a specific problem of the laminar flow through a helical pipe with circular cross section is considered, where the flow characteristics are presented studying over a wide range of curvature, torsion, Dean number and Reynolds number based on spectral study, experimental data. The same problem has been extended for the effect of torsion on the friction factor of helical pipe flow in Chapter 5. In Chapter 6, we presented the turbulent flow characteristics through a helical pipe with circular cross section with best turbulent models. In Chapte7, forced

convective heat transfer in helical pipe flow is considered. Finally, general conclusions on all the problems dealt are given in Chapter 8.

Chapter 1

Introduction

Fluid flow in helical pipes occurs in many industrial systems such as heat exchangers, chemical reactors, well drilling, and stimulation operations in the petroleum industry, kidney dialysis devices, food and dairy processes, and blood oxygenators, among many others. The investigation of helical pipe flows motivated scientific engineering applications interest, but later their relevance to industrial applications such as chemical reactors and heat exchangers (Huang *et al.*, 2001 and Rennie & Raghavan, 2006). The realization that physiological flows, such as that in the human aorta, involve both curvature and torsion (Caro *et al.*, 1996), provided additional motivation to studying helical pipes (Zabieliski & Mestel, 1998), they being the simplest of such geometries.

Due to the curvature of the helical pipes the flowing fluid experiences a centrifugal force, which depends on the local axial velocity of the fluid particle. As a result of the difference in the axial velocity between the particles owing in the center of the pipe cross section and those flowing near the pipe wall with the effect of the boundary layer, a secondary flow develops which forces the fluid from the core region of the pipe towards the outer wall of the pipe, and moves the fluid near the pipe outer wall to the pipe inner wall, and the flowing fluid; however, it also may increase the pressure gradient required to obtain a given mass flux, and thus may increase the amount of energy. Therefore, an extensive knowledge of the heat transfer and hydrodynamic characteristics of flow through a helical pipe is essential for a better design of heat exchangers and systems that contains such pipes. The first observations of the curvature effect on flow in coiled tubes were noted at the beginning of the 20th century. Grindley and Gibson (1908) noticed the curvature effect on flow in a coiled pipe while performing experiments on air viscosity. Since then, numerous experimental as well as theoretical and numerical studies including helical coils, which is a subset of curved pipes, have been reported on the flow and heat transfer through curved pipes. These studies have shown the effect of different parameters on the flow pattern, pressure drop and heat transfer characteristics through helical and curved passages. These parameters include the

geometrical values such as; effect of pitch and curvature of the pipe, and the physical properties of the flowing fluid, covering a wide range of flow fields, laminar or turbulent, steady or unsteady, and with or without heat transfer.

Before examining the existing literature on helical pipe flow, a basic summary of flow in curved pipes is first presented. This background provides a useful basis from which to understand both the solution techniques, and the results, found in the helical pipe literature. The main objective of this chapter is to discuss the development of Dean vortices which play an important role in curved pipe flows and reduces the pair of Dean vortices which goes to the single vortex in helical pipe flows. The Dean vortices arising in curved pipe single-phase flows are known as an effective means for heat and mass transfer enhancement and without convective heat transfer in laminar-transition-turbulent region. On the other hand, the flow in a helical pipe shows similarity with a number of other centrifugally unstable system where no torsion has been induced in helical pipe flows. This review, therefore, focuses on the steady, time-dependent flow behavior and transition to fully developed turbulent flow with forced convective heat transfer flow phenomena in helical pipe.

1.1 Definition of some useful parameters

Dean number

The Dean number κ is defined as:

$$\kappa = 2 \left(\frac{a}{R} \right) \left(\frac{2a\bar{w}}{\nu} \right)^2 \quad (1.1)$$

where $\bar{w}$ is the mean axial velocity in the pipe, a is the radius of the pipe, R is the radius of curvature and ν is the viscosity while the original form of the Dean number was defined by Dean (1928) as:

$$K = 2 \left(\frac{a}{R} \right) \left(\frac{aw}{\nu} \right)^2 = \left(\frac{2a^3 w^2}{\nu^2 R} \right), \quad (1.2)$$

where w is defined as a constant having the dimensions of a velocity. If we take $\bar{w} = w$, then K and κ are related by $\kappa = (2K)^{1/2}$. For fully developed flow, the axial pressure gradient is

constant, say $\frac{1}{R} \frac{\partial p}{\partial z} = -G$ (G constant), in dimensional form. We can then define a non-dimensional constant, $C = Ga^2/\nu w$, and rewrite (1.2) as

$$K = \frac{2a^3}{\nu^2 R} \left(\frac{Ga^2}{\mu C} \right)^2. \quad (1.3)$$

If, following Dean (1928), we specify w as the maximum velocity (w_{max}) in a straight pipe of the same radius and with the same pressure gradient, then $C = 4$ and Eq. (1.3) becomes

$$K = \frac{2a^3}{\nu^2 R} \left(\frac{Ga^2}{4\mu} \right)^2 = 2 \left(\frac{a}{R} \right) \left(\frac{Ga^3}{4\mu\nu} \right)^2 = \frac{G^2 a^7}{8\mu^2 \nu^2 R} \quad (1.4)$$

Whereas if we simply set $C = 1$, Eq. (1.3) becomes

$$K = \frac{2a^3}{\nu^2 R} \left(\frac{Ga^2}{\mu} \right)^2. \quad (1.5)$$

The various versions of the Dean number based on $\bar{w}$, the mean axial velocity, are natural for the experimentalist because this quantity, being readily measured provided a more convenient characterization of the flow than the more difficult measurement of pressure gradient. For fully developed flow it is little different whether one uses a form of the Dean number based on G , i.e. Eq. (1.3), since $C = \text{constant}$ for such a flow. For these flows most theoretical and numerical investigations, beginning with McConlogue & Srivastava (1968), have used the square root of (1.5) and denoted this by Dn

$$Dn = \sqrt{\frac{2a^3}{\nu^2 R}} \left(\frac{Ga^2}{\mu} \right) = \frac{Ga^3 \sqrt{2a/R}}{\nu\mu} = 4 \sqrt{\frac{2a}{R}} \frac{Ga^2}{4\nu\mu}, \quad (1.6)$$

which is related to the original Dean number, (1.4), by $D = 4\sqrt{K}$.

Nusselt number

Convection is one of the basic mechanisms of heat transfer between a solid body and a fluid. It can be quantified by the convective heat transfer coefficients. This coefficient is, in turn, determined by a quantity known as the Nusselt number, abbreviated as Nu . The Nusselt number is dependent on the geometry of the heat transfer system and on physical properties of the fluid. It is a dimensionless parameter which measures the enhancement of heat transfer from a surface which exists in a real situation. Typically, it is used to measure the

enhancement of heat transfer when convection takes place. One of the first quantities introduced about heat transfer is the convective heat transfer coefficient

$$h = \frac{q_w}{T_w - T_b}$$

where, $q_w = -k \frac{\partial T}{\partial y}$, T_w is the wall temperature, and T_b is the bulk temperature of the fluid

The Nusselt number is defined as

$$Nu = \frac{hd}{k} \quad (1.7)$$

Where, h is the convection heat transfer coefficient, d the characteristic length, and k the thermal conductivity of the fluid. The Nusselt number can also be viewed as being a dimensionless temperature gradient at the surface. $Nu > 0$ means that heat transfer occurs from solid body to the fluid, whereas $Nu < 0$ indicates heat transfer in the opposite direction.

Prandtl number

The Prandtl number, Pr , is a dimensionless parameter which approximates the ratio of kinematic diffusivity to thermal diffusivity. It describes the effect of fluid thermo-physical properties on heat transfer and is combined traditionally with other dimensionless parameters for evaluating fluid-surface interfacial heat transfer coefficient. It is defined as

$$Pr = \frac{\nu}{\alpha} \quad (1.8)$$

where ν is the kinematic viscosity and α is the thermal diffusivity. The Prandtl number shows the relative importance of heat conduction and viscosity of a fluid.

1.2 Flow in curved pipes

Secondary flow in curved pipes was first reported by Eustice (1910, 1911), in his experiments injecting ink into water flowing through a coiled pipe. The first theoretical treatment of the problem was given by Dean (Dean 1928), which used the toroidal coordinate system (r', α, θ) . The non-dimensional Navier-Stokes equations are written in this coordinate system, and then simplified. First, the velocities are rescaled to make the centrifugal-force terms the same order of magnitude as the viscous and inertial terms, as it is the centrifugal force that drives

the secondary motion. Two parameters characterize the flow in curved pipes, the Dean number, Dn and the curvature, δ . They are defined:

$$\delta = a/R$$

$$Re = \frac{\bar{w} 2a}{\nu}, Dn = Re\sqrt{\delta}. \quad (1.9)$$

The Dean number is essential, which is modifying defined by the Reynolds number. The loose coiling assumption that the pipe has very small curvature, i.e. $\delta = a/R \ll 1$ is applied by setting the δ terms to zero. The flow is also assumed to be fully developed, that is, independent of the axial position, which gives the final formulation of the equations as:

$$\nabla_1^2 w - \frac{Dn}{2} \frac{\partial p_0}{\partial z} = \frac{Dn}{2r} (\psi_\alpha w_r - \psi_r w_\alpha), \quad (1.10)$$

$$\frac{2}{Dn} \nabla_1^4 \psi + \frac{1}{r} \left(\psi_r \frac{\partial}{\partial \alpha} - \psi_\alpha \frac{\partial}{\partial r} \right) \nabla_1^2 \psi = -2w \left(\sin \alpha w_r + \frac{\cos \alpha}{r} w_\alpha \right), \quad (1.11)$$

with the boundary conditions

$$\psi = \psi_r = w = 0 \text{ at } r = 1,$$

where

$$\nabla_1^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \alpha^2}.$$

For small Dean numbers the solution can be explained in a series expansion of powers of Dn .

$$w = \sum_{n=0}^{\infty} Dn^{2n} w_n(r, \alpha), \quad \psi = Dn \sum_{n=0}^{\infty} Dn^{2n} \psi_n(r, \alpha), \quad (1.12)$$

The primary flow, w_0 , is the Poiseuille flow, and the secondary term, ψ_1 , shows a symmetric pair of counter-rotating helical vortices. A centrifugal pressure gradient is formed, which drives slower-moving fluid near the wall inward. The faster moving fluid in the core is moved, which is the second-order correction to the axial velocity, w_2 . There are several consequences of the secondary flow formation; the flow rate is reduced from the equivalent Poiseuille flow, extremes of shear stress form at the inner and outer bends and the transition to turbulence is delayed.

In consideration of their importance, flows in a curved pipe have been studied extensively in the literature for several decades. Most of the studies were focused on the Engineering applications, like the friction factor correlation (Ito, 1959; Manlapaz & Churchill, 1980; Ramshankar & Sreenivassan, 1983; Liu *et al.* 1994) and heat transfer (Mori & Nakayama,

1967; Yao & Berger, 1978). In the last few decades, the emphasis has been shifted towards more fundamental studies of flow development (Hille *et al.*, 1985; Bara *et al.*, 1992), transition and bifurcation phenomena (Winters, 1987; Kao, 1992; Yanase *et al.*, 1989, 2002). Recent works in this area deal with laminar to turbulence transition and the oscillating behavior of the unsteady flows (Canton *et al.*, 2015; Kuhnén *et al.* 2015, 2016).

1.3 Flow in helical pipes

A helical pipe is a three-dimensional object that can be described by the parametric equations:

$$x = R \cos\theta, y = R \sin\theta, z = h\theta, \quad 0 \leq \theta \leq 2\pi \quad (1.13)$$

where R is the radius or amplitude of the helix and h is a constant parameter, such that the wavelength, or one pitch length, of the helical pipe equals $2\pi h$. The radius of curvature of the helical pipe center line is $R_h = R + h^2/R$, the nondimensional curvature of the pipe $\delta = \kappa a = aR/(h^2 + R^2)$, the nondimensional torsion $\tau = ah/(h^2 + R^2)$ and the torsion parameter is defined by $\beta = \tau/\sqrt{2\delta}$ as introduced by Kao (1987). The length of the pipe center line for one turn is $2\pi\sqrt{R_h R}$.

These expressions clearly show that the ratio of curvature to torsion is a constant, equal to R/h , indeed Lancret's theorem (Scofield 1995) states that this is a necessary and sufficient condition for a curve to be helical.

The equation of the helical centerline can be written as a vector $R(t)$

$$R(\theta) = R \cos\theta \mathbf{i} + R \sin\theta \mathbf{j} + h\theta \mathbf{k} \quad (1.14)$$

Alternatively, this can be parameterized in terms of the arc-length s , which is given by

$$s = \int_0^\theta \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 + \left(\frac{dz}{d\theta}\right)^2} d\theta \quad (1.15)$$

For a helical pipe, the arc length simply increases linearly with the parameter θ , and evaluates to

$$s = \sqrt{R^2 + h^2} \theta$$

The Frenet triad of tangent $\mathbf{T}$, normal $\mathbf{N}$ and binormal $\mathbf{B}$ unit vectors of a curve $R(s)$ are defined as:

$$\mathbf{T} = \frac{dR}{ds}, \mathbf{N} = \frac{1}{\kappa} \frac{d\mathbf{T}}{ds}, \mathbf{B} = \mathbf{T} \times \mathbf{N}$$

where $\delta = \epsilon = \kappa a$ is the curvature introduced so far. Figure 1 shows several different helical pipe geometries for varying parameter values, the properties of which are given in Table 1.1. This figure indicates that one limit of a helical geometry is a straight pipe, and the other limit is a toroidal pipe.

Table 1.1: Curvature and torsion data for helical pipes shown in Figure 1.1

Pipe	$2a$ [mm]	h [mm]	R [mm]	δ	τ	β	n
a	50.5	0	252.5	0.1	0	0	0
b	50.5	22.41	250.5	0.1	0.009	0.02	3.2
c	50.5	65.46	18.3	0.1	0.36	0.8	3.2
d	50.5	45.47	8.47	0.1	0.54	1.2	3

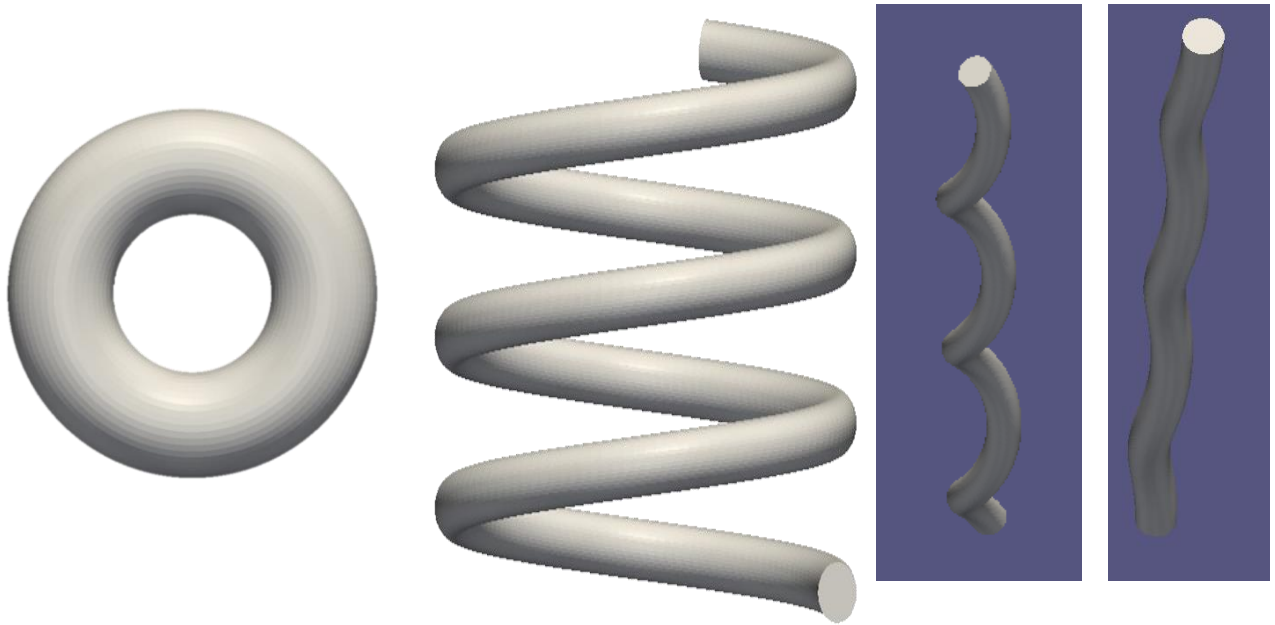


Figure 1.1: Helical pipe with circular cross section

1.3.1 Non-orthogonal coordinate system

In the theoretical perspectives of the helical pipe flows, Wang (1981) formulated the Navier-Stokes equations in a non-orthogonal helical coordinate system. This is shown in Figure 1.2, and the key points are as follows.

The first axis of the coordinate system is given by the vector equation of the centerline:

$$\mathbf{R} = X(s)\mathbf{i} + Y(s)\mathbf{j} + Z(s)\mathbf{k} \quad (1.16)$$

where s is the arc length along the axis, and $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are the standard unit vectors in a Cartesian frame. A coordinate system (r, θ, s) can then be defined, such that a Cartesian vector $\mathbf{x}$ is given by:

$$\mathbf{x} = R(s)\mathbf{T} + r\cos\theta \mathbf{N}(s) + r\sin\theta \mathbf{B}(s) \quad (1.17)$$

The definition of the pipe boundary follows naturally by setting $r = D/2$, an additional parameter that is the internal radius of the pipe. That is:

$$\mathbf{x} = R(s)\mathbf{T} + (D/2)\cos\theta \mathbf{N}(s) + (D/2)\sin\theta \mathbf{B}(s) \quad (1.18)$$

Taking the dot product of the incremental change of eqn (1.17) with itself gives:

$$d\mathbf{x} \cdot d\mathbf{x} = (dr)^2 + r^2(d\theta)^2 + [(1 - \kappa r \cos\theta)^2 + \tau^2 r^2](ds)^2 + \underbrace{2\tau r^2 ds d\theta}_{\text{non-orthogonality}} \quad (1.19)$$

It is the last term in Eq. (1.19) which highlights the non-orthogonal nature of this coordinate system. The physical consequence of this non-orthogonality is that the axial velocity w is not perpendicular to the (r, θ) plane. Wang's analysis assumes that the pipe is long enough for end effects to be ignored, that is, the velocities are independent of s , or fully developed. In addition, both the curvature and torsion are assumed to be small, with the flow steady and laminar, for the Reynolds number of order unity.

For small curvature and torsion the axial flow w is dominant, and is found to be Poiseuille flow. This allows a series expansion of the form

$$w = \mathbf{U} \sum_{i=0}^n \epsilon^i w_i \quad (1.20)$$

for the axial velocity, where $\mathbf{U}$ is a velocity scale proportional to the axial pressure gradient, and $\epsilon = \kappa a \ll 1$, for pipe radius a . The secondary flow velocity is expanded in the following way, using u as an example:

$$u = \mathbf{U} \sum_{i=1}^n \epsilon^i u_i \quad (1.21)$$

The secondary flow is therefore order-1, with stream function ψ defining the velocities as

$$u_1 = \frac{1}{\eta} \frac{\partial \psi}{\partial \eta}, u_2 = -\frac{\partial \psi}{\partial \eta} \quad (1.22)$$

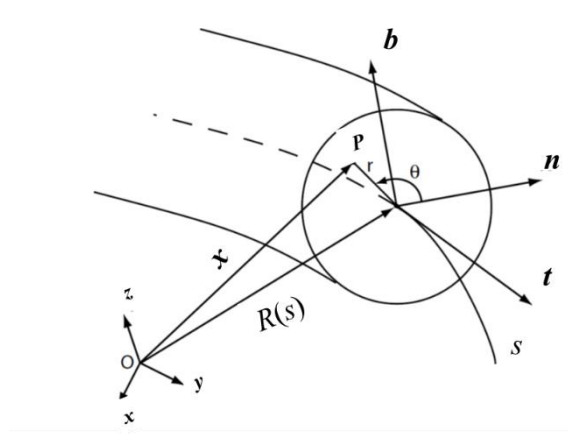


Figure 1.2: Helical coordinate system used by Wang

where η is the normalised radial position, r/a . The stream function solution is:

$$\frac{\psi}{Re} = \underbrace{\frac{\sin \theta}{288} (\eta^7 - 6\eta^5 + 9\eta^3 - 4\eta)}_{\text{Dean vortices}} - \underbrace{\frac{1}{4} \left(\frac{\lambda}{Re} \right) (\eta^4 - 2\eta^2 + 1)}_{\text{Torsion rotation}} \quad (1.23)$$

The first term in Eq. (1.23) is the stream function for the Dean vortices; a pure curvature effect. The second term is a pure rotation, indicated by the lack of θ dependence, which arises due to the effect of torsion on the flow, where $\lambda = \tau/\kappa$ and is a constant of order unity. Wang found that for $\lambda/Re \geq 1/24$ torsion dominates, causing a single recirculation cell in the secondary flow. It is important to note that this torsion effect is of first-order, which is the same order as curvature. This is because ψ is an $O(\epsilon)$ term, a first order term in this expansion, and $\lambda\epsilon = \tau a$

1.3.2 Orthogonal coordinate system

Germano showed that it is possible to construct an orthogonal coordinate system by replacing θ , from Wang's system, with $\theta + \phi(s) + \phi_0$, in the plane normal to the axis (Germano 1982), as shown in Figure 1.3. The Cartesian vector x therefore becomes:

$$\mathbf{x} = R(s)\mathbf{T} + r\cos(\theta + \phi(s) + \phi_0)\mathbf{N}(s) + r\sin(\theta + \phi(s) + \phi_0)\mathbf{B}(s) \quad (1.24)$$

where $\phi(s) = -\int_{s_0}^s \tau(s')ds'$

with ϕ_0 and s_0 taking arbitrary values. Again performing the dot product of the incremental change as before:

$$d\mathbf{x} \cdot d\mathbf{x} = (dr)^2 + r^2(d\theta)^2 + [1 + \kappa r \sin(\theta + \phi)]^2(ds)^2 \quad (1.25)$$

which shows that the system is now orthogonal. The key result reported in this

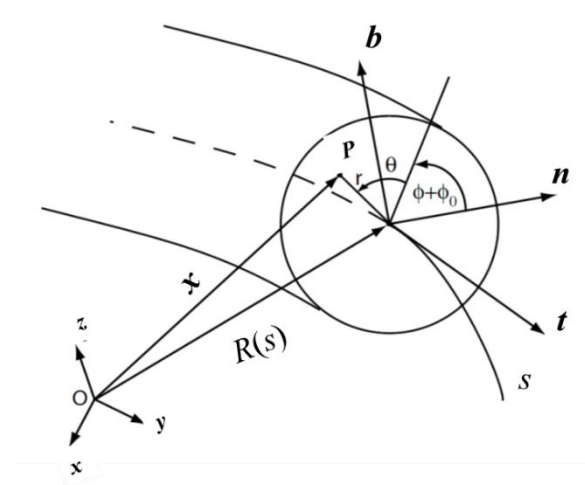


Figure 1.3: Helical coordinate system used by Germano

paper is that the torsion has only a second order effect on the flow, contrary to the findings of Wang (Wang 1981). Germano's orthogonal formulation was subsequently used by many researchers. It has also been used to analyse flow in pipes with non-uniform curvature and torsion. Kao (1987) used it to obtain a series expansion for small Dean number flows, but with the term $\beta^{1/2} = O(1)$, where $\beta^{1/2} = \lambda/(2\epsilon)^{1/2}$. The series expansions are of the form:

$$w = w_0 + \sum_{i=1}^{\infty} Dn^{(1+i)/2} w_i \quad (1.26)$$

for the axial velocity w , and for the streamfunction ψ

$$\psi = \psi_0 + \sum_{i=1}^{\infty} Dn^{(1+i)/2} \psi_i \quad (1.27)$$

where the leading order solution is the Poiseuille flow, $w_0 = 1 - r^2$ and $\psi = 0$. It should be noted that the stream function ψ is of a modified form, shown in Eq. (1.27), which is introduced to replace the continuity equation (to make the system analogous with Dean's formulation, and identical if $\beta = 0$).

$$u = \frac{1}{r} \frac{\partial \psi}{\partial \alpha} - \beta^{1/2} K^{1/2} \frac{1}{r} \int_0^r r \frac{\partial \bar{w}}{\partial \alpha} dr, \quad v = -\frac{\partial \psi}{\partial r} \quad (1.28)$$

It is clear that the second term in u , in Eq. (1.28), shows that the stream function is not a ‘true’ stream function. This shows that the secondary flow cannot be represented by the contours of the stream function, but must be represented by velocity vector plots of (u, v) . This is a direct consequence of using Germano’s orthogonal coordinate system, whereas Wang’s non-orthogonal system did allow a pure stream function representation of the secondary flow. The results suggest that if $\beta^{1/2} = O(1)$, torsion term is in the order of $3/2$ terms in the expansion, otherwise, it is second order, as reported by Germano (1982).

Kao (1987) then presented numerical solutions, valid for moderate Dean numbers, which show that if curvature is not small, even for values of $\beta^{1/2}$ as low as 0.1, significant distortion of the secondary flow is found, to the extent that the second vortex is confined to a very small region near the wall. This is qualitatively similar to the results reported by Wang (1981). Germano returned to the problem of helical pipes (Germano 1989), but on this occasion, used an expansion in the Dean number, rather than curvature as in the original paper (Germano 1981). The formulation is such that if there is no torsion, the original Dean equations are recovered. Investigating further the influence of torsion, Germano also examined geometries that possessed an elliptical cross-section, rather than a circle. The solutions obtained show that for an elliptical boundary, the effect of torsion on the secondary flow appears in the first-order terms of the expansion. In the special case of a circular boundary, the first-order terms disappear, and the effect is reduced to second-order.

Clearly, the literature examined so far is regarding the question of the order of the effect of torsion. The problem was finally resolved by Tuttle (Tuttle 1990). It is explained that Wang and Murata (1981) both used non-orthogonal co-ordinate systems, requiring a covariant tensor treatment to derive the Navier-Stokes equations. However, Wang focused on the contravariant velocities (which permitted a stream function definition) whereas Murata examined the covariant components. Since Germano’s co-ordinate system is orthogonal, the co- and contravariant vectors are equivalent. Tuttle shows that simple transformations relate the velocities defined in each co-ordinate system, so that Wang and Germano were both actually considering the same velocity field, but from different perspectives.

Tuttle concludes that the appropriate co-ordinate system for understanding the effect of torsion is Wang's system, whilst acknowledging that it is more convenient to work with Germano's formulation, as the co- and contravariant vectors are identical and easier to understand. Tuttle's analysis therefore supports Wang's original conclusion that the effect of torsion is first order that is of the same order as curvature.

Liu & Masliyah (Liu 1993) analyzed the Navier-Stokes equations for the flow in a loosely coiled helical pipe. Loose coiling can be achieved through a large helical radius or large pitch, specifically they assume:

$$\kappa a \rightarrow 0, \tau a \rightarrow 0, Re \rightarrow +\infty, \kappa a \neq 0$$

This reveals two dimensionless quantities that govern the flow. The first, the Dean number has already been discussed, and the second is the Germano number, Gn . Physically, Gn expresses the relative strengths of the effects of twisting, $\rho\eta U^2$, and viscosity, μU , that act on the flow, thus measuring the torsion effect. It is defined as:

$$Gn = Re\tau a$$

By considering the effect of Gn in terms of the secondary flow structure, two further dimensionless groups become significant. For large Dean numbers the dimensionless parameter γ must be large for Gn to have a noticeable effect on the flow, where γ is defined as:

$$\gamma = \frac{Gn}{Dn^{\frac{3}{2}}} = \frac{\tau a}{(\kappa a Dn)^{\frac{1}{2}}} \quad (1.29)$$

For small Dean numbers, $Dn < 20$, the equivalent group is γ^*

$$\gamma^* = \frac{Gn}{Dn^2} = \frac{\tau a}{(\kappa a)^{\frac{1}{2}} Dn} = \frac{\eta}{\kappa a Re} \quad (1.30)$$

Their results show that the secondary flow transition from two vortices to one vortex at $\gamma^* > 0.039$, when $Dn < 20$, and at $\gamma > 0.2$ for $Dn \geq 20$. Unique by amongst the studies discussed so far, Liu & Masliyah also examined the axial wall shear rate distribution, for varying torsion (keeping curvature ratio and Dean number fixed). It was shown that for small γ the shear rate is largest near the outer wall, but as γ increases, the maximum value moves to the inner upper

wall. For large γ the distribution is approximately uniform, which would be valuable in a biological context, suggesting one possible evolutionary advantage to non-planar geometries.

1.4 Flow pattern through curved and helical pipes

Although the systematic theoretical and experimental exploration of flow in curved conduits is of a fairly recent origin, it has long been appreciated that the flow is considerably more complex than that in straight conduits. The earliest observation of this was in fact for open channel flow, where the effects of curvature are most evident and striking (Thomson 1876-77]. In this century, Grindley and Gibson (1908) noticed the curvature effect on the flow through helical pipes during experiments on the viscosity of air. It was noted by Williams et al. (1902) that the location of the maximum axial velocity is shifted toward the outer wall of a curved pipe. Later, an increase in resistance to flow for the curved tube compared to the straight tube was observed by Eustice (1910) and this increase in resistance could be correlated to the curvature ratio. However, in coiling the tubes, considerable deformation occurred in the cross section of the tubes for some of the trials, causing an elliptical cross-sectional shape.

Eustice (1911) also noted that the curvature, even slight, tended to modify the critical velocity that is a common indicator of the transition from laminar to turbulent flow. By using ink injections into water owing through coiled tubes, U-tubes and elbows, He observed the pattern of the secondary flow. This secondary flow appears whenever a fluid flows in a curved pipe or channel. Motivated by these findings, Dean (1927-28) undertook an analytic study of steady, fully developed flows in curved pipes of circular cross section using regular perturbation methods. Dean simplified the governing equations (later termed the “Dean approximation”) by neglecting all effects arising due to pipe curvature except the centripetal acceleration terms. This approximation reduced the number of non-dimensional parameters from two (Reynolds number and the curvature) to a single non-dimensional parameter, the Dean number, $Dn = Re \delta^{0.5}$. The experimental work of Eustice and the mathematical formulations of Dean set the foundations for, essentially, all subsequent work in this area. Although Dean's approach was limited to the lower range of laminar flow in a torus of small

curvature, he clearly noticed these limitations and suggested approaches to extend the results to turbulent flow. Even at this early stage, Dean and Eustice had recognized several of the consequences of tube curvature on fluid flow that received more detailed attention in the following decades. Among these were the facts that there is no clear critical Reynolds number at which the demarcation between laminar and turbulent flow arises and that the onset of turbulence occurs at a substantially higher Reynolds number than in a straight pipe. White (1929) extended the work of Dean to coiled pipes and confirmed that the transition from laminar to turbulent flow depends on the coil curvature. Masliyah (1980) studied the flow field under the laminar condition in a helical semicircular duct by numerically solving the Navier-Stokes equations. With the plane wall of the duct being the outer wall, the solution of the momentum equations for $Dn < 105$ gave, for the secondary flow, twin counter-rotating vortices. However, for $Dn = 105$, two solutions were obtained. One solution was similar to that obtained for $Dn < 105$ and the other solution revealed four vortices for the secondary flow. Flow visualization confirmed the possibility of the presence of both types of secondary flow patterns. Nandakumar and Masliyah (1982) further studied this phenomena and reported that using a bipolar-toroidal coordinate system worked better for predicting the four-vortex solution. They showed that, for flow in helical tubes bifurcation exists irrespective of the shape of the tube. However, it was much easier to obtain a dual solution when the outer surface is nearly flat. Dennis and Ng (1982) performed numerical solutions of the Navier-Stokes equations for steady laminar flow through a slightly curved tubes of constant circular cross-section, using a finite difference method within the Dean number range of $96 < Dn < 5000$. For $Dn < 956$ only one solution was found for a given Dean number. The secondary flow consists of a symmetrical pair of counter-rotating vortices. While, for values of Dean numbers above 956, two solutions were obtained one of them with two vortices and the second has a four-vortex pattern. An analytical study on the flow Reynolds number flows of a magnetic fluid through a helical pipe was conducted by Verma and Ram (1993). They found that to the order considered, torsion does not affect the flow rate and also acknowledged that, owing to the non-zero curvature, the flow rate cannot be expressed in terms of Dean number alone. The unsteady flow in a pipe of circular cross-section which is coiled in a circle was studied by Lyne (1970), the pressure gradient along the pipe was varying sinusoidally in time

with frequency. It was found that the secondary flow in the interior of the pipe was in the opposite sense to that predicted for a steady pressure gradient, and this was verified qualitatively by an experiment. A parallel effort by Zalosh and Nelson's (1973) has appeared to study unsteady pulsating flows in curved tubes for fully developed laminar flows with pressure gradients that oscillated sinusoidally in time. Lyne (1970) has treated the case where the frequency parameter is large, while Zalosh and Nelson study was dealing with small, moderate and large values of the frequency parameter. Three different solutions have been presented. The results revealed that the secondary flow is composed of a steady component and a component oscillating at the second harmonic of the applied pressure gradient. The secondary-flow reversal noted by Lyne (1970) has also been found. Whalley (1980) studied experimentally air-water two-phase flow in a helically coiled tube. The flow pattern transition between stratified and annular flow was examined, and a series of measurements were then taken in the annular flow regime. Local values of the liquid thickness and liquid flow rate around the tube periphery were obtained. The variations of these values around the periphery was similar. The results showed that, for most of the cases studied the liquid flow rate was greatest on the inside of the bend, but in some results a subsidiary peak at the outside position was also obtained. There was little net entrained flow because of the centrifugal forces tending to move drops very quickly.

An experimental investigation was carried out by Anwer et al. (1989) to study the effect of bend curvature on a fully developed turbulent pipe flow. The pipe Reynolds number and the pipe-to-bend radius ratio were 50 000 and 0.077, respectively. Results showed that a Dean-type secondary motion was established in the bend and was confirmed by a numerical modeling of the flow based on a two-equation closure. Evidence of a second cell was obtained within the bend and again verified by numerical results and previous total pressure measurements. They explained that the turbulent kinetic energy was greatly enhanced by additional production due to extra strain rates created by bend curvature and that viscous dissipation was the only way in which this extra energy can be destroyed, therefore, recovery from bend curvature takes a long distance. Anwer and So (1993) conducted an experimental study of a swirling turbulent flow through a curved pipe. The measurements were used to analyze the competing effects of swirl and bend curvature on curved-pipe flows, particularly

their influence on the secondary flow pattern in the cross stream plane of the curved pipe. They found that the superimposed solid-body rotation (swirl) completely dominated the secondary flow, though they used a strong swirl and this may not be the case for weaker swirls. They also found that wall static pressure was lower on the outer bend than in the inner bend, which contradicts with the normal secondary flow caused by curvature. Boersma and Nieuwstadt (1996) used large-eddy simulation (LES) to study the flow pattern of a fully-developed turbulent flow in a curved pipe. The effect of curvature on the mean velocity profile was presented and also various turbulent statistics. They found that the secondary flow patterns consist of two strong counter rotating vortices near the inside of the bend which are driven by the centrifugal force. The results showed that the turbulence intensities were enhanced at the outside of the bend and suppressed at the inside, which was consistent with observations in a curved channel flow. Furthermore, it was shown that the turbulent Reynolds stresses were large in the core region of the pipe, in contrast to straight pipe flow where the Reynolds stresses are small in this region. A reasonable agreement with the experimental results from literature was found. Park et al. (1999) measured the axial and radial velocity fields in the plane of symmetry of a 90° curved tube using laser photo chromic velocimetry combined with a technique involving a simple interpolation and local modeling. This technique was also used to estimate the wall shear stress, the vorticity, and the pressure field. Experimental investigations on oil-air-water three-phase flow were carried out by Chen and Guo (1999) in helically coiled tubes to provide a basis for the invention and development of a new kind of separation technology for gasoil- water mixtures with low oil fraction. The flow patterns of oil-water two-phase flow and oil-air-water three-phase flow were directly observed in test sections made of plexiglas tubes. The flows observed in coiled tubes could be classified into more than four flow patterns. The results were compared with some results in horizontal flow. Correlations for the predictions of pressure drop were obtained. Huttel and Friedrich (2000 and 2001) used direct numerical simulations to study fully-developed, steady turbulent flow in straight, curved and helically coiled pipes. It was shown that pipe curvature which induces secondary flow has a strong effect on the flow pattern. Turbulence was significantly inhibited by streamline curvature and the flow almost relaminarized for large

values of the curvature. The results revealed that, the torsion effect was found to be weaker than the curvature effect, but cannot be neglected.

1.5 Influence of pitch on the flow pattern

An orthogonal coordinate system along a generic spatial curve has been introduced by Germano (1982) to study the effect of torsion and curvature on the flow in a helical pipe. The result showed that for curvatures and torsion are of the same order and for low Reynolds numbers the curvature induces a first-order effect on the flow, while the effect of the torsion on the flow is of the second order. This last result disagrees with those of Wang (1981), who, adopting a non-orthogonal coordinate system, found a first-order effect of torsion on the flow. The difference between the two studies was attributed to the difference in the coordinate system used. Further studies by Germano (1989) confirmed that the torsion had no first order effect on the flow in a helical pipe with circular cross-section. However, in the case of a helical pipe of elliptical cross section there was a first order effect of the torsion on the secondary flow. A study of the torsion effect on the fully developed laminar flow in a helical pipe of constant circular cross section was conducted by Kao (1987). The results indicated that, as far as the secondary flow patterns are concerned, the presence of torsion can produce a large effect if the ratio of the curvature to the torsion is of order unity. In these cases the secondary flow, though still consisting of a pair of vortices, can be very much distorted. Under extreme conditions one vortex was so prevalent as to squeeze the second one into a narrow region. However, generally the torsion effect was small and the secondary flow has the usual pattern of a pair of counter-rotating vortices of nearly equal strength. He also showed that, concerning the flow resistance in the pipe the effect of torsion is always small in all the circumstances that have been considered.

Xie (1990) investigated the steady, fully developed laminar flow in a helical pipe by the perturbation method using a helical coordinate system. Results showed that the torsion of the pipe has only a second-order effect, which causes the secondary vortices to turn and deform.

In some cases of low Reynolds numbers, the dividing line between the two vortices could be inclined so much that it is almost vertical. A study of the effect of pitch and torsion on the secondary flow fields for fully developed laminar flow was performed by Liu and Masliyah (1994) and showed that the critical value for the transition of two vortices to a single vortex was based on the Dean number, the normalized curvature ratio, and the normalized torsion. The pressure drop and friction factors were also studied for fully developed laminar flow. The results for the pressure drop and the friction factor were further validated by experimental results of Liu et al. (1993). Huttel and Friedrich (1999) used direct numerical simulations to study the effects of curvature and torsion on turbulent flow in helical coils. It was shown that the torsion increased the secondary flow effect and tended to change its pattern, while having negligible effects on the axial velocity. On the other hand, curvature tended to decrease the turbulent kinetic energy compared to a straight pipe flow.

1.6 Stability of the flow in a helical pipe

Fluid flow in helical pipes appears in many engineering processes particularly those involving pipe systems for transport and treatment of gases and liquids. As an initiator, Dean first studied the flow in a curved pipe in 1927 using a concentric toroidal coordinates system. Since then a lot of researches have been made theoretically, numerically, and experimentally and attracted great interest. For example, Nandakumar & Masliyah (1982) dealt with the flow in a torus and documented the flow and dimensionless radius of curvature. One of the interesting features of the flow through a toroidal pipe has been found unexpectedly by Dennis & Ng (1982) and Yanase et al. (1989), which is the appearance of dual or more solutions if the Dean number exceeds a certain critical value. The stability of flows in a torus has been studied by Yanase et al. (1989).

Yanase et al. (1989) numerically performed the dual solution of the laminar flow through a toroidal pipe with circular cross section. In the steady calculations, they obtained a four-vortex structure of secondary flow tends to an almost two-vortex structure due to the instability of toroidal pipe flow by use of the spectral method assuming that the flow field is

uniform in the axial direction of the toroidal pipe. They obtained the formula for the friction factor of a toroidal pipe, which covers the wide region from low to high Dean numbers.

Laminar flow in helical pipes has been treated less extensively. Although a helical pipe involves an additional parameter compared with the curved pipe, which naturally increases its complexity. The main reason impeding the progress of the research in a helical pipe is using a non-orthogonal coordinate system. Owing to the non-orthogonality of the coordinates, problems may arise in the analysis, numerical calculation and the interpretation of the results between two coordinate systems. To avoid the complexity associated with the non-orthogonal helical coordinate system, Germano (1982, 1989) introduced a helical coordinate system. The advantages of the orthogonal helical coordinate system are not only the simplification of the basic equations but also it makes the comparison with that of a curved pipe much clearer. Based on the helical coordinate system, Yamamoto *et al.* (1994) made some numerical analysis on the helical pipe flow over the wide range of the Dean number.

The literature examined so far has only considered laminar flow, but there are some papers that have investigated the transition to turbulence in helical pipe flow. It is known that the flow in curved pipes is far more stable than that in a straight pipe (Taylor 1929) (White 1929).

Sreenivasan and Strykowski (1982) studied the stabilization effects in flow through helically coiled pipes with negligible torsion and showed that, it is possible that the curvature in a pipe flow somehow acts as a filter that removes the most critical disturbances, or at least diminishes their amplitude, alters the frequency or both, in such a way that the remainder of the disturbances does not become unstable until after a fairly high value of the Reynolds number is attained.

Yamamoto *et al.* (1994) numerically investigated helical pipe flow by use of the spectral method assuming that the field is uniform in the axial direction of the helical pipe. In their comprehensive numerical study, vector plots of the secondary flow and axial flow distributions were shown for large Dean numbers with fairly large curvature and torsion. In the study, they found that the two-vortex structure of secondary flow in a toroidal pipe tends to almost single-vortex structure as torsion increases. They also found that if the pipe is toroidal,

the maximum axial velocity lies on the horizontal line, whereas the maximum velocity position moves away from the horizontal line if torsion increases.

Yamamoto *et al.* (1995) experimentally investigated the laminar to turbulent transition of the helical pipe flow. They found that the addition of torsion had a destabilizing effect on the laminar flow. The critical Reynolds number of the laminar to turbulent transition decreased with increasing torsion until it reached minimum. Then, it increased with further increasing torsion. The minimum critical Reynolds number was far below that of a straight pipe. More specifically, for three different dimensionless curvatures (0.01, 0.05, 0.1), increasing a torsion parameter reduces the critical Reynolds number, with a minimum occurring for β in the range 1.3–1.4. The lowest critical Reynolds number was found to be approximately 400 at $\beta = 1.4$ when $\delta = 0.1$ (note, that here, the critical Reynolds number indicates the start of transition, rather than flow is turbulent). They suggest that this behavior is directly linked with the transition from two vortices to one vortex in the secondary flow.

Yamamoto *et al.* (1998) further numerically studied, by way of the 2D linear stability analysis with the spectral method as the initial condition of the steady solution is also performed, and obtained results that were in general agreement with the experiment data of Yamamoto *et al.* (1995). However, the critical Reynolds number is found to be consistently lower in the numerical study. It is thought that this is due to the time needed for the disturbances at the inlet to grow large enough for the hot-wire to detect in the experimental configuration. By use of periodic in- and outflow boundary conditions, Huttel *et al.* (1999) obtained secondary and axial flow for a helical pipe at low torsion parameter by the finite volume method. However, they did not discuss the destabilizing effect of torsion on the helical pipe flow.

In another paper by Yamamoto *et al.* (2002) experimental and numerical results are presented that visualize the flow structures in helical pipes. Once again, increasing torsion causes a transition from a two vortex structure to a single vortex. The smoke injection results confirm their previous work on turbulent transition, as when the Reynolds number reached 900 the smoke is mixed by turbulence, and no longer provides effective visualization of the flow. Tracking particles through a numerical velocity field (obtained in Yamamoto *et al.* (2000)) gives good agreement with the experiments.

An experimental investigation regarding the laminar to turbulent flow transition in helically coiled pipes with negligible torsion was performed by Cioncolini and Santini (2006). The influence of curvature on the laminar to turbulent flow transition was analyzed from direct inspection of the experimental friction factor profiles obtained for twelve coils. They found that, coil curvature was effective in smoothing the emergence of turbulence and in increasing the value of the Reynolds number required to attain fully turbulent flow, with respect to straight pipes. The results showed that, with strongly curved coils, namely for ratios of a coil diameter to a tube diameter ranging from 6.9 to 24, the process of turbulence emergence is so gradual that only one irregular behavior were observed in the friction factor profile, actually marking the end of the turbulence emergence process. With intermediate curvature coils, namely for ratios of a coil diameter to a tube diameter ranging from 35.3 to 103.7, the process of turbulence emergence is still very gradual but the friction factor profiles exhibit a more complicated pattern a part of which was apparently not observed in the previous research. A mild curvature, namely a ratio of coil 25 diameter to a tube diameter ranging from 153 to 369, was found effective in smoothing the emergence of turbulence only in the very beginning of the emergence process.

Recently, Hayamizu *et al.* (2008) reported more accurate experimental data as for the critical Reynolds number of the laminar to turbulent transition in helical pipe flow and found good agreement with 2D spectral results of Yamamoto *et al.* (1998) for limited dimensionless curvature.

In summary, in the steady calculations, the secondary flow of helical pipes is either a pair of counter-rotating vortices with asymmetry, or a single vortex, and the axial velocity is non-axisymmetric. Since curvature has a stabilizing influence and torsion has a destabilize effect on the laminar flow in a helical pipe, the critical transition Reynolds number is reduced significantly.

1.7 Pressure drop and friction factor

The main disadvantage of using curved and helical pipes is the increase of the flow resistance due to the curvature, which mean greater pressure drop for the same volume flux as compared

to straight pipes. Pressure drop usually calculated using the friction factor, f , for which many correlation formulae have been developed.

Ito (1959) presented data for isothermal flow of water through smooth curved pipes with curvature ratios from 1/16.4 to 1/648 to determine the friction factors for turbulent flow. The flow Reynolds number ranged for $10^3 < Re < 10^5$ covering both laminar and turbulent regions. He used both the 1/7th power velocity distribution law and the logarithmic velocity distribution law to deduce resistance formulae for turbulent flow. It was found that for values of $Re \delta^2 < 0.34$ the friction factor in curved and helical pipes, f_c , was equivalent to that of a straight pipe, f_s . For $0.34 < Re \delta^2 < 300$ the resistance law is as follows:

$$f_c \delta^{-0.5} = 0.00725 + 0.076[Re \delta^2]^{-0.25} \quad (1.31)$$

However, for large values of $Re \delta^2$, the following empirical equation was presented:

$$f_c \delta^{-0.5} = \frac{0.079}{[Re \delta^2]^{0.2}} \quad (1.32)$$

Barua (1963) analyzed the motion of flow in a stationary curved pipe for large Dean numbers. The analyses assumed a non-turbulent core where fluid moved towards the outer periphery, and within a thin boundary layer fluid moved back to the inner periphery of the tube. A relationship between the friction factor of a curved tube and a straight tube was made based on Dn . This was compared to the experimental observations of other authors and was found to be in fairly good agreement. The agreement was better for higher values of Dn than for low values.

Akagawa et al. (1971) measured the frictional pressure drop of an air-water two-phase flow in helically coiled tubes with different curvatures and found that the frictional pressure drop in these coils was 1.1 to 1.5 times as much as that in a straight pipe. Kasturi and Stepanek (1972) carried out measurements of pressure drop and void fraction for the gas-liquid two-phase flow in a helical coil. The pressure drop results were fitted with the Lockhart-Martinelli correlation but there was a systematic displacement of the curves for the various systems with the Lockhart-Martinelli plot. Tarbell and Samuels (1973) solved the equations of motion and heat transfer in curved tubes numerically by the alternating direction-implicit technique. A friction factor correlation which properly accounts for the effect of curvature is developed to be valid in the range $20 < Dn < 500$. The resulting correlation is given below.

$$\frac{f_c}{f_s} = 0.25 + [2.0697 \times 10^{-4} + 1.991 \times 10^{-3} \delta] Re - 0.524 \times 10^{-7} Re^2, \quad (1.33)$$

Mishra and Gupta (1979) presented pressure drop data in the laminar and turbulent region for Newtonian fluids flowing through helical coils. Coils of wide range of diameter and pitch were investigated. The following correlations were obtained for $0.003 < \delta < 0.15$ and $0.0 < H/D_c < 25.4$:

For laminar flow:

$$\frac{f_c}{f_s} = 1.0 + 0.033[\log(Dn)]^4, \quad (1.34)$$

where $1.0 < Dn < 10^5$

Another correlation for the friction factor in laminar flow through helical and curved pipes was developed by Manalpez and Churchill (1980):

$$f_c = f_s \left[1 + \left(1 + \frac{\delta}{3} \right)^2 \frac{Dn}{88.33} \right]^{0.5}, \quad (1.35)$$

where $Dn > 40$

Grundmann (1985) published a technical note on a friction diagram for hydraulic smooth pipes and helical coils. The friction factors for the helical pipes were based on previously published friction factor correlations. Hart et al. (1988) also produced a friction factor chart for helically coiled tubes, covering both laminar and turbulent flow regions, $0 < Re < 2.0 \times 10^5$. Experiments have been performed on an adiabatic two-phase flow in a helically coiled tube by Czop et al. (1994). They found that the results were different from those calculated using the Lockhart-Martinelli correlation, but they were in a good agreement of the Chisholm correlation.

Xin et al. (1996) performed an experimental investigation and comparative study on the pressure drop and void fraction of air-water two-phase flow in vertical helicoidal pipes. Eight coils have been tested. The results showed that the pressure drop of the two-phase flow in vertical helicoidal pipes depends on both the Lockhart-Martinelli parameter and the flow rates for low flow rates in small ratio of coil diameter to pipe diameter. They reported that the geometric parameters have no apparent effects on the void fraction, but affected the

frictional pressure drop. A correlation for frictional pressure drop multiplier for small coils was provided based on the experimental data.

Sarin (1997) studied the fully developed steady laminar flow of an idealized elastico-viscous liquid through a curved tube with elliptic cross section. It was shown that if the curvature of the pipe was increased, there will be delay in setting up a secondary motion. The shear stress was found to be larger at the major axis and the outer bend than at the minor axis and the inner bend. The curvature seemed to increase the wall shear stress on the outside wall and to decrease it on the inside wall. The influence of helix axial inclination angle to the horizontal plane on the fractional pressure drop of single phase water and steam-water two-phase flows were studied in a pressure range of 3.0-3.5 MPa by Guo *et al.* (2001). Two helically coiled tubes were employed as test sections and their four different helix axial inclinations were examined. It is found that helix axial angles have little influence on the single-phase frictional pressure drop, while variation of the steam-water two-phase flow frictional pressure drop was enlarged to 70%. In a further study the same authors examined the two-phase flow of steam-water to characterize pressure drop oscillations in a closed loop steam generator system (2001).

Ali (2001) developed a correlation between pressure drop and flow rate for helical coils by using the Euler number, the Reynolds number and a new geometrical number which is a function of the equivalent diameter of the coil (taking pitch into account), the inside diameter of the tube, and length of the coiled tube. He suggested that there are four regions of flow, a laminar, a turbulent and two ranges of transitional flow.

1.8 Heat transfer of flow through curved and helical pipes

Seban and McLaughlin (1963) performed an investigation of heat transfer in coiled tubes for both laminar and turbulent flows. They used two Type 321 stainless steel tubes of 7.4 mm inside diameter and 0.3 mm wall thickness. One tube was formed into a coil with 6.5 turns and 125.2 mm diameter, the other was formed into 1.5 turns with 764.5 mm diameter. Freezene oil was used to study heat transfer in laminar flow, whereas water was used to study the turbulent range. Heat input and flow rate were varied in each case. For the oil, the Prandtl

number ranged from 100 to 657, and the Reynolds number ranged from 12 to 5600. For water, the Prandtl range was 2.9 to 5.7 and the Reynolds range was 6000 to 65600.

For laminar flow the results showed that the outer periphery had higher Nusselt numbers than the inner (ratio of roughly 4:1 for the curvature ratio of 1/17), with both being substantially higher than theoretical Nusselt numbers for a straight pipe under the same conditions. The results also indicated the effect on the Nusselt numbers due to a thermal entry length. One of the interesting results of this study was that although the local heat transfer coefficients dropped from inlet towards outlet, there was a tendency for them to rise at the last measurement position (near the outlet). This was attributed to a buoyancy effect because the increase appeared to be relatively greater at higher heat flux input. They proposed the following equation for case of laminar flow:

$$\frac{hd_p}{k}(Pr)^{-1/3} = C \left[\frac{f}{2} Re^2 \right]^{1/3}, \quad (1.36)$$

where C is a constant suggested to be 0.13 for the small coil, and 0.74 for the larger coil.

The same experimental setup and methods were used to determine the friction factor and the Nusselt number relationships for turbulent flow of water in the same coils as mentioned above. The ratios of the inside to outside periphery heat transfer coefficients were of the order of 2 and 4 for the coils with curvature ratios of 1/104 and 1/17, respectively. Seban and McLaughlin (1963) compared their average coefficients to the following relationship and found that there was no deviation greater than 10% and 15% for the coils with curvature ratios of 1/104 and 1/17:

$$\frac{hd_p}{k}(Pr)^{-0.4} = \frac{f}{2} Re \quad (1.37)$$

Data on pressure drop and heat transfer to aqueous solutions of glycerol flowing in different types of coiled pipes were presented for laminar flow by Kubair and Kuloor (1966). The Reynolds numbers was in the range of 80 to 6000 with curvature ratios in the range of 1/10.3 to 1/27 with 7 to 12 turns. A steam bath was used to provide constant temperature wall

conditions on the outside of the coil. The authors also noted that the results for the interaction between heat transfer rates and the Graetz number of Berg and Bonilla (1950) were opposite to those of Seban and McLaughlin (1963). The authors (1966) speculated that this difference might have been due to the fact that the two studies used different boundary conditions, one being constant wall temperature (1950) and the other at constant heat flux (1963). One of the coils used by Kubair and Kuloor had the same curvature ratio as one of the coils used by Seban and McLaughlin. The results of Kubair and Kuloor coincided with those of Seban and McLaughlin at low Graetz numbers, but deviated at higher Graetz numbers. An empirical correlation was developed based for curvature, δ , and the Graetz number (Gz):

$$Nu = [1.98 + 1.8 \delta] Gz^{0.7} \quad (1.38)$$

in the range of $10 < Gz < 1000$, $80 < Re < 6000$ and $20 < Pr < 100$.

The fully developed laminar convective heat transfer and fluid flow in a curved pipe with a uniform heat flux for large Dean numbers was investigated both theoretically and experimentally by Mori and Nakayama (1965).

They assumed that the flow was divided into two sections, a small boundary layer near the pipe wall, and a large core region making up the remaining flow. In the core region, the secondary flow was assumed to be completely horizontal, transporting the fluid to the outside of the pipe. The fluid returned to the inner wall through the boundary layer. This pattern differed for that of small Dean numbers. By their theoretical analysis, they deduced that the additional flow resistance of a curved pipe is caused by stress due to the secondary flow. They also developed a ratio between the thickness of the temperature and velocity boundary layers, expressed in terms of the Prandtl number. The Nusselt number ratio between straight and curved pipes was shown to be a function of the Dean number and the ratio between the thickness of the thermal and hydrodynamic boundary layers. Experiments performed with a curvature of $1/40$ were used to validate the theory. Profiles of both temperature and velocity supported the theoretical analysis. Nusselt numbers calculated from experimental data were shown to agree with those obtained theoretically. In their second report, Mori and Nakayama (1965) studied the effect of curvature on flow field and heat transfer in fully developed turbulent flow both theoretically and experimentally. Heat transfer was studied for the constant heat flux system and the variations of physical properties with temperature were not

taken into account in the development. The analysis was performed assuming a thin boundary layer along the pipe wall, similar to that in their previous report (1965).

Nusselt numbers averaged around the periphery were calculated. It was shown that the difference between the Nusselt numbers for a straight and a curved pipe decreased slightly as the Prandtl number became large, which is opposite to the results found for laminar flow [1965, 1967]. Experiments were performed with air flowing through curved pipes with curvature of 1/18.7 and 1/40. They extended their work (1967) to the theoretical analysis of uniform wall temperature rather than uniform heat flux. The results show that the formula for the Nusselt number of the uniform wall temperature is the same as that for uniform heat flux. The Nusselt number relationships developed in the previous work (1965) for laminar flow were presented in a more practical form. The change of physical properties with temperature was studied for air, water, and oil. With the exception of laminar flow of water, the mean temperature of the physical properties between the inlet and outlet could be used. The correlations proposed for Nusselt number calculations were as follows:

I-Laminar flow:

$$Nu = \frac{0.864}{\zeta} Dn^{0.5} (1 + 2.35 Dn^{-0.5}) \quad (1.39)$$

where,

$$\zeta = \begin{cases} \frac{2}{11} \left[1 + \sqrt{\left(1 + \frac{77}{4} \frac{1}{Pr^2} \right)} \right] & \text{for } Pr \geq 1, \\ \frac{1}{5} \left[2 + \sqrt{\left(\frac{10}{Pr^2} \right)} \right] & \text{for } Pr \leq 1. \end{cases} \quad (1.40)$$

II-Turbulent flow:

For gases

$$Nu = \frac{Pr}{26.2(Pr^{2/3} - 0.074)} Re^{0.8} \delta^{0.1} \left[1 + \frac{0.098}{(Re\delta^2)^{0.2}} \right], \quad (1.41)$$

where $Pr \approx 1$ and $Re\delta^2 > 0.1$, while for liquids

$$Nu = \frac{Pr Re^{5/6} \delta^{1/2}}{41.0} \left[1 + \frac{0.061}{(Re\delta^{2.5})^{1/6}} \right], \quad (1.42)$$

where, $Pr > 1$ and $Re\delta^{2.5} > 0.4$.

Akiyama and Cheng (1971) studied the hydrodynamically and thermally fully developed laminar forced convection in curved pipes subjected to the thermal boundary conditions of axially uniform wall heat flux and peripherally uniform wall temperature at any axial position. The Prandtl number effect on the heat transfer result was clarified, and it was shown that all the heat transfer results for $Pr \geq 1.0$ can be correlated by a single curve using a new parameter with reasonable accuracy. The results showed that as the Dean number increased, the center of the secondary circulation moved towards the outer wall. However, in the horizontal direction, it first begins to move towards the outer wall, but with further increase of the Dean number, the center begins to move back towards the inner wall. From the streamlines produced at the Dean number of 196, They suggested that the boundary layer approximation that has been used by other researchers [Barua (1963), Mori and Nakayama, 1965] could not be applied. Akiyama and Cheng (1972) extended their work to include the thermal boundary condition of uniform wall temperature. It has been shown that the heat transfer results for fully developed laminar flow in curved pipes with the two basic thermal boundary conditions of uniform wall temperature and uniform wall heat flux are quite similar. For given Prandtl and Dean numbers, the Nusselt number for the uniform wall temperature case became higher than that for the uniform wall heat flux case after reaching a certain Dean number. This situation is opposite to that of a pure forced convection in a straight pipe.

Kalb and Seader (1972) studied theoretically the heat transfer to steady viscous flow in curved pipe of circular cross section for fully developed velocity and temperature fields under the thermal boundary condition of axially uniform wall heat flux with peripherally uniform wall temperature. They found that the curvature parameter had negligible effect on the average Nusselt number for any Prandtl number. However, the curvature ratio did have an effect on the peripheral variation of the Nusselt number, though this effect was still not large. For $Pr > 0.7$, it was shown that the local Nusselt number in the area of the inner wall was always less than that of a straight tube, and increasing as the Dean number increased till it reached a limiting value. The local Nusselt numbers on the outer wall continued to increase with increasing Dean number. They also noted that the fractional increase in the heat transfer coefficient is significantly greater than the fractional increase in friction losses, except for

liquid metals. The following Nusselt number relationship was developed for low Prandtl numbers (0.005 to 0.05) with Dean numbers in the range of 20 to 1200:

$$Nu = (3.31Dn^{0.115})Pr^{0.0108} \quad (1.43)$$

For Prandtl numbers in the range of 0.7 to 5 and Dean numbers of 80 to 1200, the following average Nusselt number relationship was developed:

$$Nu = (0.913Dn^{0.476})Pr^{0.20} \quad (1.44)$$

Kalb and Seader (1974) advanced this work by applying the method to the case of a uniform wall-temperature. Their results showed that there exists a slight effect of curvature on the peripheral variation of the Nusselt number. However, it did not affect the average Nusselt number. As Akiyama and Cheng (1971) did, Kalb and Seader (1974) found that the thermal boundary condition had a significant effect on the temperature field.

Austin and Seader (1973) numerically solved the equations of fluid motion for the case of steady, fully-developed, isothermal, incompressible, viscous Newtonian flow in toroidal-type coiled-tube geometry. However, the effect of coil pitch was not taken into consideration. They speculated that the pitch effect might become important as the radius of curvature approaches the radius of the tube. For low Dean numbers, the authors showed that the axial velocity profile was essentially parabolic as in the straight tube case. As the Dean number is increased, the maximum velocity began to be skewed toward the outer periphery. It was shown that for $Dn > 10.0$, the maximum axial velocity on the horizontal axis tends to move towards the outer periphery. Also they indicated that the pressure distribution is dependent mainly on the Dean number and that the pressure increases smoothly from the inner to the outer periphery. The axial pressure gradient was correlated to the friction factor, which showed a slight dependence on the curvature ratio. The numerical results of the secondary flow the center of the secondary flow vortex was close to the center of the tube horizontally and situated slightly closer to the middle than the upper tube wall in the vertical direction. The streamlines around the vortex were rather symmetrical. However, when the Dean number was increased, the center of the vortex was seen to move upwards and inwards and the streamlines lost their symmetrical form. Changing the curvature ratio had a slight effect on the secondary flow streamlines for the same Dean number, indicating that the effect of

curvature ratio was almost completely accounted for in the Dean number. The numerical results of Austin and Seader (1973) differed significantly from the model of Mori and Nakayama (1965).

Patankar et al. (1974) investigated the three dimensional parabolic flows in a helical pipe while they predict the velocity and temperature fields in helically coiled pipes. Predictions were presented of flow and heat transfer in the developing and fully developed regions. Comparisons of the developing and fully developed velocity profiles with experimental data of Mori and Nakayama (1965) exhibited good agreement. The development of the wall temperature for the case of axially uniform heat flux with an isothermal periphery has been compared with experimental data and the agreement was also good. Predictions of fully developed temperature profiles and heat transfer coefficients also exhibited close agreement with experimental data of Mori and Nakayama (1965). Effects of the Dean number on the friction factor and heat transfer were presented.

Yao and Berger (1978) conducted an analytical study of the fully developed laminar flow in a heated curved pipe under the influence of both centrifugal and buoyancy forces. The pipe was assumed to be heated so as to maintain a constant axial temperature gradient. Both horizontal and vertical pipes were considered. Solutions for these two cases were obtained by regular perturbations in the Dean number and the product of the Reynolds and the Rayleigh numbers; the solutions were therefore limited to small values of these parameters. Predictions of the axial and secondary flow velocities, streamlines, shear stress, temperature distribution and heat transfer were given for a representative case.

The effect of buoyancy on the flow field and heat transfer was studied numerically by Lee et al. (1985) for the case of fully developed laminar flow and axially constant heat flux with a peripherally constant wall temperature. They found that the buoyancy acts to increase the average Nusselt number, change the local Nusselt number distribution around the periphery and rotate the orientation of the secondary flow velocity. Heat transfer coefficients of helical coils in agitated vessels have been investigated by Havas et al. (1987). A modified Reynolds number has been introduced into the heat transfer equation and a correlation was developed for the Nusselt number on the outer wall based on the modified Reynolds number, the Prandtl

number, and the ratio of the diameter of the tube to the diameter of the vessel. In developing this correlation, the calculations of the heat transfer coefficient on the outside of the pipe was done by taking the total heat flux, temperature difference, and the internal heat transfer coefficient.

Austen and Soliman (1988) studied the influence of pitch on the pressure drop and heat transfer characteristics of helical coils for the condition of uniform input heat flux. The results included the isothermal and adiabatic friction factors, wall temperature, and local and fully developed Nusselt numbers. Significant pitch effects were observed in the friction factor and Nusselt number results at low Reynolds numbers. These effects were attributed to free convection, and they diminished as the Reynolds number increased.

Yang et al. (1995) presented a numerical model to study the fully developed laminar convective heat transfer in a helicoidal pipe having a finite pitch. The effects of the Dean number, torsion, and the Prandtl number on the laminar convective heat transfer were discussed. The helicoidal pipe was assumed to have uniform wall heat flux with a uniform peripheral wall temperature. The studied laminar flow of the incompressible Newtonian fluid was subjected to be hydrodynamically and thermally fully developed. The results revealed that the temperature gradient increased on one side of the pipe wall and decreased on the other side with increasing torsion. In the case of a fluid with a large Prandtl number, the Nusselt number significantly decreased as torsion increased, on the other hand, for a small Prandtl number, the Nusselt number decreased slightly as the torsion increased.

Yang and Ebadian (1996) extended this research to turbulent flow using $k - \varepsilon$ model. As in the laminar case, the torsion rotated and distorted the temperature profiles. The effects of torsion could be enhanced by increasing the axial flow velocity.

Xin and Ebadian (1997) carried out experiments to investigate the effects of the Prandtl number and geometric parameters on the local and averaged convective heat transfer characteristics in helical pipes, with three different fluids air, water, and ethylene glycol on five uniformly heated helical pipes. The ratios of the pipe diameter and pitch to coil diameter ranged from 0.0267 to 0.0884 and 0.20 to 2.56, respectively. The peripheral and average, fully developed Nusselt numbers were evaluated in the experiments. Experimental findings indicated that after two turns the temperature distributions all dimensionless peripheral wall

temperatures are nearly identical, implying that both the flow and temperature distribution within the helical pipes are fully developed. These results revealed that the peripheral Nusselt number varies significantly for higher Prandtl numbers and Dean numbers in the laminar flow region. A new set of empirical expressions established for the averaged fully developed Nusselt number. No obvious effects of the coil pitch or torsion were observed in the scope of the investigation.

Numerical studies were made of the flow and heat transfer characteristics of a fully-developed pulsating flow in a strongly curved pipe by Chung and Hyun (1994). By using a toroidal coordinate system, the complete, time-dependent incompressible Navier Stokes equations were formulated. The peripherally uniform temperature condition was imposed on the pipe wall. Particular attention is given to heat transfer properties over substantially extended parameter ranges of the Reynolds number and the Womersley number (ratio of transient inertial to viscous forces), which is a function of the pipe radius, the kinematic viscosity, and the frequency of the pulsation. The computed results on the flow field were in close agreement with the experimental data in the covering parameter ranges. The numerical results showed that when Womersley number was small, the time and space averaged Nusselt numbers, were lower for a pulsating flow than for a corresponding non-pulsating flow. At moderate and high Womersley numbers, however, the difference in between a pulsating and a non-pulsating flow was insignificant.

A further work on pulsating flow was performed by Guo et al (1998) for fully developed turbulent flow in a uniformly heated helical coiled tube. In their work they studied both pulsating flow and steady state flow. A series of correlations were proposed for the average and local heat transfer coefficients both under steady and oscillatory flow conditions. They noted that as the Reynolds number increased to very large values ($Re > 140000$), the heat transfer coefficient for coils began to match the heat transfer coefficient for a straight pipe. Correlation of the Nusselt number as a function of the Womersley number were also given. Goering et al. (1997) used a two-dimensional numerical representation of the fully-elliptic Navier-Stokes and energy equations to study the fully developed laminar flow through a heated horizontal curved tube. The combined effects of curvature and buoyancy were presented. Two thermal boundary conditions were used, both were with constant heat fluxes,

but one was with a constant peripheral heat flux while the other had a constant peripheral wall temperature. Regime maps were presented for both boundary conditions which split the heat transfer into three regimes; curvature dominant, buoyancy dominant, and an area where both factors were important.

Bai et al. (1999) performed an experimental study on the turbulent heat transfer from horizontal helically coiled pipe over a wide range of experimental parameters. They found that as the Reynolds number increased, the effect of secondary flow on heat transfer decreased, and the heat transfer approaches that of a straight pipe. This is due to the fact that as the Reynolds number increases the boundary layer becomes smaller. They also found that the local heat transfer coefficients on the outside wall was three or four times a than that on the inside wall. A correlation was proposed to describe the local distribution of the of the Nusselt number in the periphery.

Comparisons of the heat transfer coefficients between a straight pipe and helically coiled pipe immersed in a water bath were performed by Prabhanjan et al. (2002). The purpose of this study was to determine the relative advantage of using a helically coiled heat exchanger versus a straight tube heat exchanger for heating liquids. Results showed that the heat transfer coefficient was affected by the geometry of the heat exchanger and the temperature of the water bath surrounding the heat exchanger. All tests were performed in the transitional and turbulent regimes.

Inagaki et al. (1998) studied the thermal hydraulic characteristics of the gas to gas heat exchanger with helically coiled tube bundles for Reynolds numbers in the range of 6000 to 22000. They developed a correlation for the Nusselt number on the outside wall as a function of both the Reynolds and the Prandtl numbers.

$$Nu = (0.78Re^{0.51})Pr^{0.30} \quad (1.45)$$

A numerical study of the developing convective heat transfer in a coiled heat exchanger with the effect of buoyancy was performed by Sillekens et al. (1998). Results showed that heat transfer was highly influenced by the secondary flow and buoyancy forces. For low Grashof numbers a splitting phenomenon of the temperature field was observed due to large secondary velocities, resulting in two separated areas of fluid. However, for higher values of

the Grashof number, the fluid became almost linearly stratified which resulted in small secondary velocities.

Zheng et al. (2000) studied the interaction phenomena between laminar forced convection and thermal radiation in a participating medium inside a helical pipe numerically. The results showed that under the conditions examined in this study, although the thermal radiation had no significant influence on flow and temperature fields, it substantially enhanced the total heat transfer in the helical pipe especially in a fully developed region.

Prabhanjan et al. (2004) conducted heat transfer studies of a helical coil immersed in a water bath. A method to predict outlet temperatures from the helical coil was proposed which took into consideration the flow rates and geometry of the coil. The heat transfer on the outside of the coils was caused by natural convection. In this type of system, neither constant wall temperature, nor constant wall heat flux could be assumed.

Simultaneous development of laminar Newtonian flow and heat transfer in helical pipes was numerically studied by Liu and Masliyah (1994). The governing equations were fully parabolized in the axial direction and were written in an orthogonal helical coordinate system. The Nusselt number in the developing region was found to be oscillatory. The asymptotic Nusselt number and the thermal entrance length are correlated with the Prandtl number and the Dean number. They showed that when torsion was dominant, the asymptotic Nusselt number decreased while the thermal development length increased.

Developing laminar forced flow and heat transfer in the entrance region of helical pipes have been investigated using a fully elliptic numerical method by Lin and Ebadian (1997b). The results presented in this work covered a Reynolds number range of 250 to 2000, a pitch range of 0.0 to 0.6, and a curvature range of 0.025 to 0.20. It has been found that both the friction factor and the Nusselt number were oscillatory in the entrance region of helical pipes. These oscillations were higher for lower curvature. The pitch and the Reynolds number exerted different effects on the developments of the friction factor and the Nusselt number than the curvature ratio. The same effects were numerically studied by Lin and Ebadian (1997b) for the case of developing turbulent flow and observed the same oscillatory behavior of both the Nusselt number and friction factor. However, they found that as the pitch, curvature or the Reynolds was increased, these oscillations became stronger. Lin and Ebadian (1999)

numerically studied the effect of inlet turbulence intensity on the development of the turbulent flow and heat transfer in helically coiled pipes. Their findings were that increased inlet turbulence intensity tended to reduce the velocity gradient at the walls of the pipe although it had negligible effect on the maximum axial velocity. Increasing the intensity level also increased the intensity of the secondary flow but did not change its pattern. The thermal boundary layer developed quicker with increased inlet turbulence intensity.

A fully elliptic numerical study was performed to investigate the buoyancy affected, three-dimensional turbulent flow and heat transfer in the entrance region of a curved pipe by Li et al. (1998). It was found that, at higher Grashof numbers, the developing secondary flow field showed the existence of three vortices. The peripherally averaged Nusselt number and the friction factor exhibited oscillatory behavior along in streamwise direction. The augmentation of the average Nusselt number and the friction factor resulting from buoyancy was prominent at the entrance region of the pipe, but gradually became weaker further downstream.

Rindt et al. (1999) conducted a numerical study on the development of mixed convective flow in a helically coiled heat exchanger with an axially varying wall temperature for Reynolds number of 500, the Prandtl number of 5 and curvature of 1/14 and compared to the constant wall temperature boundary condition. The influence of buoyancy forces on heat transfer and secondary flow was presented. It was found that, for all Grashof numbers studied it appeared that both heat transfer, quantified by the Nusselt number, and secondary flow, quantified by the relative kinetic energy, and exhibited a wavy behavior in axial direction. For higher Grashof numbers, however, this phenomenon diminished for the case with an axially varying wall temperature due to stabilizing effects of stratification.

Chapter 2

2.1 Mathematical formulation and governing equations

The governing equations described below are the Navier-Stokes equations and heat conduction equations. OpenFOAM reduces those PDE's into a set of linear algebraic equations and solves them. In order to validate a CFD code, computational results must be compared with some prepossessed knowledge (i.e., previous experimental data or numerical result by spectral study). In the case that steady and transient solutions of helical circular pipes can be derived, a foundation for validation is provided.

Any point of a helical pipe with circular cross section can be described by the coordinate system (x, y, z) , as shown in Fig. 2. The cross section of the helical pipe is a circle with the internal diameter $d_p (= 2a)$, and H is one pitch length of the helical pipe. The unit tangential vector, $\mathbf{t}$, the unit normal vector, $\mathbf{n}$, and the unit binormal vector, $\mathbf{b}$, of the center line of the pipe are defined, where $\mathbf{b} = \mathbf{t} \times \mathbf{n}$ (see Fig. 1(b)). The x-axis is to the direction of $-\mathbf{n}$, the y-axis to the direction of $-\mathbf{b}$ and the z-axis is a taken along the center line of the pipe. The x , y and z coordinates constitute the orthogonal coordinate system. The flow is driven by constant pressure gradient along the center line of the pipe and the main flow direction is to the positive z . A thick broken line denotes the center line of the helical pipe and R is the distance of the center line from the axis of the whole system.

Then the continuity equation, the Navier-Stokes equations and heat conduction equation can be written as:

$$\nabla \cdot \mathbf{U} = 0, \quad (2.1)$$

$$\frac{\partial \mathbf{U}}{\partial t} + (\mathbf{U} \cdot \nabla) \mathbf{U} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{U}, \quad (2.2)$$

$$\frac{\partial T}{\partial t} + (\mathbf{U} \cdot \nabla) T = \alpha \nabla^2 T. \quad (2.3)$$

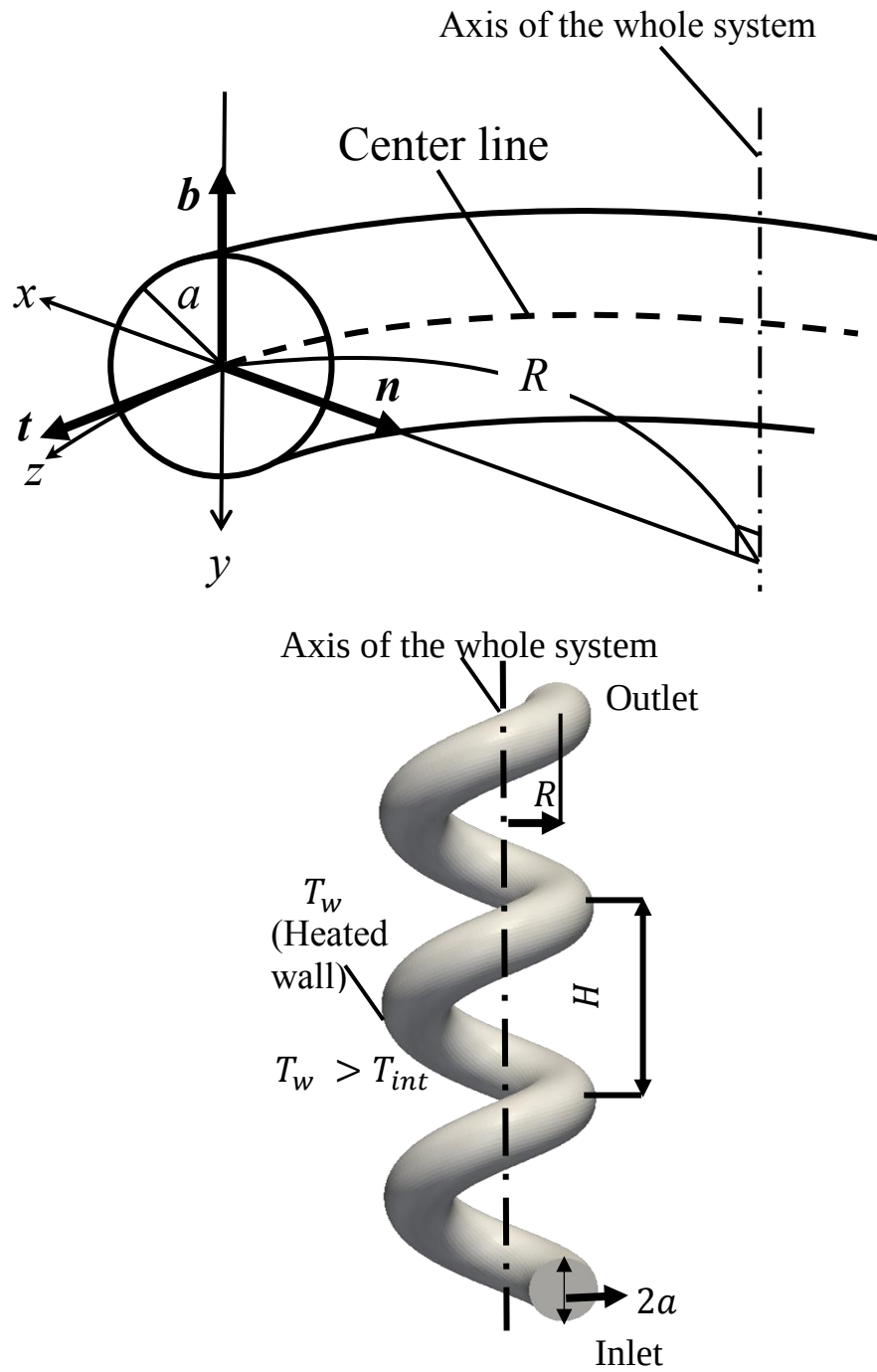


Fig. 2.1. (a) Helical pipe with circular cross section, (b) coordinate system.

The aforementioned numerical solutions can be found by reducing the governing equations under these assumptions:

1. The flow is incompressible Newtonian fluid.
2. The pipe has a rigid, symmetric, circular cross-section.
3. Convective heat transfer affected for heat transfer case.
4. The time derivative is zero for steady flow.
5. No slip condition is applied to the velocity of a helical pipe wall.
6. A wall of the helical pipe is uniformly heated.
7. Wall temperature of the helical pipe always greater than the inlet temperature.

For fully-developed flow, the derivative in the z direction is zero.

$$\frac{\partial}{\partial z} = 0$$

Here, $\mathbf{U} = (u, v, w)$ is the velocity vector, where u, v, w are the velocity of the x, y and z components, respectively, t the time, P the pressure, $\nu (= \mu/\rho)$ the kinematic viscosity, ρ the density, T the temperature and α the thermal diffusivity of the fluid.

The radius of curvature of the center line of the helical pipe $R_h = R [1 + (H/2\pi R)^2]$. Then, the non-dimensional curvature of the pipe $\delta = a/R_h$, the non-dimensional torsion $\tau = a(H/2\pi)/R_h^2$ and the torsion parameter $\beta = \tau/\sqrt{2\delta}$ as introduced by Kao [10].

The non-dimensional variables are defined by

$$u = (a/\nu)u', \quad v = (a/\nu)v', \quad w = (a/\nu)\sqrt{2\delta}w'$$

$$p = (a/\nu)^2 p'/\rho$$

The non-dimensional parameters Re , the Reynolds number, Dn , the Dean number and Pr , the Prandtl number is given by

$$Re = \frac{d_p \bar{w}}{\nu}, \quad Dn = Ga^3 \sqrt{2\delta}/\mu\nu, \quad Pr = \frac{\nu}{\alpha},$$

where $\bar{w}$ is the mean axial velocity, G the mean pressure gradient along the center line of the helical pipe, $\mu (= \rho\nu)$ the viscosity, and α the thermal diffusivity of the fluid.

2.2 Turbulence Models

The motion of a fluid is governed by the Navier Stokes equations. In the turbulent regime, the solution to these equations is chaotic, three-dimensional and unsteady. Such solutions are not easily solved, even by massively parallel supercomputers. A much simpler level of description is required: this calls for a statistical approach. As it will be seen in the following, there are no closed equations for the statistics of turbulent flow, which means that there is a larger number of unknowns than equations.

Equation (2.1) expresses the incompressibility of fluid volumes, which is equivalent to mass conservation in the present case. Equation (2.2) and (2.3) expresses the conservation of momentum and energy equation.

The total instantaneous velocity is denoted by $\mathbf{U}$ and it can be decomposed into the mean velocity $\bar{\mathbf{u}}$ and the fluctuating velocity $\mathbf{u}'$ as $\mathbf{U} = \bar{\mathbf{u}} + \mathbf{u}'$, where the overbar denotes the ensemble or time average. It should be remarked that the time-average is generally used in experiments while the ensemble-average is implied theoretically if a turbulence model is utilized. The pressure p is decomposed as $p = \bar{p} + p'$ in the same way as the velocity. Substitution of the velocity and the pressure into Eqs. (2.1) and (2.2) gives

$$\nabla \cdot \bar{\mathbf{u}} = 0, \quad (2.4)$$

$$\frac{\partial \bar{\mathbf{u}}}{\partial t} + (\bar{\mathbf{u}} \cdot \nabla) \bar{\mathbf{u}} = -\frac{1}{\rho} \nabla \bar{p} + \nu \nabla^2 \bar{\mathbf{u}} + \nabla \cdot \mathbf{R}_s, \quad (2.5)$$

where $\mathbf{R}_s$ is the Reynold stress. Equations (3) and (4) are the Reynolds-averaged Navier-Stokes (RANS) equations. In the tensor notation, $\mathbf{R}_s = (\overline{u'_i u'_j})$ is the Reynolds stress tensor. The Reynolds number, Re , is defined by

$$Re = \frac{2a \langle \bar{w} \rangle}{\nu}, \quad (2.6)$$

where $\langle \bar{w} \rangle$ is the mean axial velocity averaged over the cross section and $\mu = \rho \nu$ is the viscosity.

The turbulent viscosity, ν_t , is also known as the eddy viscosity of the turbulent flow appearing in Equations (2.4) - (2.5), which depends on the mean flow rate of the deformation.

Time averaging is carried out to the above governing equations (2.4) - (2.5) to obtain a Reynolds averaged Navier-Stokes (RANS) equations, which are solved by using different turbulence models as described later. Turbulent flow can be numerically solved by large computers, although it is expensive.

RANS models are formulated to predict the effect of turbulence on the mean field. Since there is no single turbulence model that can predict all turbulent flows with sufficient accuracy, we used three turbulent eddy viscosity models to calculate the mean flows to compare with the experimental data. In the present study, brief discussions of three different $k-\varepsilon$ models such as Launder-Sharma $k-\varepsilon$, Lien-Cubic $k-\varepsilon$ and RNG $k-\varepsilon$ are given to estimate critical the critical Reynolds number of transition to fully developed turbulent flow.

2.2.1 RNG $k-\varepsilon$ model

The renormalization group (RNG) was derived by Yakhot et al. (1992) with an extensive analysis of the eddy viscosity model. Compared with the standard $k-\varepsilon$ model, the RNG model possesses an additional factor in the turbulent dissipation energy, ε , equation, which leads to improving the accuracy of the flow field. In addition, the RNG $k-\varepsilon$ model is appropriate for a low Reynolds number flow as well, while the standard model was derived at a high Reynolds number flow.

The RNG - model has shown improved results for high streamline curvature and strain rate, transitional flows and wall heat and mass transfer, but shows no improvement over the standard model for predicting vortex evolution. The RNG - model relaxes the assumption of isotropy inherent in the standard - model.

The RNG $k-\varepsilon$ model has a similar form of the kinetic energy k and dissipation energy ε equations as in the standard model but with some modifications shown in below equations.

The turbulent kinetic energy, k , and its rate of dissipation, ε , are

$$\frac{\partial k}{\partial t} + \frac{\partial}{\partial r_j} (ku_i) = \frac{\partial}{\partial r_j} \left\{ \left(\nu + \frac{\nu_t}{\sigma_k} \right) \frac{\partial k}{\partial r_j} \right\} + P_k - \varepsilon \quad (2.7)$$

$$\frac{\partial}{\partial t} (\rho \varepsilon) + \frac{\partial}{\partial r_j} (\rho \varepsilon u_i) = \frac{\partial}{\partial r_j} \left\{ \left(\nu + \frac{\nu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial r_j} \right\} + c_{\varepsilon 1} \frac{\varepsilon}{k} P_k - c_{\varepsilon 2} \frac{\varepsilon^2}{k} \quad (2.8)$$

The turbulent viscosity ν_t of turbulent flow is computed by combining ε and k as describe below with $C_\mu = 0.0845$

$$\nu_t = C_\mu \frac{k^2}{\varepsilon}$$

The constants of the RNG model (Yakhot and Orszag (1986);Yakhot et al. (1992)) are: $C_\mu = 0.0845$, $c_{\varepsilon 1} = 1.42$, $c_{\varepsilon 2} = 1.68$, $\sigma_k = 0.7194$, $\sigma_\varepsilon = 0.7194$.

2.2.2 Lien-Cubic k - ε model

In the present investigation a number of low-Reynolds number k - ε turbulence models were examined. It was, however, decided to present only the results for the low Reynolds number Lien Cubic k - ε turbulence model (Lien et al. (1996)). Being a low- Re k - ε model, it uses damping functions for resolving the boundary layer, unlike the high-Reynolds number models, which use a wall function instead. In addition, the used k - ε model is an eddy viscosity model. These models are known to be unable of accounting for the non-isotropic nature of the turbulence. Therefore, for their improvement, series expansions of functional terms can be added to the expression for the linear eddy viscosity model. By this approach nonlinear eddy viscosity models are created. Several functional terms, known as quadratic terms, cubic terms and so forth, can be added, based on the desired theoretical accuracy. Consequently, the general expression for the Reynolds stresses is expressed as below.

$$\begin{aligned} \overline{u_i u_j} = & \frac{2}{3} k \delta_{ij} - 2\mu_t S_{ij} + 4C_1 \mu_t \beta^* \left(S_{ik} S_{kj} - \frac{1}{3} S_{kl} S_{kl} \delta_{ij} \right) \\ & + 4C_2 \mu_t \beta^* (W_{ik} S_{kj} + W_{jk} S_{ki}) + 4C_3 \mu_t \beta^* \left(W_{ik} W_{jk} - \frac{1}{3} W_{kl} W_{kl} \delta_{ij} \right) \\ & + 8C_4 \mu_t \beta^{*2} (W_{lj} S_{ki} + W_{li} S_{kj}) S_{kl} + 8C_5 \mu_t \beta^{*2} (S_{kl} S_{kl} - W_{kl} W_{kl}) S_{ij}, \end{aligned} \quad (2.9)$$

where S_{ij} and W_{ij} are strain and vorticity rates, respectively, and $\beta^* = k/\varepsilon$ is the turbulent dissipation time scale. As the explanation above, in addition to the linear expression which appears in the first line in Eq. (2.9), quadratic and cubic products of S_{ij} and W_{ij} appear in the second and third line, respectively. By using only the first line, or the first two lines or the entire expression, the linear, quadratic or cubic versions of the eddy viscosity model are

expressed, respectively. In the preset work, where the cubic version is used, all the three lines are employed.

The turbulent kinetic energy, k , and its rate of dissipation, ε , are

$$\frac{\partial}{\partial t}(k) + \frac{\partial}{\partial r_j}(ku_i) = \frac{\partial}{\partial r_j} \left\{ \left(\nu + \frac{\nu_t}{\sigma_k} \right) \frac{\partial k}{\partial r_j} \right\} + \nu_t P_k - \varepsilon, \quad (2.10)$$

$$\frac{\partial}{\partial t}(\varepsilon) + \frac{\partial}{\partial r_j}(\varepsilon u_i) = \frac{\partial}{\partial r_j} \left\{ \left(\nu + \frac{\nu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial r_j} \right\} + c_{\varepsilon 1} \frac{\varepsilon}{k} \nu_t P_k - c_{\varepsilon 2} \frac{\varepsilon^2}{k}, \quad (2.11)$$

where $P_k = -\overline{u_i u_j} \frac{\partial u_i}{\partial r_j}$ and the constants of the Lien-cubic model are: $C_\mu = 0.0845$, $c_{\varepsilon 1} =$

1.44, $c_{\varepsilon 2} = 1.92$, $\sigma_k = 1.0$, $\sigma_\varepsilon = 1.22$.

The main reason for using the cubic model here is its capability of accounting for non-isotropy of turbulence. Because the low-Reynolds number version is used, it is expected to give a detailed prediction in the boundary layer region. The cubic model has the property of being able to account for the anisotropy of turbulence. It means that it is capable of predicting, for instance, the stress-induced secondary motions. The cubic models are also expected to perform better than quadratic and linear models in predicting curved flows.

2.2.3 Launder-Sharma $k - \varepsilon$ model

LaunderSharma $k - \varepsilon$ (1974) model is based on an eddy viscosity model which assumes that the Reynolds stresses in the Navier stokes equation are proportional to the mean velocity gradient with the coefficient of constant turbulent viscosity. The $k - \varepsilon$ model gives reasonably good results for free-shear-layer flow with relatively small pressure gradient. For wall bounded flow, the model gives good agreement with experimental results for zero and small mean pressure gradients, but is less accurate for large adverse pressure gradient.

The turbulent kinetic energy, k , and its rate of dissipation, ε , are

$$\frac{\partial k}{\partial t} + \frac{\partial}{\partial r_j}(ku_i) = \frac{\partial}{\partial r_j} \left\{ \left(\nu + \frac{\nu_t}{\sigma_k} \right) \frac{\partial k}{\partial r_j} \right\} + P_k - \varepsilon, \quad (2.12)$$

$$\frac{\partial \varepsilon}{\partial t} + \frac{\partial}{\partial r_j}(\varepsilon u_i) = \frac{\partial}{\partial r_j} \left\{ \left(\nu + \frac{\nu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial r_j} \right\} + c_{\varepsilon 1} \frac{\varepsilon}{k} P_k - c_{\varepsilon 2} \frac{\varepsilon^2}{k} (1 - 0.3e^{(-R_T^2)}), \quad (2.13)$$

where $R_T = \frac{k^2}{\nu \left(\varepsilon - 2\nu \left(\frac{\partial \sqrt{k}}{\partial r_j} \right)^2 \right)}$ and the constants of the Launder-Sharma $k-\varepsilon$ model are: $C_\mu =$

$$0.09, c_{\varepsilon 1} = 1.44, c_{\varepsilon 2} = 0.92, \sigma_k = 1.0, \sigma_\varepsilon = 1.22$$

LaunderSharma $k - \varepsilon$ model predicts high accurately for large pressure gradient and insensitive to freestream values of the turbulence. This model requires explicit wall-damping functions and the use of fine grid spacing near solid walls. The main disadvantage of this model is the assumptions that the Reynolds stresses are isotropic especially for free shear flows. This model sometimes referred in this study as the standard $k - \varepsilon$ model.

2.3 Boundary conditions

As the inflow boundary condition, we imposed a uniform axial flow determined by the Reynolds number for the velocity and the Neumann condition for the pressure. Zero gradient for the velocity and a fixed value (atmospheric pressure) for the pressure were imposed as outflow boundary conditions. As for the temperature boundary conditions, a uniform heated temperature T_w is assumed on the pipe wall. Uniform axial flow at temperature T_0 enters into the helical pipe at the inlet and zero gradient was imposed for temperature at the outlet. The physical properties of working fluid depends on the bulk temperature given at each axial position. For unsteady solutions, we imposed zero gradient for the pressure and a uniform value determined by the Reynolds number for the velocity as an inflow boundary condition. No slip boundary condition was imposed on the wall for steady and unsteady case. Note that for Dean number calculation, we change the inflow boundary condition for steady calculation, we imposed a fixed value for the pressure and zero gradient for the velocity as an inflow boundary condition. Zero gradient for the velocity was imposed as an outflow boundary condition.

Chapter 3

3.1 Numerical Methods

3.2 Finite Volume Discretization

The purpose of any discretization method is to transform one or more partial differential equations into a corresponding system of algebraic equations. The solution of this system produces a set of values which correspond to the solution of the original equations at some pre-determined locations in space and time, provided certain conditions, to be defined later, and are satisfied. The discretization process can be divided into two steps: the discretization of the solution domain and equation discretization.

The discretization of the solution domain produces a numerical description of the computational domain, including the positions of points in which the solution is sought and the description of the boundary. The space is divided into a finite number of discrete regions, called control volumes or cells. For transient simulations, the time interval is also split into a finite number of time-steps. Equation discretization gives an appropriate transformation of terms of governing equations into algebraic expressions.

This Chapter presents the Finite Volume method (FVM) of discretization, with the following properties:

- The method is based on discretizing the integral form of governing equations over each control volume. The basic quantities, such as mass and momentum, will therefore be conserved at the discrete level.
- Equations are solved in the fixed Cartesian coordinate system on the mesh that does not change in time. The method is applicable to both steady-state and transient calculations.
- The control volumes can be of a general polyhedral shape, with a variable number of neighbors, thus creating an arbitrarily unstructured mesh. All dependent variables share the same control volumes, which is usually called the colocated or non-staggered variable arrangement (Rhie and Chow (1982), Peric (1985)).

- Systems of partial differential equations are treated in the different terminology way (Patankar and Spalding (1972)), meaning that they are solved one at a time, with the inter-equation coupling treated in the explicit manner. Non-linear differential equations are linearized before the discretization and the non-linear terms are lagged.

The details of the discretization practice and implementation of boundary conditions will be described in the remainder of this Chapter. In Section 3.2 the mesh arrangement is described. Section 3.3 presents a discretization procedure for a scalar transport equation. The discretization of the convection, diffusion and source terms is presented in section 3.3.1.1-3.3.1.4. Several possibilities for the discretization of the temporal derivative are given in Section 3.4. A new second order accurate and bounded convection differencing scheme and some details of the solution procedure, including the treatment of mesh non-orthogonality, deferred correction and the under-relaxation are given in Section 3.5. The solution procedure for systems of partial differential equations and the generalized segregated approach for pressure-velocity coupling is described in Section 3.6-3.7.

3.2.1 Discretization of the Solution Domain

Discretization of the solution domain produces a computational mesh on which the governing equations are subsequently solved. It also determines the positions of points in space and time where the solution is sought. The procedure can be split into two parts: discretization of time and space.

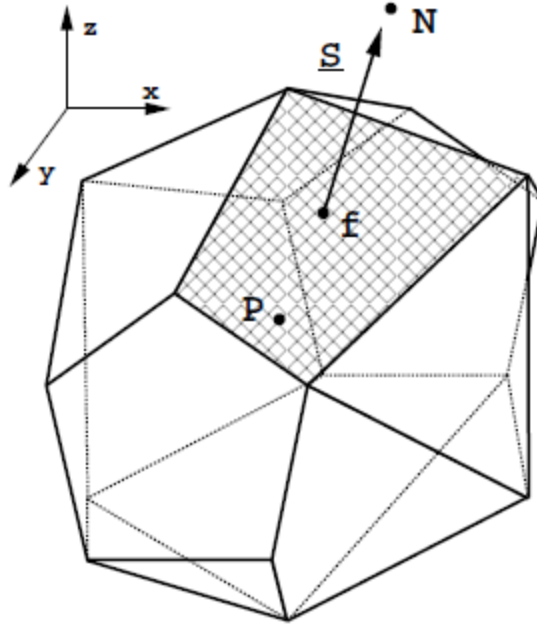


Figure 3.1: Control volume (H. Jasak (1996))

Since time is a parabolic coordinate (Patankar (1981)), the solution is obtained by marching in time from the prescribed initial condition. For the discretization of time, it is therefore sufficient to prescribe the size of the time-step that will be used during the calculation. The discretization of space for the finite volume method used in this study requires a subdivision of the domain into control volumes (CV). Control volumes do not overlap and completely fill the computational domain. In the present study all variables share the same control volumes.

A typical control volume is shown in Fig. 3.1. The computational point P is located at the centroid of the control volumes, such that:

$$\int_{V_p} (x - x_p) dV = 0. \quad (3.1)$$

The control volume is bounded by a set of flat faces and each face is shared with only one neighboring control volume. The topology of the control volume is not important (it is a general polyhedron).

The cell faces in the mesh can be divided into two groups - internal faces (between two control volumes) and boundary faces, which coincide with the boundaries of the domain. The face area vector S_f is constructed for each face in such a way that it points outwards from the cell with the lower label, is normal to the face and has the magnitude equal to the area of the face. The cell with the lower label is called the “owner” of the face - its label is stored in the “owner” array. The label of the other cell (“neighbor”) is stored in the “neighbor” array. Boundary face area vectors point outwards from the computational domain - boundary faces are “owned” by the adjacent cells. For the shaded face in Fig. 3.1, the owner and neighbor cell centers are marked with P and N, as the face area vector S_f points outwards from the P control volume. For simplicity, all faces of the control volume will be marked with f , which also represents the point in the middle of the face (see Fig. 3.1).

The capability of the discretization practice to deal with arbitrary control volumes gives considerable freedom in mesh generation. It is particularly useful in meshing complex geometrical configurations in three spatial dimensions. Arbitrarily unstructured meshes also interact well with the concept of local grid refinement, where the computational points are added in the parts of the domain where high resolution is necessary, without disturbing the rest of the mesh.

3.3 Discretization of the Transport Equation

The standard form of the transport equation for a scalar property ϕ is:

$$\underbrace{\frac{\partial \phi}{\partial t}}_{\text{temporal derivative}} + \underbrace{\nabla \cdot (U\phi)}_{\text{convection term}} - \underbrace{\nabla \cdot (\Gamma_\phi \nabla \phi)}_{\text{diffusion term}} = \underbrace{S_\phi(\phi)}_{\text{source term}} \quad (3.2)$$

Equation (3.2) is a second-order partial differential equation, as the diffusion term includes the second derivative of ϕ in space. For good accuracy, it is necessary for the order of the discretization to be equal to or higher than the order of the equation that is being discretized. The individual terms of the transport equation will be treated separately.

In certain parts of the discretization it is necessary to relax the accuracy requirement, either to accommodate the irregularities in the mesh structure or to preserve the boundedness of the solution. Any deviation from the prescribed order of accuracy creates a discretization error, which is of the order of other terms in the original equation and disappears only in the limit of excessively mesh.

The accuracy of the discretization method depends on the assumed variation of the function $\phi = \phi(x, t)$ in space and time around the point P . In order to obtain a second-order accurate method, this variation must be linear in both space and time, i.e. it is assumed that:

$$\phi(x) = \phi_p + (x - x_p) \cdot (\nabla \phi)_p, \quad (3.3)$$

$$\phi(t + \Delta t) = \phi^t + \Delta t \left(\frac{\partial \phi}{\partial t} \right)^t \quad (3.4)$$

where

$$\phi_p = \phi(x_p), \quad (3.5)$$

$$\phi^t = \phi(t). \quad (3.6)$$

Let us consider the Taylor series expansion in space of a function around the point x :

$$\begin{aligned} \phi(x) = \phi_p + (x - x_p) \cdot (\nabla \phi)_p + \frac{1}{2} (x - x_p)^2 : (\nabla \nabla \phi)_p + \frac{1}{2!} (x - x_p)^3 :: (\nabla \nabla \nabla \phi)_p + \dots + \\ \frac{1}{n!} \underbrace{(x - x_p)^n}_n :: \underbrace{(\nabla \nabla \dots \nabla \phi)_p}_n + \dots \end{aligned} \quad (3.7)$$

In Eq. (3.7) consists the expression $(x - x_p)^n$ represents the n th tensorial product of the vector $(x - x_p)$ with itself, producing an n th rank tensor. The operator “ $::$ ” is the inner product of two n th rank tensors, creating a scalar.

Comparison between the assumed variation, Eq. (3.3), and the Taylor series expansion, Eqn. (3.7), shows that the first term of the truncation error scales with $|(x - x_p)^2|$, which is for a 1-D situation equal to the square of the size of the control volume. The assumed spatial variation is therefore second-order accurate in space. An equivalent analysis shows that the truncation error in Eqn. (3.4) scales with $(\Delta t)^2$, resulting in the second-order temporal accuracy.

The Finite Volume method requires that Eqn. (3.2) is satisfied over the control volume V_p around the point P in the integral form:

$$\int_t^{t+\Delta t} \left[\frac{\partial}{\partial t} \int_{V_p} \phi dV + \int_{V_p} \nabla \cdot (U\phi) dV - \int_{V_p} \nabla \cdot (\Gamma_\phi \nabla \phi) dV \right] dt = \int_t^{t+\Delta t} \left(\int_{V_p} S_\phi(\phi) dV \right) dt \quad (3.8)$$

The discretization of above equation will now be examined term by term.

3.3.1 Discretization of Spatial Terms

Let us first examine the discretization of spatial terms. The generalized form of Gauss' theorem will be used throughout the discretization procedure, involving these identities:

$$\int_V \nabla \cdot a dV = \oint_{\partial V} dS \cdot a, \quad (3.9)$$

$$\int_V \nabla \phi dV = \oint_{\partial V} dS \phi, \quad (3.10)$$

$$\int_V \nabla a dV = \oint_{\partial V} dS a, \quad (3.11)$$

where ∂V is the closed surface bounding the volume V and dS represents an infinitesimal surface element with associated outward pointing normal on ∂V .

A series of volume and surface integrals needs to be evaluated. Taking into account the prescribed variation of ϕ over the control volume P , Eq. (3.3), it follows:

$$\begin{aligned} \int_{V_p} \phi(x) dV &= \int_{V_p} [\phi_p + (x - x_p) \cdot (\nabla \phi)_p] dV \\ &= \phi_p \int_{V_p} dV + \left[\int_{V_p} (x - x_p) dV \right] \cdot (\nabla \phi)_p \\ &= \phi_p V_p \end{aligned} \quad (3.12)$$

where V_p is the volume of the cell. The second integral in Eq. (3.12) is equal to zero because the point P is the centroid of the control volume.

Let us now consider the terms under the divergence operator. Having in mind that the control volume is bounded by a series of flat faces, Eq. (3.9) can be transformed into a sum of integrals over all faces:

$$\begin{aligned}
\int_V \nabla \cdot a \, dV &= \oint_{\partial V_p} dS \cdot a \\
&= \sum_f \left(\int_f dS \cdot a \right).
\end{aligned} \tag{3.13}$$

The assumption of linear variation of ϕ leads to the following expression for the face integral in Eqn. (3.13):

$$\oint_f dS \cdot a = S \cdot a_f. \tag{3.14}$$

Combining Eqs. 3.12, 3.13 and 3.14, a second-order accurate discretized form of the Gauss' theorem is obtained:

$$(\nabla \cdot a) V_p = \sum_f S \cdot a_f \tag{3.15}$$

Here, the subscript f implies the value of the variable (in this case, a) in the middle of the face and S is the outward-pointing face area vector. In the current mesh structure, the face area vector S_f point outwards from P only if f is “owned” by P . For the “neighboring” faces S_f points inwards, which needs to be taken into account in the sum in Eq. (3.15). The sum over the faces is therefore split into sums over “owned” and “neighboring” faces:

$$\sum_f S \cdot a_f = \sum_{owner} S_f \cdot a_f - \sum_{neighbour} S_f \cdot a_f. \tag{3.16}$$

This is true for every summation over the faces. In the rest of the text, this split is automatically assumed.

3.3.1.1 Convection Term

The discretization of the convection term is obtained using Eq. (3.15):

$$\begin{aligned}
\int_{V_p} \nabla \cdot (U\phi) \, dV &= \sum_f S \cdot (U\phi)_f \\
&= \sum_f S \cdot (U)_f \phi_f \\
&= \sum_f F \phi_f
\end{aligned} \tag{3.17}$$

where F in Eq. (3.17) represents the mass flux through the face

$$F = \mathcal{S} \cdot (U)_f \quad (3.18)$$

Equations (3.17 and 3.18) also require the face value of the variable ϕ calculated from the values in the cell centers, which is obtained using the convection differencing scheme.

Before we continue the formulation of the convection differencing scheme, it is necessary to examine the physical properties of the convection term. Irrespective of the distribution of the velocity in the domain, the convection term does not violate the bounds of ϕ given by its initial distribution. If, for example, ϕ initially varies between 0 and 1, the convection term will never produce the values of ϕ that are lower than zero or higher than unity. Considering the importance of boundedness in the transport of scalar properties of interest, it is essential to preserve this property in the discretized form of the term.

3.3.1.2 Convection Differencing Scheme

The role of the convection differencing scheme is to determine the value of ϕ on the face from the values in the cell centers. In the framework of arbitrarily unstructured meshes, it would be impractical to use any values other than ϕ_P and ϕ_N , because of the storage overhead associated with the additional addressing information. We shall therefore limit ourselves to differencing schemes using only the nearest neighbors of the control volume.

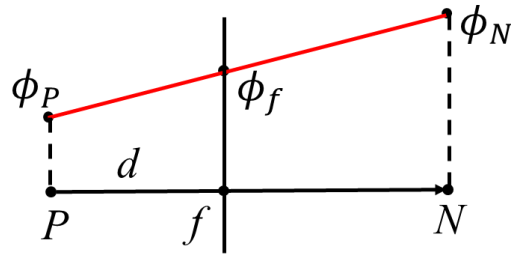


Fig. 3.2 Face interpolation

Assuming the linear variation of ϕ between P and N , Fig. 3.2, the face value is calculated according to:

$$\phi_f = f_x \phi_P + (1 - f_x) \phi_N. \quad (3.19)$$

Here, the interpolation factor f_x is defined as the ratio of distances $\overline{fN}$ and $\overline{PN}$:

$$f_x = \frac{\overline{fN}}{PN}. \quad (3.20)$$

The differencing scheme using Eqn. (3.19) to determine the face value of ϕ is called central differencing (CD). Although this has been the subject of some debate, Ferziger and Peric (1995) show that it is second order accurate even on non- uniform meshes. This is consistent with the overall accuracy of the method. It has been noted, however, that CD causes unphysical oscillations in the solution for convection-dominated problems (Patankar, (1972)), thus violating the boundedness of the solution.

An alternative discretization scheme that guarantees boundedness is upwind differencing (UD). The face value of ϕ is determined according to the direction of the flow:

$$\phi_f = \begin{cases} \phi_f = \phi_P & \text{for } F \geq 0 \\ \phi_f = \phi_N & \text{for } F < 0 \end{cases} \quad (3.21)$$

Boundedness of the solution is guaranteed through the sufficient boundedness criterion for systems of algebraic equations (see e.g. Patankar, (1972)). Boundedness of UD is effectively insured at the expense of the accuracy, by implicitly introducing the numerical diffusion term. This term violates the order of accuracy of the discretization and can severely distort the solution.

Blended Differencing (BD) (Peric (1985)) represents an attempt to preserve both boundedness and accuracy of the solution. It is a linear combination of upwind differencing, Eqn. (3.21) and central differencing, Eqn. (3.19):

$$\phi_f = \gamma(\phi_f)_{CD} + (1 - \gamma)(\phi_f)_{UD}, \quad (3.22)$$

The blending factor γ , $0 \leq \gamma \leq 1$, determines how much numerical diffusion will be introduced. Peric (1985) proposes a constant γ for all faces of the mesh. For $\gamma = 0$ the scheme reduces to UD. The most promising approach at this stage combines a higher-order scheme with upwind differencing on a face-by- face basis, based on different boundedness criteria.

3.3.1.3 Diffusion Term

The diffusion term will be discretized in a similar way. Using the assumption of linear variation of ϕ and Eqn. (3.15), it follows:

$$\begin{aligned} \int_{V_p} \nabla \cdot (\Gamma_\phi \nabla \phi) dV &= \sum_f \mathbf{S} \cdot (\Gamma_\phi \nabla \phi)_f \\ &= \sum_f (\Gamma_\phi)_f \mathbf{S} \cdot (\nabla \phi)_f \end{aligned} \quad (3.23)$$

If the mesh is orthogonal, i.e. vectors $\mathbf{d}$ and $\mathbf{S}$ in Fig. 3.3 are parallel, it is possible

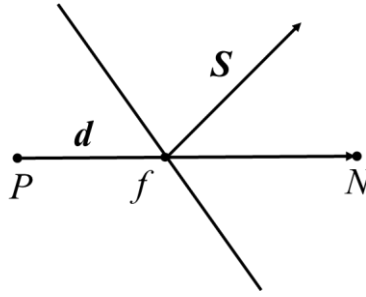


Fig. 3.3 Vector $\mathbf{d}$ and $\mathbf{S}$ on a non-orthogonal mesh

to use the following expression:

$$\mathbf{S} \cdot (\nabla \phi)_f = |\mathbf{S}| \frac{\phi_N - \phi_P}{|d|}. \quad (3.24)$$

Using Eqn. (3.24), the face gradient of ϕ can be calculated from the two values around the face. An alternative would be to calculate the cell-centred gradient for the two cells sharing the face as:

$$(\nabla \phi)_P = \frac{1}{V_p} \sum_f \mathbf{S} \phi_f, \quad (3.25)$$

interpolate it to the face:

$$(\nabla \phi)_f = f_x (\nabla \phi)_P + (1 - f_x) (\nabla \phi)_N. \quad (3.26)$$

Although both of the above-described methods are second-order accurate, Eqn. (3.26) uses a larger computational module. The first term of the truncation error is now four times larger than in the first method, which in turn cannot be used on non-orthogonal meshes.

In order to make use of the higher accuracy of Eqn. (3.24), the product $\mathbf{S} \cdot (\nabla \phi)_f$ is split into two parts:

$$\mathbf{S} \cdot (\nabla \phi)_f = \underbrace{\Delta \cdot (\nabla \phi)_f}_{\text{Orthogonal contribution}} + \underbrace{\mathbf{k} \cdot (\nabla \phi)_f}_{\text{non-orthogonal correction}} \quad (3.27)$$

The two vectors introduced in Eq. (3.27), Δ and $\mathbf{k}$, have got to satisfy the following condition:

$$\mathbf{S} = \Delta + \mathbf{k}$$

Vectors Δ is chosen to be parallel with $\mathbf{d}$. This allows us to use Eqn. (3.24) on the orthogonal contribution, limiting the less accurate method only to the non-orthogonal part which cannot be treated in any other way.

Orthogonal correction approach. This approach keeps the contribution from ϕ_P and ϕ_N the same as on the orthogonal mesh irrespective of the non-orthogonality, Fig. 3.4. To achieve this we define:

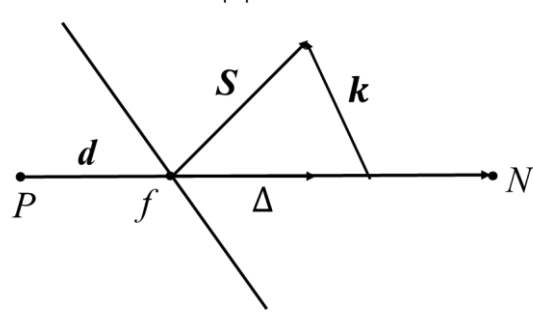
$$\Delta = \frac{d}{|d|} |\mathbf{S}| \quad (3.28)$$


Fig.3.4 Non-orthogonality treatment in the “orthogonal correction” approach

The diffusion term, Eq. (3.24), in its differential form exhibits the bounded behavior. Its discretized form will preserve this property only on orthogonal meshes. The non-orthogonal correction potentially creates unboundedness, particularly if mesh non-orthogonality is high. If the preservation of boundedness is more important than accuracy, the non-orthogonal correction has got to be limited or completely discarded, thus violating the order of accuracy of the discretization.

3.3.1.4 Source Terms

All terms of the original equation that cannot be written as convection, diffusion or temporal terms are treated as sources. The source term, $S_\phi(\phi)$, can be a general function of ϕ . When deciding on the form of the discretization for the source, its interaction with other terms in the equation and its influence on boundedness and accuracy should be examined. Some

general comments on the treatment of source terms are given in Patankar (1972). A simple procedure will be explained here.

Before the actual discretization, the source term needs to be linearized:

$$S_\phi(\phi) = Su + Sp\phi, \quad (3.29)$$

where Su and Sp can also depend on ϕ . Following Eq. (3.12), the volume integral is calculated as:

$$\int_{V_p} S_\phi(\phi) dV = SuV_p + SpV_p \phi_p \quad (3.30)$$

The importance of the linearization becomes clear in implicit calculations. It is desirable to treat the source term as “implicitly” as possible.

3.4 Temporal discretization

In the previous Section, the discretization of spatial terms has been presented. This can be split into two parts, the transformation of surface and volume integrals into discrete sums and expressions that give the face values of the variable as a function of cell values. Let us again consider the integral form of the transport equation:

$$\int_t^{t+\Delta t} \left[\frac{\partial}{\partial t} \int_{V_p} \phi dV + \int_{V_p} \nabla \cdot (U\phi) dV - \int_{V_p} \nabla \cdot (\Gamma_\phi \nabla \phi) dV \right] dt = \int_t^{t+\Delta t} \left(\int_{V_p} S_\phi(\phi) dV \right) dt \quad (3.31)$$

Using Eqns. 3.17 and 3.30, and assuming that the control volumes do not change in time, so, transport equation can be written as:

$$\int_t^{t+\Delta t} \left[\left(\frac{\partial \phi}{\partial t} \right)_p V_p + \sum_f F \phi_f - \sum_f (\Gamma_\phi)_f S \cdot (\nabla \phi)_f \right] dt = \int_t^{t+\Delta t} (SuV_p + SpV_p \phi_p) dt \quad (3.32)$$

The above expression is usually called the “semi-discretized” form of the transport equation. Having in mind the prescribed variation of the function in time, Eq. (3.32), the temporal integrals and the time derivative can be calculated directly as:

$$\left(\frac{\partial \phi}{\partial t} \right)_p = \frac{\phi_p^n - \phi_p^0}{\Delta t}, \quad (3.33)$$

$$\int_t^{t+\Delta t} \phi(t) dt = \frac{1}{2} (\phi^0 + \phi^n) \Delta t, \quad (3.34)$$

where

$$\phi^n = \phi(t + \Delta t), \quad (3.35)$$

$$\phi^0 = \phi(t). \quad (3.36)$$

Assuming that the density and diffusivity do not change in time, Eqs. 3.32, 3.32 and 3.34 give:

$$\begin{aligned} \frac{P\phi_p^n - P\phi_p^0}{\Delta t} V_p + \frac{1}{2} \sum_f F\phi_f^n - \frac{1}{2} \sum_f (\Gamma_\phi)_f S. (\nabla\phi)_f^n \\ + \frac{1}{2} \sum_f F\phi_f^0 - \frac{1}{2} \sum_f (\Gamma_\phi)_f S. (\nabla\phi)_f^0 = SuV_p + \frac{1}{2} SpV_p\phi_p^n + \frac{1}{2} SpV_p\phi_p^0. \end{aligned} \quad (3.37)$$

This form of temporal discretization is called the Crank-Nicholson method. It is second-order accurate in time. It requires the face values of ϕ and $\nabla\phi$ as well as the cell values for both old and new time-step. The face values are calculated from the cell values on each side of the face, using the appropriate differencing scheme for the convection term. Our task is to determine the new value of ϕ_p . Since ϕ_f and $(\nabla\phi)_f$ also depend on values of ϕ in the surrounding cells, Eq. (3.37) is reduced to an algebraic equation:

$$a_p\phi_p^n + \sum_N a_N\phi_N^n = R_p \quad (3.38)$$

For every control volume, an equation of this form is assembled. The value of ϕ_p^n depends on the values in the neighboring cells, thus creating a system of algebraic equations:

$$[A][\phi] = [R], \quad (3.39)$$

where $[A]$ is a sparse matrix, with coefficients a_p on the diagonal and a_N off the diagonal, $[\phi]$ is the vector of ϕ -s for all control volumes and $[R]$ is the source term vector. The sparseness pattern of the matrix depends on the order in which the control volumes are labelled, with every off-diagonal coefficient above and below the diagonal corresponding to one of the faces in the mesh. In the rest of this study, Eq. (3.39) will be represented by the typical equation for the control volume, Eq. (3.38).

When this system is solved, it gives a new set of ϕ values, the solution for the new time-step. As will be shown later, the coefficient a_p in Eq. (3.38) includes the contribution from all terms corresponding to ϕ_p^n , the temporal derivative, convection and diffusion terms as well as the linear part of the source term. The coefficients a_N include the corresponding terms for each of the neighboring points. The summation is done over all CVs that share a face with

the current control volume. The source term includes all terms that can be evaluated without knowing the new ϕ , namely, the constant part of the source term, and the parts of the temporal derivative, convection and diffusion terms corresponding to the old time- level.

The Crank-Nicholson method of temporal discretization is unconditionally stable, but does not guarantee boundedness of the solution. Examples of unrealistic solutions given by the Crank-Nicholson scheme can be found in Patankar and Baliga (1978). As in the case of the convection term, boundedness can be obtained if the equation is discretized to first order temporal accuracy.

It has been customary neglected the variation of the face values of ϕ and $\nabla\phi$ in time (Patankar (1972)). This leads to several methods of temporal discretization. The new form of the discretized transport equation combines the old and new time-step convection, diffusion and source terms, leaving the temporal derivative unchanged:

$$\frac{\phi_P^n - \phi_P^0}{\Delta t} V_p + \sum_f F \phi_f - \sum_f (\Gamma_\phi)_f S. (\nabla\phi)_f = SuV_p + SpV_p \phi_P \quad (3.40)$$

The resulting equation is only first-order accurate in time and a choice has to be made about the way the face values of ϕ and $\nabla\phi$ are evaluated.

In explicit discretization, the face values of ϕ and $\nabla\phi$ are determined from the old-time field:

$$\phi_f = f_x \phi_P^0 + (1 - f_x) \phi_N^0, \quad (3.41)$$

$$S. (\nabla\phi)_f = |\Delta| \frac{\phi_N^0 - \phi_P^0}{|d|} + \mathbf{k}. (\nabla\phi)_f^0 \quad (3.42)$$

The linear part of the source term is also evaluated using the old-time value. Eq. (3.44) can be written in the form:

$$\phi_P^n = \phi_P^0 + \frac{\Delta t}{V_p} \left[\sum_f F \phi_f - \sum_f (\Gamma_\phi)_f S. (\nabla\phi)_f = SuV_p + SpV_p \phi_P^0 \right]. \quad (3.43)$$

The consequence of this choice is that all terms on the r.h.s. of Eq. (3.43) depend only on the old-time field. The new value of ϕ_P can be calculated directly - it is no longer necessary to solve the system of linear equations. The drawback of this method is the Courant number limit (Courant et al. (1928)). The Courant number is defined as:

$$C_0 = \frac{U_f \cdot d}{\Delta t}. \quad (3.44)$$

If the Courant number is larger than unity, the explicit system becomes unstable. This is a severe limitation, especially if we are trying to solve a steady-state problem.

3.5 Solution techniques

Let us again consider the system of algebraic equations created by the discretization, Eq. (3.42):

$$a_p \phi_p^n + \sum_N a_N \phi_N^n = R_p \quad (3.45)$$

This system can be solved in several different ways. Existing solution algorithms are classified into two main categories: direct and iterative methods. Direct methods give the solution of the system of algebraic equations in a finite number of arithmetic operations. Iterative methods start with an initial guess and then continue to improve the current approximation of the solution until some “solution tolerance” is met. While direct methods are appropriate for small systems, the number of operations necessary to reach the solution raises with the number of equations, making them prohibitively expensive for large systems. Iterative methods are more economical, but they usually pose some requirements on the matrix.

The matrix resulting from Eq. (3.38) is sparse-most of the matrix coefficients are equal to zero. If it were possible to choose a solver which preserves the sparsity pattern of the matrix, the computer memory requirements would be significantly decreased. Unlike direct solvers, some iterative methods preserve the sparseness of the original matrix. These properties make the use of iterative solvers very attractive.

Iterative solvers require diagonal dominance in order to guarantee convergence. A matrix is said to be diagonally equal if the magnitude of the diagonal coefficient is equal to the sum of magnitudes of off-diagonal coefficients. The additional condition for diagonal dominance is $|a_p| > \sum_n |a_N|$ for at least one row of the matrix.

In order to improve the solver convergence, it is desirable to increase the diagonal dominance of the system. Discretization of the linear part of the source term, Eq. (3.32), is closely related

to this issue. If $S_p < 0$, its contribution increases diagonal dominance and S_p is included into the diagonal. In the case of $S_p > 0$, diagonal dominance would be decreased. It is more effective to include this term into the source and update it when the new solution is obtained. This measure is, however, not sufficient to guarantee the diagonal dominance of the matrix.

The analysis of the structure of the matrix leads us back to the issue of boundedness. The sufficient boundedness criterion for systems of algebraic equations mentioned states that the boundedness of the solution will be preserved for diagonally equal systems of equations with positive coefficients. This allows us to examine the discretized form of all the terms in the transport equation from the point of view of boundedness and diagonal dominance and identify the troublesome parts of discretization.

The convection term creates a diagonally equal matrix only for upwind differencing. Any other differencing scheme will create negative coefficients, violate the diagonal equality and potentially create unbounded solutions. In the case of central differencing on a uniform mesh, the problem is further complicated by the fact that the central coefficient is equal to zero. In order to improve the quality of the matrix for higher-order differencing schemes, Khosla and Rubin (1974) propose a correction implementation for the convection term. Here, any differencing scheme is treated as an upgrade of upwind differencing. The part of the convection term corresponding to upwind differencing is treated implicitly (i.e. built into the matrix) and the other part is added into the source term. This, however, does not affect the boundedness in spite of the fact that the matrix is now diagonally equal, as the “troublesome” part of the discretization still exists in the source term.

The diffusion term creates a diagonally equal matrix only if the mesh is orthogonal. On non-orthogonal meshes, introduces the “second neighbors” of the control volume into the computational molecule with negative coefficients, thus violating diagonal equality. As a consequence of mesh non-orthogonality, the boundedness of the solution cannot be guaranteed. The increase in the computational molecule would result in a higher number of non-zero matrix coefficients, implying a considerable increase in the computational effort

required to solve the system. On the other hand, the non-orthogonal correction is usually small compared to the implicit part of the diffusion term. It is therefore reasonable to treat it through the source term. In this study, the diffusion term will therefore be split into the implicit orthogonal contribution, which includes only the first neighbors of the cell and creates a diagonally equal matrix and the non-orthogonal correction, which will be added to the source. If it is important to resolve the non-orthogonal parts of the diffusion operators (like in the case of the pressure equation), non-orthogonal correctors are included. The system of algebraic equations, Eqn. (3.38), will be solved several times. Each new solution will be used to update the non-orthogonal correction terms, until the desired tolerance is obtained. It should again be noted that this practice only improves the quality of the matrix but does not guarantee boundedness. If boundedness is essential, the non-orthogonal contribution should be discarded.

The decomposition of the face area vector into the orthogonal and non-orthogonal part determines the split between the implicit and explicit part of the term, with the consequences on the accuracy and convergence of non-orthogonal correctors. The comparison of different treatments is based on several criteria:

- It is necessary to introduce non-orthogonal correctors - how good an approximation of the converged solution can be obtained after only one solution of the system?
- How many non-orthogonal correctors are needed to meet a certain tolerance?
- How does the number of solver iterations change with the number of correctors?
- If the non-orthogonal correction needs to be discarded for the sake of boundedness, which approach causes the smallest discretization error?

The discretization of the temporal derivative creates only the diagonal coefficient and a source term contribution, thus increasing the diagonal dominance. Unfortunately, the sufficient boundedness criterion cannot be used to establish the boundedness of the discretization, as it does not take into account the influence of the source term.

The above discussion is concentrated on the analysis of the discretization on a term- by-term basis. In reality, all of the above coefficients contribute to the matrix, thus influencing the properties of the system. It has been shown that the only terms that actually enhance the diagonal dominance are the linear part of the source and the temporal derivative.

In steady-state calculations, the beneficial influence of the temporal derivative on the diagonal dominance does not exist. In order to enable the use iterative solvers, the diagonal dominance needs to be enhanced in some other way, namely through under-relaxation. Consider the original system of equations, Eq. (3.38):

$$a_p \phi_p^n + \sum_N a_N \phi_N^n = R_p, \quad (3.46)$$

Diagonal dominance is achieved by an artificial term added to both left and right-hand side of Eqn. (3.38):

$$a_p \phi_p^n + \frac{1-\alpha}{\alpha} a_p \phi_p^n + \sum_N a_N \phi_N^n = R_p + \frac{1-\alpha}{\alpha} a_p \phi_p^0 \quad (3.47)$$

or

$$\frac{a_p \phi_p^n}{\alpha} + \sum_N a_N \phi_N^n = R_p + \frac{1-\alpha}{\alpha} a_p \phi_p^0 \quad (3.48)$$

ϕ_p^0 here represents the value of ϕ from the previous iteration and α is the under-relaxation factor ($0 < \alpha < 1$). Additional terms cancel out when steady-state is reached ($\phi_p^0 = \phi_p^n$)

In present study, the iterative solution procedure used to solve the system of algebraic equations is the conjugate gradient (CG) method, originally proposed by Hestens and Steifel (1952). It guarantees that the exact solution will be obtained in the number of iterations smaller or equal to the number of equations in the system. The convergence rate of the solver depends on the dispersion of the eigenvalues of the matrix $[A]$ in Eq. (3.39) and can be improved through pre-conditioning. For symmetric matrices, the incomplete cholesky preconditioned conjugate gradient (ICCG) solver will be used. The method is described in detail by Jacobs(1980).

3.6 Discretization procedure for the Navier-Stokes system

In this Section, a discretization procedure for the Navier-Stokes equations will be presented. We shall start with the incompressible form of the system, Eqns. (2.1 and 2.2):

$$\nabla \cdot \mathbf{U} = 0, \quad (3.49)$$

$$\frac{\partial \mathbf{U}}{\partial t} + (\mathbf{U} \cdot \nabla) \mathbf{U} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{U}. \quad (3.50)$$

Two issues require special attention: non-linearity of the momentum equation and the pressure-velocity coupling.

The non-linear term in Eq. (2.1) is $(\mathbf{U} \cdot \nabla) \mathbf{U}$, i.e. velocity is “being transported by itself”. The discretized form of this expression would be quadratic in velocity and the resulting system of algebraic equations would therefore be non-linear. There are two possible solutions for this problem - either use a solver for non-linear systems, or linearize the convection term.

$$\begin{aligned} (\mathbf{U} \cdot \nabla) \mathbf{U} &= \sum_f \mathcal{S} \cdot (\mathbf{U})_f (\mathbf{U})_f = \sum_f F (\mathbf{U})_f, \\ &= a_P U_P + \sum_N a_N U_N, \end{aligned} \quad (3.51)$$

where F , a_P and a_N are a function of $\mathbf{U}$. The important issue is that the fluxes F should satisfy the continuity equation, Eqn. 2.1 and 2.2 should therefore be solved together, resulting in an even larger (non-linear) system. Having in mind the complexity of nonlinear equation solvers and the computational effort required, linearization of the term is preferred. Linearization of the convection term implies that an existing velocity (flux) field that satisfies Eq. (2.1) will be used to calculate a_P and a_N .

The linearization does not have any effect in steady-state calculations. When the steady-state is reached, the fact that a part of the nonlinear term has been lagged is not significant. In transient flows two different approaches can be adopted: either to iterate over non-linear terms or to neglect the lagged nonlinearity effects. Iteration can significantly increase the computational cost, but only if the time-step is large. The advantage is that the nonlinear system is fully resolved for every time-step, whose size limitation comes only from the temporal accuracy requirements. If it is necessary to resolve the temporal behavior well, a small time-step is needed. On the other hand, if the time-step is small, the change between consecutive solutions will also be small and it is therefore possible to lag the nonlinearity without any significant effect.

In this study, the PISO procedure proposed by Issa (1986) is used for pressure-velocity coupling in transient calculations. For steady-state calculations, a SIMPLE pressure-velocity coupling procedure by Patankar (1972) is used.

In Section 3.6.1 the problem of pressure-velocity coupling is presented. The pressure equation is derived for the incompressible Navier-Stokes equations. Section 3.6.2 describes the pressure-velocity coupling algorithms.

3.6.1 Derivation of the pressure equation

In order to derive the pressure equation, a semi-discretized form of the momentum equation is used:

$$a_p U_p = \mathbf{H}(\mathbf{U}) - \nabla p. \quad (3.52)$$

In the spirit procedure of the Rhie and Chow (1982), the pressure gradient term is not discretized at this stage. Eq. 3.52 is obtained from the integral form of the momentum equation, using the discretization procedure described previously. It has been consequently divided through by the volume in order to enable face interpolation of the coefficients. The $\mathbf{H}(\mathbf{U})$ term consists of two parts: the “transport part”, including the matrix coefficients for all neighbors multiplied by corresponding velocities and the “source part” including the source part of the transient term and all other source terms apart from the pressure gradient (in our case, there is no additional source terms):

$$\mathbf{H}(\mathbf{U}) = -\sum_N a_N U_N + \frac{\mathbf{U}^0}{\Delta t}. \quad (3.53)$$

The discretized form of the continuity equation (Eqn. 2.23) is:

$$\nabla \cdot \mathbf{U} = \sum_f \mathbf{S} \cdot \mathbf{U}_f = 0. \quad (3.54)$$

Eqn. (3.52) is used to express $\mathbf{U}$:

$$U_p = \frac{\mathbf{H}(\mathbf{U})}{a_p} - \frac{1}{a_p} \nabla p. \quad (3.55)$$

Velocities on the cell face are expressed as the face interpolate of Eqn. 3.55:

$$U_f = \left(\frac{\mathbf{H}(\mathbf{U})}{a_p} \right)_f - \left(\frac{1}{a_p} \right)_f (\nabla p)_f. \quad (3.56)$$

This will be used later to calculate the face fluxes.

When Eq. (3.56) is substituted into Eqn. 3.54, the following form of the pressure equation is obtained:

$$\nabla \cdot \left(\frac{\nabla p}{a_p} \right) = \nabla \cdot \left(\frac{\mathbf{H}(\mathbf{U})}{a_p} \right) = \sum_f \mathbf{S} \cdot \left(\frac{\mathbf{H}(\mathbf{U})}{a_p} \right)_f. \quad (3.57)$$

The Laplacian on the L.H.S. of Eqn. 3.57 is discretized in the standard way.

The final form of the discretized incompressible Navier-Stokes system is:

$$a_p U_p = \mathbf{H}(\mathbf{U}) - \sum_f \mathbf{S}(p)_f, \quad (3.58)$$

$$\sum_f \mathbf{S} \cdot \left[\left(\frac{1}{a_p} \right)_f (\nabla p)_f \right] = \sum_f \mathbf{S} \cdot \left(\frac{\mathbf{H}(\mathbf{U})}{a_p} \right)_f. \quad (3.59)$$

The face flux F is calculated using Eqn. 3.56:

$$F = \mathbf{S} \cdot U_f = \mathbf{S} \cdot \left[\left(\frac{\mathbf{H}(\mathbf{U})}{a_p} \right)_f - \left(\frac{1}{a_p} \right)_f (\nabla p)_f \right]. \quad (3.60)$$

When Eq. (3.141) is satisfied, the face fluxes are guaranteed to be conservative.

3.6.2 Pressure-velocity coupling

Consider the discretized form of the Navier-Stokes (Eqns. 3.58 and 3.59). The form of the equations shows linear dependence of velocity on pressure and vice-versa. Simultaneous algorithms operate by solving the complete system of equations simultaneously over the whole domain. Such a procedure might be considered when the number of computational points is small and the number of simultaneous equations is not too large. The resulting matrix includes the inter-equation coupling and is several times larger than the number of computational points. The cost of a simultaneous solution is great, both in the number of operations and memory requirements.

A special treatment is required in order to establish the necessary inter-equation coupling. PISO (Issa (1986)), SIMPLE (Patankar (1972)), and their derivatives are the most popular methods of dealing with inter-equation coupling in the pressure-velocity system.

3.8.2.1 The PISO algorithm for transient flows

This pressure-velocity treatment for transient flow calculations has been originally proposed by Issa (1986). Let us again consider the discretized Navier-Stokes equations for incompressible flow, (Eqns. 3.58 and 3.59). The PISO algorithm can be described as follows:

- The momentum equation is solved first. The exact pressure gradient source term is not known at this stage - the pressure field from the previous time-step is used instead.

This stage is called the momentum predictor. The solution of the momentum equation, Eq. (3.58) gives an approximation of the new velocity field.

- Using the predicted velocities, the $\mathbf{H}(\mathbf{U})$ operator can be assembled and the pressure equation can be formulated. The solution of the pressure equation gives the first estimate of the new pressure field. This step is called the pressure solution.
- Eqn. 3.60 gives a set of conservative fluxes consistent with the new pressure field. The velocity field should also be corrected as a consequence of the new pressure distribution. Velocity correction is done in an explicit manner, using Eqn. 3.55). This is the explicit velocity correction stage.

A closer look to Eqn. 3.55 reveals that the velocity correction actually consists of two parts - a correction due to the change in the pressure gradient ($\nabla p/a_p$ term) and the transported influence of corrections of neighboring velocities ($\mathbf{H}(\mathbf{U})/a_p$ term). The fact that the velocity correction is explicit means that the latter part is neglected - it is effectively assumed that the whole velocity error comes from the error in the pressure term. This, of course, is not true. It is therefore necessary to correct the $\mathbf{H}(\mathbf{U})$ term to formulate the new pressure equation and repeat the procedure. In other words, the PISO loop consists of an implicit momentum predictor followed by a series of pressure solutions and explicit velocity corrections. The loop is repeated until a pre-determined tolerance is reached.

Another issue is the dependence of $\mathbf{H}(\mathbf{U})$ coefficients on the flux field. After each pressure solution, a new set of conservative fluxes is available. It would be therefore possible to recalculate the coefficients in $\mathbf{H}(\mathbf{U})$. This, however, is not done: it is assumed that the non-linear coupling is less important than the pressure-velocity coupling, consistent with the linearization of the momentum equation. The coefficients in $\mathbf{H}(\mathbf{U})$ are therefore kept constant through the whole correction sequence and will be changed only in the next momentum predictor.

3.6.2.2 The SIMPLE algorithm

If a steady-state problem is being solved iteratively, it is not necessary to fully resolve the linear pressure-velocity coupling, as the changes between consecutive solutions are no longer

small. Nonlinearity of the system becomes more important, since the effective time-step is much larger.

The SIMPLE algorithm by Patankar (1981) is formulated to take advantage of these facts:

- An approximation of the velocity field is obtained by solving the momentum equation. The pressure gradient term is calculated using the pressure distribution from the previous iteration or an initial guess. The equation is under-relaxed in an implicit manner, with the velocity under-relaxation factor α_U .
- The pressure equation is formulated and solved in order to obtain the new pressure distribution.
- A new set of conservative fluxes is calculated using Eqn. 3.60. As it has been noticed before, the new pressure field includes both the pressure error and convection-diffusion error. In order to obtain a better approximation of the “correct” pressure field, it is necessary to solve the pressure equation again. On the other hand, the non-linear effects are more important than in the case of transient calculations. It is enough to obtain an approximation of the pressure field and recalculate the $\mathbf{H}(\mathbf{U})$ coefficients with the new set of conservative fluxes. The pressure solution is therefore under-relaxed in order to take into account of the velocity part of the error:

$$p^{new} = p^{old} + \alpha_p(p^p - p^{old}) \quad (3.61)$$

where

- p^{new} is the approximation of the pressure field that will be used in the next momentum predictor,
- p^{old} is the pressure field used in the momentum predictor,
- p^p is the solution of the pressure equation,
- α_p is the pressure under-relaxation factor, ($0 < \alpha_p < 1$).

If the velocities are needed before the next momentum solution is obtained, the explicit velocity correction, Eqn. 3.55, is performed.

Peric(1985) gives an analysis of the under-relaxation procedure based on the expected behavior of the second corrector in the PISO sequence. The recommended values of under-relaxation factors are (Peric(1985)):

- $\alpha_p = 0.2$ for the pressure
- $\alpha_p = 0.8$ for momentum.

3.7 Solution procedure for the Navier-Stokes equations

It is now possible to describe the solution sequence for the Navier-Stokes system with additional coupled transport equations (e.g. a turbulence model, energy equation or some other equations that influence the system).

In transient calculations, all inter-equation couplings apart from the pressure- velocity system are lagged. If it is necessary to ensure a closer coupling between some of the equations (e.g. energy and pressure in combustion), they are included in the PISO loop. A transient solution procedure for incompressible turbulent flows can be summarized as follows:

1. Set up the initial conditions for all field values.
2. Start the calculation of the new time-step values.
3. Assemble and solve the momentum predictor equation with the available face fluxes.
4. Go through the PISO loop until the tolerance for pressure-velocity system is reached.
At this stage, pressure and velocity fields for the current time-step are obtained, as well as the new set of conservative fluxes.
5. Using the conservative fluxes, solve all other equations in the system. If the flow is turbulent, calculate the effective viscosity from the turbulence variables.
6. If the final time is not reached, return to step 2.

The solution procedure for steady-state incompressible turbulent flow is similar:

1. Set all field values to some initial guess.
2. Assemble and solve the under-relaxed momentum predictor equation.
3. Solve the pressure equation and calculate the conservative fluxes. Update the pressure field with an appropriate under-relaxation. Perform the explicit velocity correction using Eqn. 3.55

4. Solve the other equations in the system using the available fluxes, pressure and velocity fields. In order to improve convergence, under-relax the other equations in an implicit manner.
5. Check the convergence criterion for all equations. If the system is not converged, start a new iteration on step 2.

Chapter 4

Laminar flow through a helical pipe with circular cross section

4.1 Objective and scope of this chapter

In this study, a comprehensive three dimensional numerical study will be presented by direct numerical simulations of a viscous incompressible fluid flow through a helical pipe with a circular cross section. Numerical calculations will be carried out over a wide range of various types of parameters such as curvature, δ , torsion parameter, β , Reynolds number, Re , and Dean number, Dn . The main objective of this study is to numerically investigate the critical Reynolds number of explosive 3D laminar-turbulent transition instead of the linear stability analysis, to compare the results with those by experiment and 2D spectral study. In this study, we will perform the time evolution of disturbances in the unstable region of unsteady solutions and the transition process from nearly two-dimensional state to nonperiodic oscillation in detail. The destabilizing behavior of the flows, which can be obtained by this study, may be helpful to discuss a good physical insight into the actual helical pipe flow and may give a firm framework for the advanced study and research in helical pipe flow. The study of fully developed flow in a helical pipe has been attracted considerable attention because of its ample applications in chemical mechanical and biological engineering. Secondary flows of the fully developed region are induced by the circular motion of the main body of the fluid in a helical pipe may have important implications for blood flow in the human arterial system. The critical Reynolds number is important in fluid mechanic's is used to predict flow patterns in different fluid flow situations, and the issue point how the disturbances of turbulence develops in these flows is closely related to the onset of an infectious disease in the human body.

The objectives of this study are:

- Develop a new model and method to investigate Newtonian fluid flow characteristics through a helical pipe.
- A way to find the appearance of the fully developed two-dimensional (2D) flow region in 3D steady calculations.
- To find the stabilizing or destabilizing effect of torsion on the secondary flow in a helical pipe.
- Find the scenario of 2D and 3D behaviors of steady solution.
- Expect a general scenario of steady to unsteady state while a parameter is changed. The flow characteristics may turn into turbulent readily and its clarify.
- Find the instability of the steady solutions of the helical pipe flow instead of the linear stability analysis.
- Play a pioneer role in finding the critical Reynolds number of the laminar flow to turbulent transition by observing the unsteady behaviors of the solution
- Find a close relationship between the disturbances of unsteady solutions (such as nearly 2D state, nonlocal 3D modification, highly nonlinear behaviours, continuous nonperiodic oscilation), and the vortical structure of steady solutions in helical pipe flow.
- To clarify the region of turbulent and how to disturbances break down and go to turbulent in unsteady calculations.
- To compare the present 3D DNS results with experimental data and 2D spectral study.
- To find the explosive 3D instability in the marginal instability state.
- Help to understand the physics and mechanism of the flow behavior inside a helical pipe.

4.2 Results

In this section, the results of DNS are presented. In the first part, the steady solutions obtained by DNS are compared with those of the 2D spectral study (Yamamoto *et al.* (1994)) in the fully developed 2D region of the helical pipe for $\delta = 0.4$. It should be remarked that steady solutions for $\delta = 0.1$ were not shown in Yamamoto *et al.* (1998) although the results of the linear stability analysis were given. That is why we compared the present DNS results of the steady solutions with those by Yamamoto *et al.* (1994) for $\delta = 0.4$. In the second part, unsteady solutions are obtained to find the critical Reynolds numbers of the laminar to turbulent transition for various β and $\delta = 0.1$. The results are compared with those by the 2D linear stability analysis (Yamamoto *et al.* (1998)) and the experimental data (Hayamizu *et al.* (2008)).

4.2.1 Two dimensional steady state and the definition of Re and Dn

In the steady calculations, the solution is assumed to be convergent if the maximum residual error in any of the primitive variables such as velocity components, pressure, over the whole grids is less than 10^{-6} between one successive iteration. In the present calculation, the Reynolds number Re was determined by the mean axial velocity. In steady solutions, the value of Re , is calculated by the mean axial velocity at the exit of the pipe. It is clear that the mean axial velocity is the same at both the entrance and exit of the helical pipe, and the value may be changed by the pressure gradient at the exit. Thus, we can calculate the helical pipe flow with the same value of the Reynolds number as the 2D theoretical study by Yamamoto *et al.* (1994). When the flow is fully developed, it is expected that there may exist an interval in the z -direction where the flow profile does not vary along the pipe axis irrespective of the entrance effects, and the pressure gradient $\partial p / \partial z$ is constant in the z -direction. In order to find the fully developed flow region, we plot the pressure of the steady calculation on the center of the pipe cross section for $\beta = 0, 0.4, 0.8$ and 1.2 , and for $\delta = 0.4$ and $Dn = 1000$, as a function of z in Fig. 4.1. It should be noted that p is the deviation from the standard atmospheric pressure, and z is the length from the entrance. For $\beta = 0$, we calculated one turn of the helical pipe, while three turns for $\beta = 0.4, 0.8$ and 1.2 . It is seen in Fig. 4.1 that

the constant pressure gradient region increases as β increases, and the pressure gradient is nearly constant up to the exit for $\beta = 0.8$ and 1.2.

The computational mesh numbers have been varied in order to check the numerical accuracy. We obtained the steady solution at $Dn = 1000$ and $\delta = 0.4$ for $\beta = 0.8$ for five types of the mesh, which are named as M1, M2, M3, M4 and M5 as listed in Table 4.1. For each mesh, the volumetric flux through the pipe, F , the maximum axial velocity, w_{max} , and the total number of grids are shown in Table 4.1.

In the present study, the axial length of the helical pipe was three turns of the helical pipe for all the cases. We conducted the calculations of the flow through the helical pipe with four turns to check the influence of the outflow condition on the instability for a few cases. It is found that there is no appreciable effect of the outflow condition for the three-turn helical pipe.

Table 4.1. Grid sensitivity analysis for steady solutions for $Dn = 1000$ and $\delta = 0.4$ at $\beta = 0.8$.

Mesh name	Mesh size			F [m ³ /s]	w_{max} [m/s]	Total number of grid
	M	N	L			
M1	8 × 11 × 360			8.565 × 10 ⁻⁵	111.44	149760
M2	10 × 13 × 360			8.457 × 10 ⁻⁵	110.11	223200
M3	12 × 16 × 360			8.329 × 10 ⁻⁵	109.05	328320
M4	14 × 18 × 360			8.245 × 10 ⁻⁵	108.65	433440
M5	16 × 21 × 360			8.249 × 10 ⁻⁵	108.45	576000

Based on Table 4.1, we concluded that M4 is sufficient for the computation of the steady solutions. The grid sensitivity analysis for unsteady solutions will be described later in section 3.

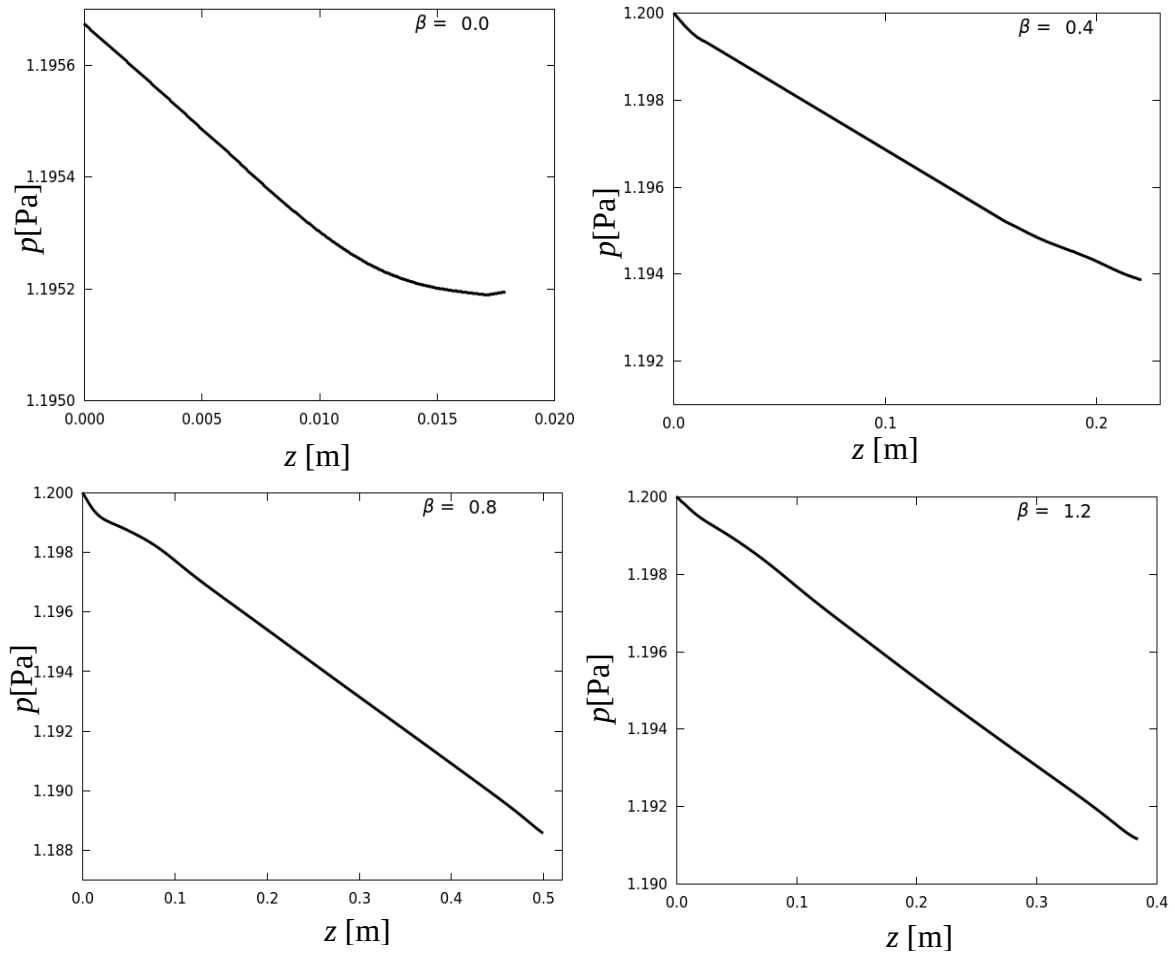


Fig. 4.1. Pressure p versus z for $\delta = 0.4$ and $Dn = 1000$.

As for the definition of the Dean number Dn , a careful treatment is needed because the mean pressure gradient G cannot be given a priori in the numerical calculation. First, the outlet pressure gradient $G_{out}(= \partial p / \partial z)$ is given at the exit of the pipe. Then, the steady solution is obtained numerically. If a constant pressure gradient region exists where $G_{int}(= \partial p / \partial z)$ is nearly constant, Dn is defined in the following way. In the theoretical study by Yamamoto *et al.* (1994), where two-dimensionality in the pipe axial direction is assumed, Dn is defined by Eq. (1.6) with $G = \partial p / \partial z$, where G is the pressure gradient in the axial direction. In the present study, G_{int} can be used for G to define Dn . We can vary Dn by changing G_{out} , the value of the pressure gradient at exit. It should be remarked that the relation $G_{int} \cong G_{out}$ holds for $\beta \geq 0.8$ in the present calculation.

4.2.2 Dean number calculated in the present study

In the present numerical calculation, we obtained Dn from a toroidal pipe to a helical pipe as β increases. We plot several lines of Dn with respect to G_{out} in Fig. 4.2. The lines having the symbols $\circ$, $\blacksquare$, $\blacktriangle$ and $\bullet$ are for $\beta = 0, 0.4, 0.8$ and 1.2 , respectively, where $\delta = 0.4$. It is found that if β increases, Dn decreases because $|G_{int}| > |G_{out}|$ for smaller β and $G_{int} \cong G_{out}$ for larger β .

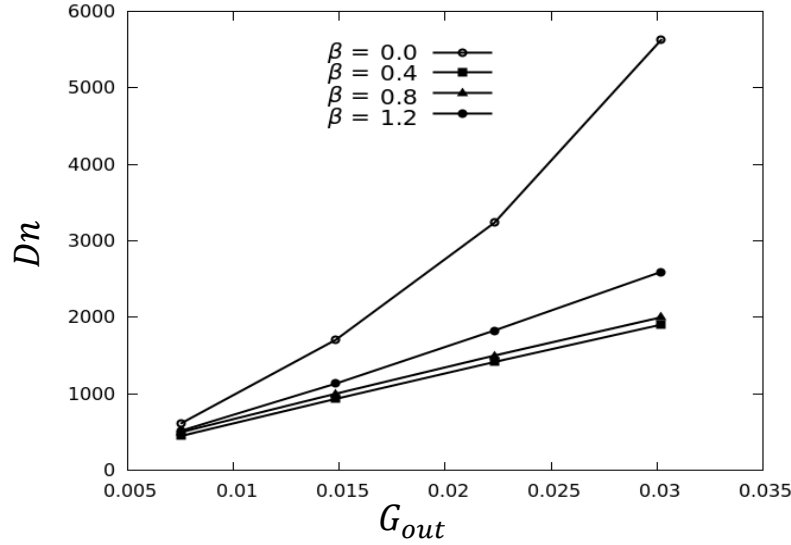


Fig. 4.2. Dn of the helical pipe flows for $\delta = 0.4$, where Dn is calculated by Eq. (1.6) with G_{int} used for G .

The results of steady calculations show that the functional relationship between Re and Dn exhibits a good agreement with that of the theoretical 2D study by Yamamoto *et al.* (1994) as shown in Fig. 4.3, for $\beta = 0.8$ and $\delta = 0.4$. It should be remarked that Dn was calculated by Eq. () with G_{int} for G in the present calculation.

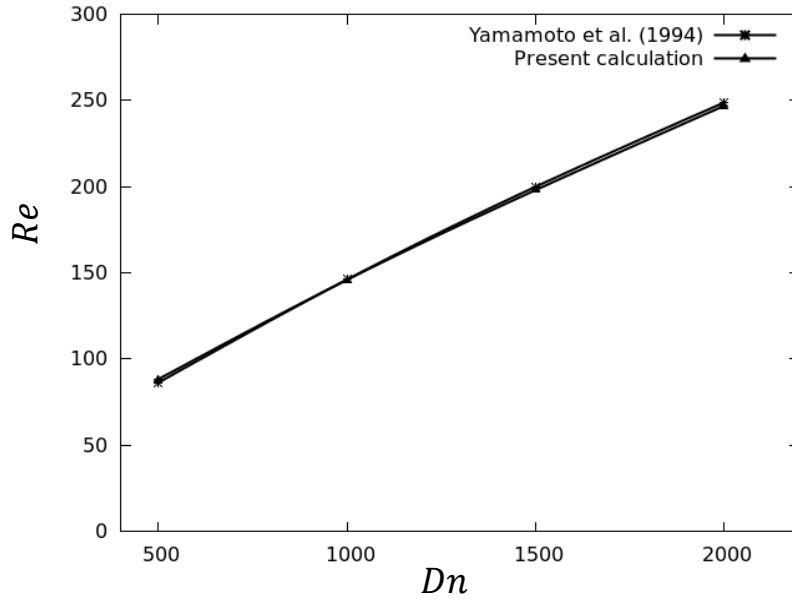


Fig. 4.3. Re versus Dn for $\beta = 0.8$ and $\delta = 0.4$.

4.2.3 Secondary flow for $\delta = 0.4$

On the cross section of the toroidal pipe ($\beta = 0$), high axial flow yields the centrifugal force toward outside of the pipe, and the fluid is moved from the inner to outer side. Because of mass conservation, the fluid returns to the inner side around the upper and lower region of the cross section. As a result, secondary flow is formed in the cross section exhibiting two symmetric circulating cells if there is no torsion effect ($\beta = 0$). If torsion effect appears and increases, however, the symmetric two-cell pattern changes into the asymmetric and finally one-cell pattern. In Fig. 4.4, we show the vector plots of the secondary flow for $\beta = 0, 0.4, 0.8$ and 1.2 at $Dn = 1000$ and $\delta = 0.4$ with those by Yamamoto *et al.* (1994). We also show the cut section of the contours of the Q -value (the second invariant of the velocity gradient tensor) surface (see Lesieur *et al.* (2005)) on the figures of the secondary flow. In Fig. 4.4(a), we observe the symmetric secondary flow with an anti-clockwise vortex in the upper hemisphere and a clockwise vortex in the lower hemisphere. For $\beta = 0.4$ in Fig. 4.4(b), a large and strong vortex of the secondary flow is produced in the lower part of the cross section and the vortex becomes small and weak in the upper part, which is in good agreement with the result of Yamamoto *et al.* (1994). This shows that, torsion of the pipe increases the strength of the vortex rotating in the same direction as torsion and vice versa. It is seen in Fig.

4.4(c) that for $\beta = 0.8$, the secondary flow consists of one large vortex, the center of which lies in the fourth quadrant of the cross section. As seen in Fig. 4.4(d), for large torsion parameter $\beta = 1.2$, the center of the single vortex moves to the middle of first and fourth quadrant of the cross section, which is also in good agreement with those by Yamamoto *et al.* (1994). This pattern change of the secondary flow as β increases is also in good agreement with the experimental results of Hayamizu *et al.* (2008).

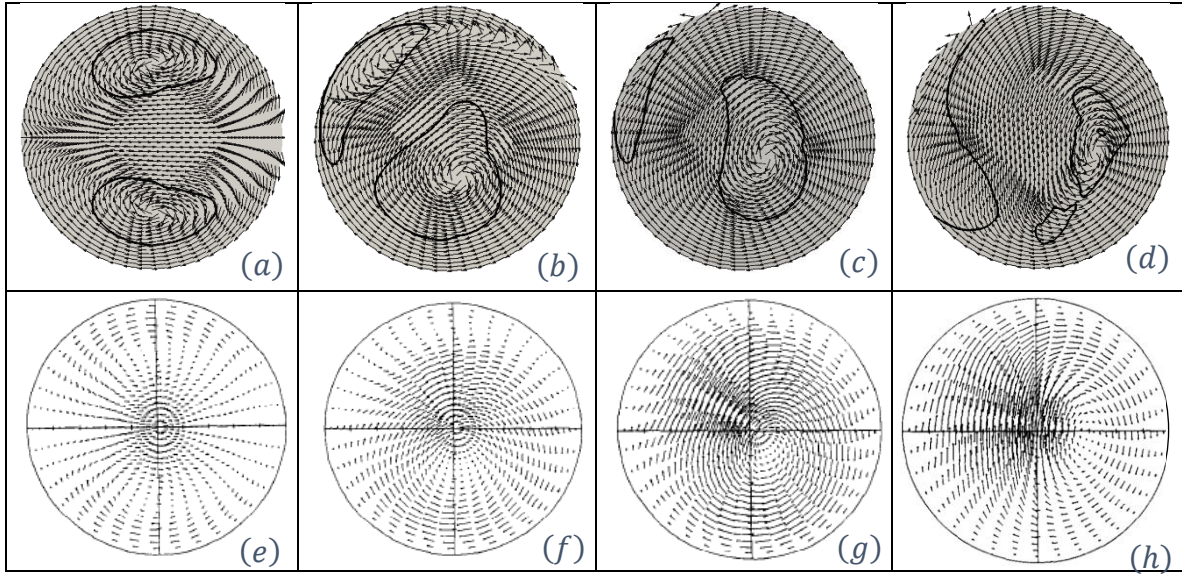


Fig. 4.4. Vector plots of the secondary flow and contours of the Q -value for $Dn = 1000$ and $\delta = 0.4$. The present DNS results are on the cross section in the fully developed 2D region, where an outer wall is on the right-hand side of each figure. The thick lines denote the contour of the Q -value, where $Q = 0.75$ for $\beta = 0.0$ and $Q = 1.5$ for other cases of β . Figure 4.4(a) is for $\beta = 0.0$ at $z = 0.008$ m, (b) $\beta = 0.4$ at $z = 0.19$ m, (c) $\beta = 0.8$ at $z = 0.36$ m and (g) $\beta = 1.2$ at $z = 0.29$ m. Figure 4.4 (d), (e), (f) and (g) are the corresponding results of the 2D spectral study (Yamamoto *et al.* (1994)).

4.2.4 Axial flow for $\delta = 0.4$

In this subsection, we study the axial flow distribution in the fully developed region. The fluid particle of high velocity in the z -direction gaining high centrifugal force is moved to outer region. In Fig. 4.5, the contour lines of the axial flow velocity are drawn at $Dn = 1000$ and $\delta = 0.4$ for over a wide range of β from 0 to 1.2 of the present calculation (left-hand side) and the theoretical study by Yamamoto *et al.* (1994) (right-hand side). The contour lines are drawn at every 20 of the axial velocity from the pipe wall, where the axial velocity is nondimensionalized by $(a/\nu)\sqrt{2\delta}$. When $\beta = 0$ (no torsion), a high axial velocity region is shifted to outer region of the cross section (Fig. 4.5(a)). The maximum of the axial velocity lies to the right of the center of the tube, where the maximum axial velocity is 78 in the present calculation. If β increases to 0.4, however, the high axial velocity region is rotated in the anticlockwise direction (Fig. 4.5(b)). The maximum axial velocity occurs at the right upper part of the cross section and the maximum axial velocity is 102 in the present calculation. As β increases further to 0.8, the high axial velocity region exists in the second quadrant (Fig. 4.5 (c)). It is interesting that the maximum axial velocity position reaches the left upper part of the cross section for $\beta = 0.8$ as shown in Fig. 4.5 (c), and the maximum axial velocity is 114 in the present calculation. In Fig. 4.5 (d), we observe that the maximum axial velocity position approaches the center line in the middle of the second and third quadrants, as β further increases to 1.2 and the maximum axial velocity is 161 in the present calculation.

It is interesting to note that when β is zero, the axial flow pattern is that of a toroidal pipe, while, as β increases, the position of the maximum axial flow velocity position turns around to the opposite direction of the torsion. When β become much larger, the axial velocity distribution may become very close to that of the Hagen-Poiseuille flow.

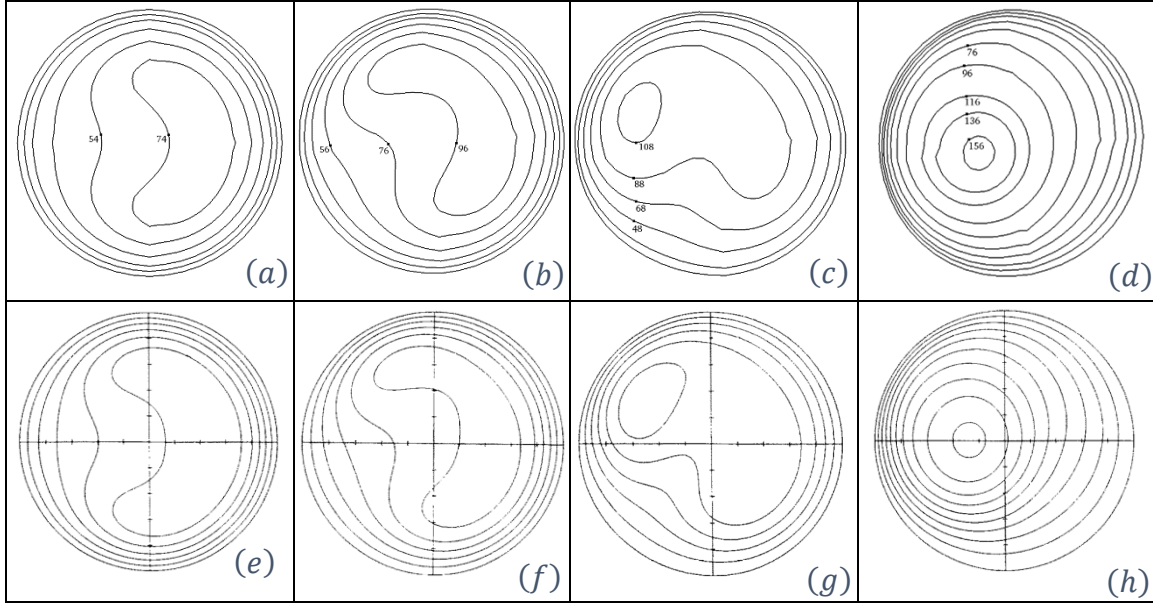


Fig. 4.5. Contours of the axial velocity for $Dn = 1000$ and $\delta = 0.4$. The present DNS results are the axial velocity on a cross section in the fully developed 2D region. Figure 4.5(a) is for $\beta = 0.0$ at $z = 0.008$ m, (b) $\beta = 0.4$ at $z = 0.19$ m (c) $\beta = 0.8$ at $z = 0.36$ m and (g) $\beta = 1.2$ at $z = 0.29$ m. Figures (d), (e) (f) and (g) are the corresponding results of the 2D study (Yamamoto *et al.* (1994)), . The increment of the velocity is 20.

In the present numerical calculations, we obtain the axial flow velocity distribution on the x -axis for $Dn = 1000$ and $\delta = 0.4$ for $\beta = 0, 0.4, 0.8, 1.2$ as shown in Fig. 4.6, where radial distance x is nondimensionalized by a and the axial velocity w by $(a/\nu)\sqrt{2\delta}$. The lines are for $\beta = 0, 0.4, 0.8, 1.2$, respectively.

We observe that maximum velocity exists near the outer wall of the pipe for $\beta = 0$ and 0.4. If β increases, however, the maximum position moves to the inner wall for $\beta = 0.8$, and slightly returns to the outer side for $\beta = 1.2$. This is consistent to Figs. 7 that the region of the maximum axial velocity on the cross section of the helical pipe moves from the outer wall side to the left upper part, and finally back to the inner wall side on the x -axis as β increases from zero to 1.2.

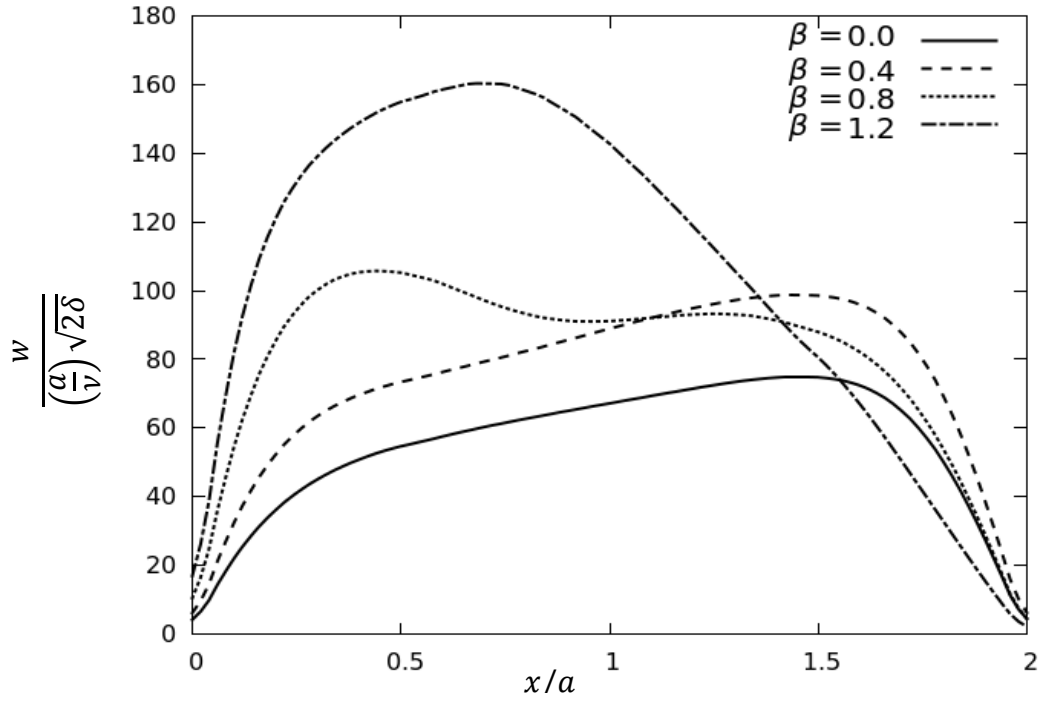


Fig. 4.6. The distribution of the axial velocity for $Dn = 1000$ and $\delta = 0.4$ on the center line ($y = 0$).

4.2.5 Flux through the pipe

Figure 4.7 shows the flux ratio Q/Q_s (where Q is the flux through the helical pipe and Q_s is the flux through the straight pipe) versus Dn with the same axial pressure gradient, obtained by the present numerical calculation and by Yamamoto *et al.* (1994) for $\beta = 0.8$ and $\delta = 0.4$. As seen in Fig. 4.7, flux ratio decreases as Dn increases. Both results agree very well with each other.

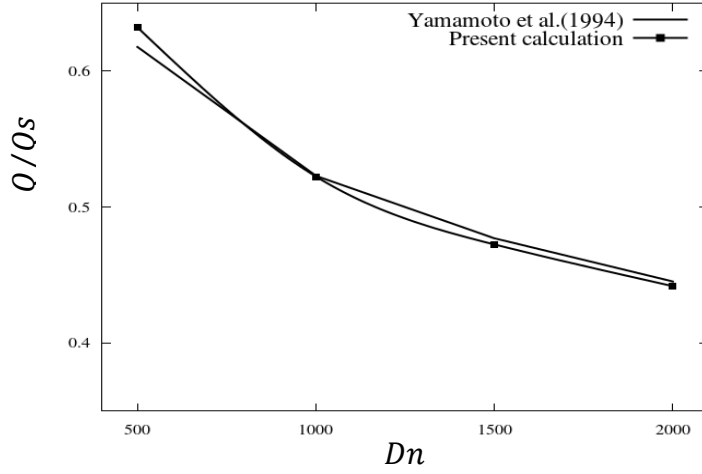


Fig. 4.7. Flux ratio Q/Q_s for $Dn = 1000$ and $\delta = 0.4$.

4.3 Unsteady solutions

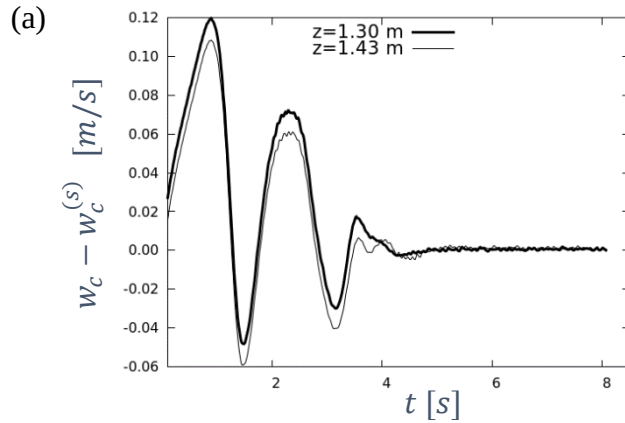
Yamamoto *et al.* (1998) performed the 2D linear stability analysis of the steady solutions obtained by themselves for $\delta = 0.1$, and found that the critical Reynolds number remarkably decreases as β increases and becomes less than 400 if $\beta \geq 1.2$. This finding is very surprising because the critical Reynolds number is very large (3331 by Canton *et al.*(2016)) for the toroidal pipe ($\beta = 0$) (Canton *et al.*(2016) and Piazza *et al.* (2011)) and the Hagen Poiseuille flow ($\beta = \infty$) is linearly stable. However, their linear stability analysis was limited to only the 2D analysis, so that the critical Reynolds number might be lower than what they obtained although it gives the upper bound. In spite of that, the experimental study (Yamamoto *et al.* (1995) and Hayamizu *et al.* (2008)) showed that the critical Reynolds numbers obtained two-dimensionally are very close to those of the experimental study.

4.3.1 Method to obtain the critical Reynolds number

In order to obtain the critical Reynolds number, we repeated unsteady calculations changing Re , where the initial condition is the steady solution obtained by the steady calculation. In the present study, disturbances are expected to be naturally introduced as a numerical error in the course of the simulation. If the unsteady solution converges to the steady solution, it is regarded to be stable, while unstable if the unsteady solution keeps to fluctuate and does not

converge to the steady solution. By the above mentioned method, the critical Reynolds number is determined as the value between stable and unstable Reynolds numbers. It should be remarked that the criterion of stability and instability did not depend on the position of z in the present study.

As an example, the results of time-evolution calculations are shown for $\beta = 0.6$ and $\delta = 0.1$. The deviation of the time-evolution of the axial velocity on z -axis (w_c) from the steady solution ($w_c^{(s)}$) is shown at several positions of z on the center line. Time-evolution of the axial velocity for $Re = 1096$ shows that its value quickly approaches zero as shown in Fig. 4.8(a). Therefore, the steady solution for $Re \leq 1096$ is thought to be stable. Next, time-evolution calculations were performed for $Re \geq 1100$. It is seen in Fig. 4.8 (b) and (c) that the value of the axial velocity does not approach zero but fluctuations for four values of z . Therefore, the steady solutions for $Re = 1100$ is thought to be unstable and the critical Reynolds number, Re_c , lies between 1096 and 1100. We shall call the flow for $Re > Re_c$ *the super critical state*. It is interesting that the magnitude of the deviation becomes larger toward the downflow direction.



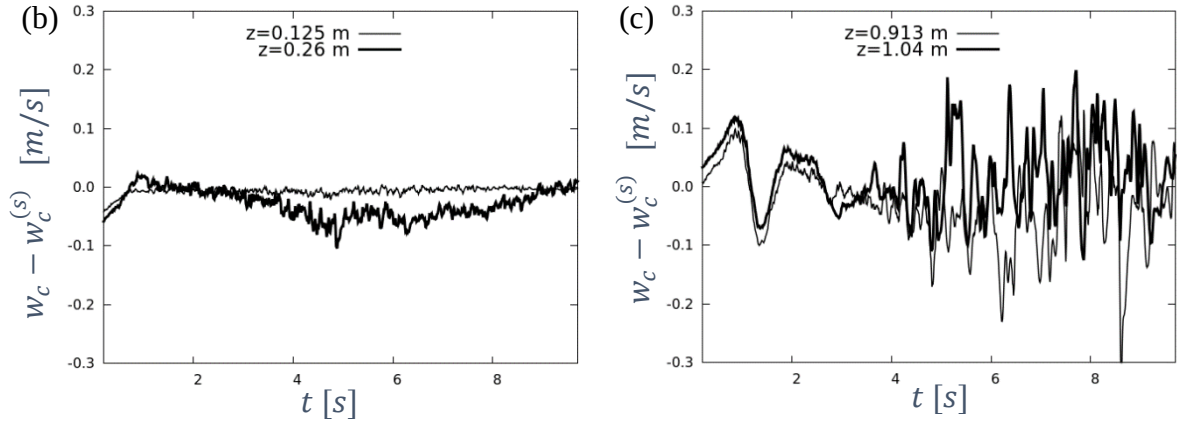


Fig. 4.8. Time-evolution of the deviation of the axial velocity from the steady solution at various values of z on the center line for $\beta = 0.6$ and $\delta = 0.1$. Figure 4.8(a) is for $Re = 1096$ and (b), (c) for $Re = 1100$.

In the assessment of the numerical accuracy for the steady solution at $\beta = 0.8$ and $\delta = 0.4$, it was found that M4 mesh ($14 \times 18 \times 360$) is sufficient for the steady calculations. To calculate unsteady solutions, however, the resolution in the z -direction may play an important role. Therefore, we performed the test calculations to check the numerical accuracy by increasing the resolution in the z -direction. As listed in Table 4.2, we tested three mesh cases P1, P2 and P3 by obtaining the critical Reynolds number within an error of 1 for $\beta = 0.6$ and $\delta = 0.1$. It is found that the results of P2 and P3 agree with each other. Thus, it is concluded that the mesh P2 is sufficient for the unsteady calculations in the present study.

Table 4.2. Grid sensitivity analysis for unsteady calculations. The symbol $\bigcirc$ indicates the stable and $\square$ the unstable.

Mesh	Mesh Size			Re	Instability of the helical pipe	Total number of grid
	M	N	L			
P1	$14 \times 18 \times 360$			1093	$\bigcirc$	433440
				1094	$\square$	
P2	$14 \times 18 \times 540$			1096	$\bigcirc$	650160
				1097	$\square$	
P3	$14 \times 18 \times 720$			1096	$\bigcirc$	866880
				1097	$\square$	

4.3.2 Critical Reynolds number for various β

We performed a comprehensive study of unsteady flow calculations over the range $0.6 \leq \beta \leq 1.8$ for $\delta = 0.1$. Figure 4.9 shows that the critical Reynolds number obtained by the present DNS (shown with the symbol $\bullet$) decreases as β increases, and that it becomes minimum at $\beta = 1.6$, then rapidly increases as β further increases. It should be noted that the 2D linear stability analysis by Yamamoto *et al.* (1998) (shown with the symbol ∇) gave the minimum critical Reynolds number at $\beta = 1.4$. We listed the values of the present critical Reynolds number for various β in Table 4.3.

As shown in Fig. 4.9, we may say that the stability characteristics obtained numerically by DNS nearly agree with those of Yamamoto *et al.* (1998) for $\beta \leq 1.4$, but not for $\beta = 1.6$. It should be remarked that no linear stability result is available for $\beta = 1.8$. The discrepancy between the 2D linear stability analysis and the present DNS in the case for $\beta = 1.6$ may suggest that 3D disturbances dominate the marginal instability state. In Fig. 4.9, the experimental results by Hayamizu *et al.* (2008) are also plotted with the symbol $\bigcirc$. Though they nearly agree with the present results, no experimental data is available for $\beta = 1.6$ and 1.8.

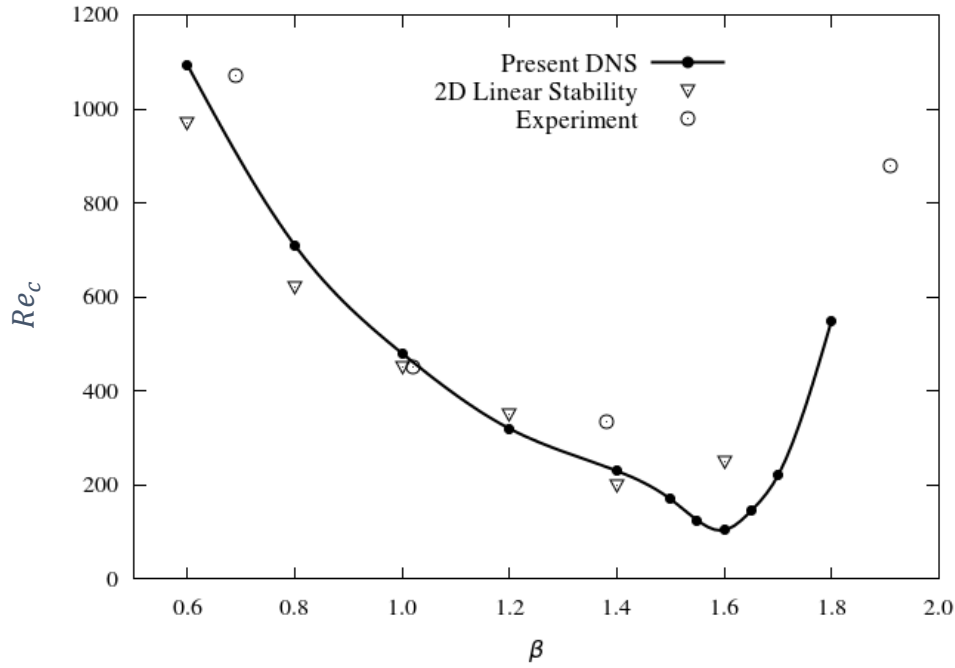


Fig. 4.9. The critical Reynolds number in the Re - β plane for $\delta = 0.1$. The symbol $\bullet$ denotes the present DNS results. The results of the 2D linear stability by Yamamoto *et al.* (1998) are shown with the symbol ∇ , and the experimental data by Hayamizu *et al.* (2008) with the symbol $\circ$.

Table 4.3. The critical Reynolds number. Re_{c1} is the Reynolds number of the marginally stable state and Re_{c2} is that of the marginally unstable state.

β	Re_{c1}	Re_{c2}	β	Re_{c1}	Re_{c2}
0.6	1096	1097	1.55	124	125
0.8	716	717	1.6	105	106
1.0	479	480	1.65	143	144
1.2	317	318	1.7	219	220
1.4	227	228	1.8	553	554
1.5	168	169			

4.3.3 Time-evolution of the equi-surface of the axial velocity for $\beta = 0.6$

In order to investigate the time-evolution of the disturbances of the supercritical state for the low torsion case, we plot equi-surfaces of the axial velocity, w , in the middle region of the helical pipe ($0.43 \text{ m} \leq z \leq 0.90 \text{ m}$) for $Re = 1100$, $\beta = 0.6$ and $\delta = 0.1$ in Fig. 4.10, where Re is slightly above the marginal instability (see Table 4.3). The figures show the time-evolution of the constant value surface of $w = 0.41 \text{ m/s}$ for $0.5 \text{ s} \leq t \leq 5 \text{ s}$. In Fig. 4.10(a), the wall of the helical pipe is shown, where the flow goes from the bottom to the top. Figures 4.10 (b)-(e) show the initial evolution of the flow for $0.8 \text{ s} \leq t \leq 1.5 \text{ s}$. It is found that nearly 2D (uniform in the z -direction) flow evolution terminates at $t \approx 0.8 \text{ s}$. After $t = 1 \text{ s}$, 3D surface modification begins as shown in Figs. 4.10(d)-(e). For $t \geq 1.8 \text{ s}$, nonlinear flow evolution seems to begin as shown in Figs. 4.10(f)-(h). It is found that nonlinear behavior of the flow seems to be stronger in the downstream region (in the upper part of the figures) because the instability grows in the main flow direction.

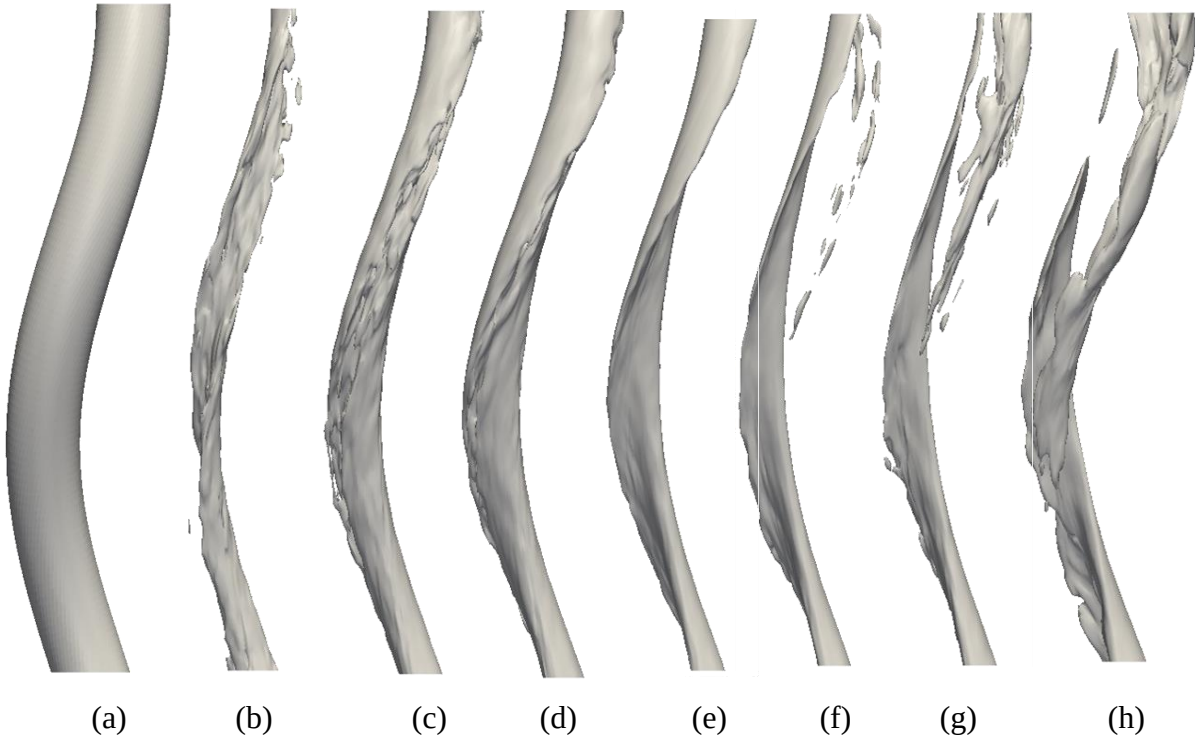


Fig. 4.10. Equi-surface of the constant velocity of w for $Re = 1090$, $\beta = 0.6$ and $\delta = 0.1$ in the region $z = 0.43 \text{ m}$ - 0.90 m . Figure 7(a) is the pipe wall, (b) at $t = 0.5 \text{ s}$, (c) $t = 0.8 \text{ s}$, (d) $t = 1.0 \text{ s}$, (e) $t = 1.5 \text{ s}$, (f) $t = 1.8 \text{ s}$, (g) $t = 3.0 \text{ s}$ and (h) $t = 5.0 \text{ s}$.

Figures. 4.11(a)-(h) shows that the flow fluctuates in the form of the two vortex solution. It is found that in the initial time period, the vortex pattern nearly consists of two vortices in the upper and lower hemispheres of the cross section. As time proceeds, the lower vortex moves to the center of the cross section, and the upper vortex becomes smaller and weaker. The maximum velocity region on the cross section does not move but stays in the second quadrant of the cross section throughout the time visualized as shown in Fig. 4.12. It is interesting that after $t = 5$, the contour lines become complicated, which is very different from those before the time $t = 5$. This is consistent to the non-periodic oscillation of the equi- U_z surface presented in Fig. 12. Fig. 4.13 (a-d) shows that, in the initial time period, the vortex nearly consists of two vortices at $z = 0.774$, which is the same as $z = 0.39$. However, for $t > 7.2$, we see that three-, four- and five- vortex patterns appear on the whole cross section. It should be remarked that flow oscillation behavior of the secondary flow is similar in the time interval $0.8 \leq t \leq 1.2$. It is found that the maximum axial velocity region stays in the left upper corner of the cross section for $t \leq 1.8$. However, a new minimum region appears in the right lower corner of the cross section for $t \geq 2$ as shown in Fig. 4.14. After $t = 5$, the contour line of the axial velocity becomes complicated as shown in Fig. 4.14(e-h), which is quite different from the behavior for $t \leq 5$ shown in Fig. 4.10(a-d). It is to be noted that at $z = 0.774$, the behavior of the axial velocity is consistent to the non-periodic oscillation of the equi- U_z surface presented in Fig. 4.10. Finally, it is concluded that two-vortex pattern of the secondary flow is kept throughout the time visualized at $z = 0.39$, while it is destroyed for $t > 7.22$ at $z = 0.774$. This implies that a strong 3D instability occurs in some region between $z = 0.39$ and $z = 0.774$.

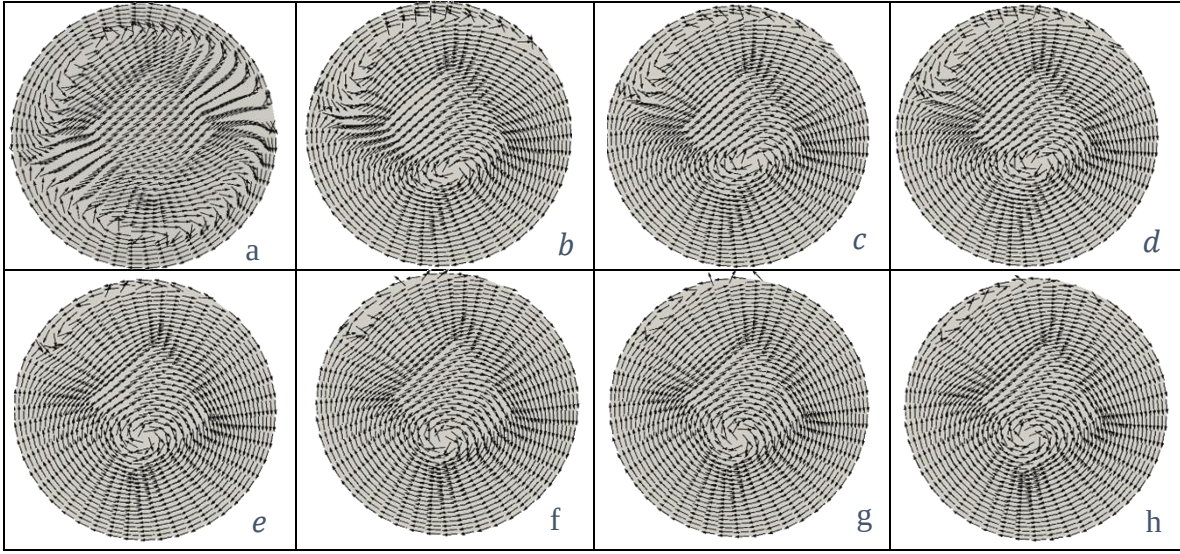


Fig. 4.11. Vector plots of the secondary flow for $Re = 1090$, and $\beta = 0.6$ at $z = 0.39$, where an outer wall is on the right-hand side of each figure.. Figure 4.11(a) is for $t = 0.2s$, (b) $t = 1.2s$, (c) $t = 1.8s$, (d) $t = 2.1s$, (e) $t = 5s$, (f) $t = 7.2s$, (g) $t = 8s$, and (g) (c) $t = 9.35s$.

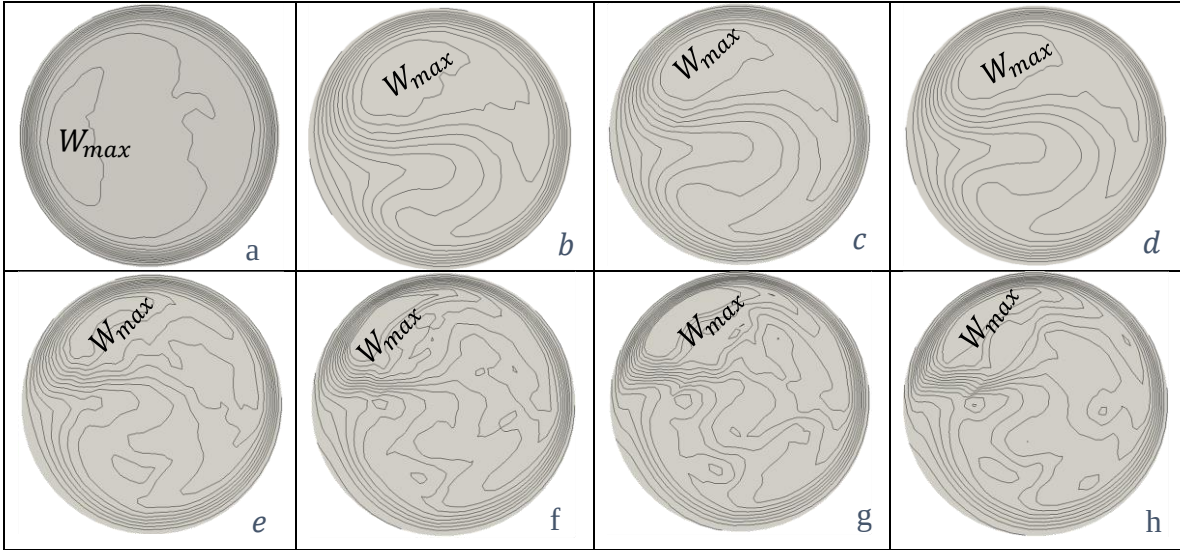


Fig. 4.12. Contours of the axial flow for $Re = 1090$, and $\beta = 0.6$ at $z = 0.39$. W_{max} is maximum position of the axial velocity., where an outer wall is on the right-hand side of each figure.. Figure 4.12(a) is for $t = 0.2s$, (b) $t = 1.2s$, (c) $t = 1.8s$, (d) $t = 2.1s$, (e) $t = 5s$, (f) $t = 7.2s$, (g) $t = 8s$, and (g) (c) $t = 9.35s$.

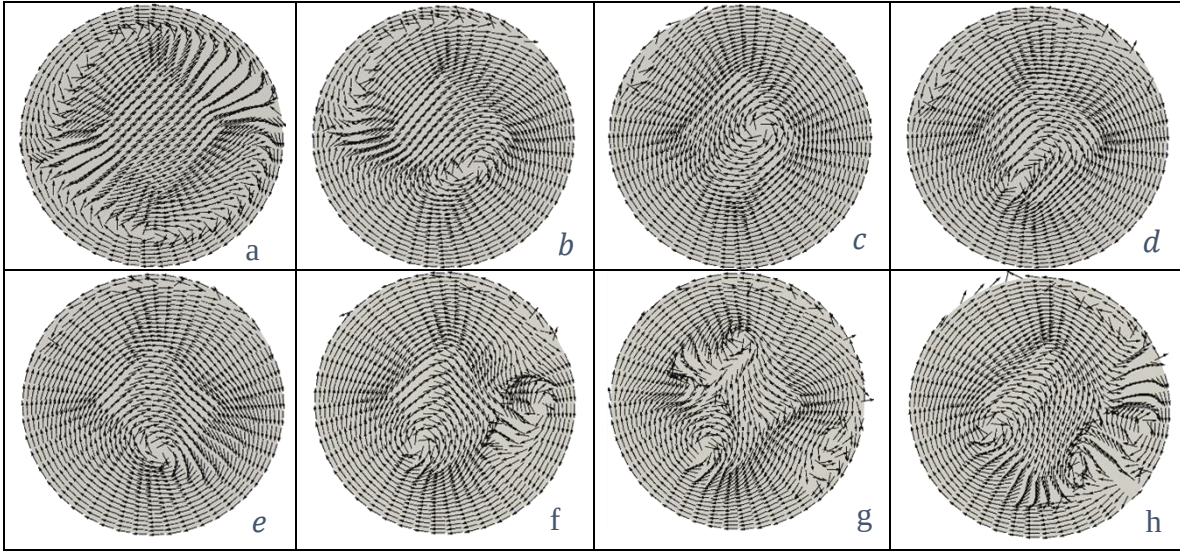


Fig. 4.13. Vector plots of the secondary flow for $Re = 1090$, and $\beta = 0.6$ at $z = 0.774$, where an outer wall is on the right-hand side of each figure.. Figure 4.13(a) is for $t = 0.2s$, (b) $t = 1.2s$, (c) $t = 1.8s$, (d) $t = 2.1s$, (e) $t = 5s$, (f) $t = 7.2s$, (g) $t = 8s$, and (g) (c) $t = 9.35s$.

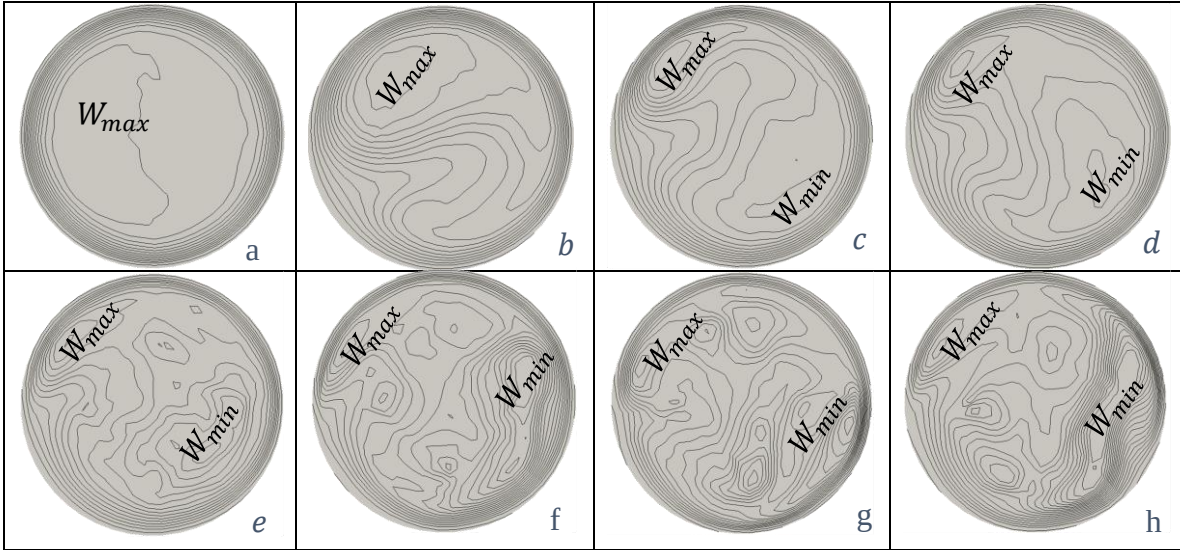


Fig. 4.14. Contours of the axial flow for $Re = 1090$, and $\beta = 0.6$ at $z = 0.774$. W_{max} is maximum and W_{min} is minimum position of the axial velocity., where an outer wall is on the right-hand side of each figure.. Figure 4.14(a) is for $t = 0.2s$, (b) $t = 1.2s$, (c) $t = 1.8s$, (d) $t = 2.1s$, (e) $t = 5s$, (f) $t = 7.2s$, (g) $t = 8s$, and (g) (c) $t = 9.35s$.

4.3.4 Time-evolution of the equi-surface of the axial velocity for $\beta = 1.6$

We plot equi-surface of w in the middle region of the helical pipe ($0.44 \text{ m} \leq z \leq 0.66 \text{ m}$) for $Re = 110$, $\beta = 1.6$ and $\delta = 0.1$ in Fig. 4.15, where Re is slightly above the marginal instability (see Table 4.3). It should be remarked again that the critical Reynolds number obtained by the present 3D DNS study is much lower than that obtained by the 2D linear stability analysis (Yamamoto et al. (1998)). The figures show the time-evolution of the constant value surface $w = 0.041 \text{ m/s}$, for $2 \text{ s} \leq t \leq 7.2 \text{ s}$. In Figs. 4.15(e)-(g), the occurrence of explosive 3D instability is clearly seen after $t = 6.8 \text{ s}$ in the downstream region.

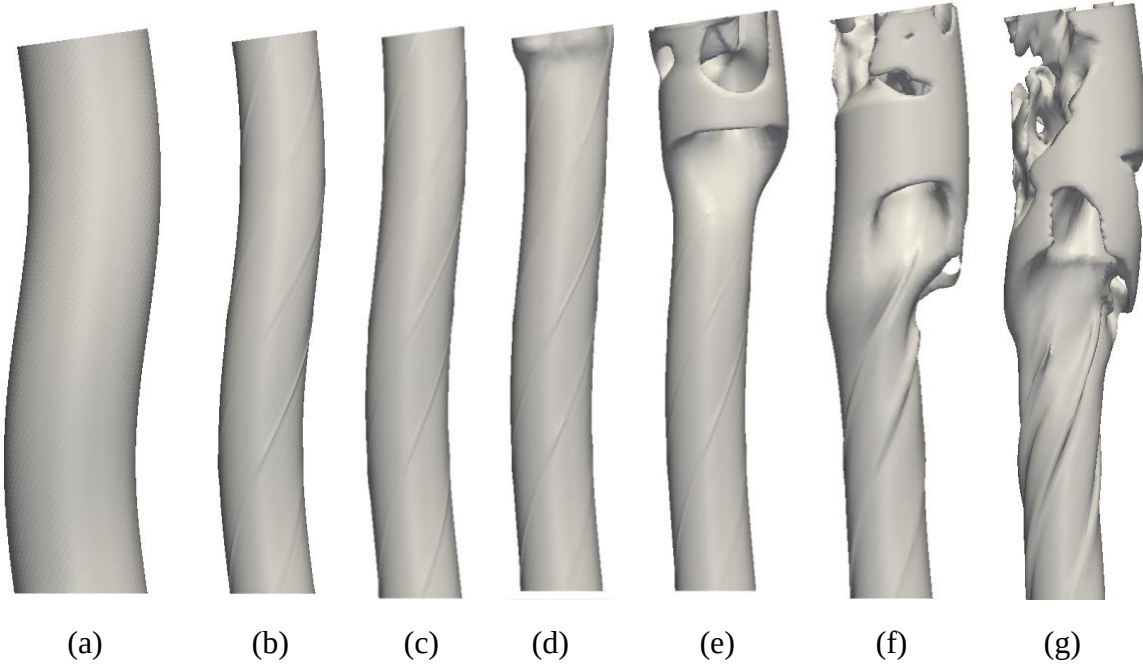


Fig. 4.15. Equi-surface of the constant velocity of w for $Re = 110$, $\beta = 1.6$ and $\delta = 0.1$ in the region $z = 0.42 \text{ m}$ - 0.66 m . Figure 8(a) is the pipe wall, (b) at $t = 2.0 \text{ s}$, (c) $t = 5.0 \text{ s}$, (d) $t = 6.6 \text{ s}$, (e) $t = 6.8 \text{ s}$, (f) $t = 7.0 \text{ s}$ and (g) $t = 7.2 \text{ s}$.

Since 3D disturbances are found to dominate the marginal instability state for $\beta = 1.6$, we visualize 3D vortical structures of the disturbance at the marginal state in Fig. 4.16, where we plot the equi- Q value surface (see Lesieur et al. (2005)) at the same region of Fig. 8 for $Re = 110$, $\beta = 1.6$ and $\delta = 0.1$. In Fig. 4.16, we clearly observe that the waveform appeared for $t \leq 5$ and it broke down as time proceeds. Therefore, it is clear that the wave-form

disturbance is a trigger of the 3D instability of the helical pipe flow at very low Reynolds numbers for $\beta = 1.6$.

For the evolution of the disturbance, the wavenumber, k , plays an important role for for $t \leq 5$. We obtained the wavenumber of the wave-form disturbance, $k = 1.5$, for $\beta = 1.6$, where the wavenumber is non-dimensionalized by a , the pipe radius. It is found that since the phase speed of the wave-form disturbance is nearly zero, it is a standing wave. In a straight pipe, however, it is well known that the low Reynolds number bifurcation occurs in the form of travelling wave as obtained by Wedin and Kerswell (2004). Therefore, it is interesting to study the relationship between the standing wave of the present calculation and the travelling wave of a straight pipe flow.

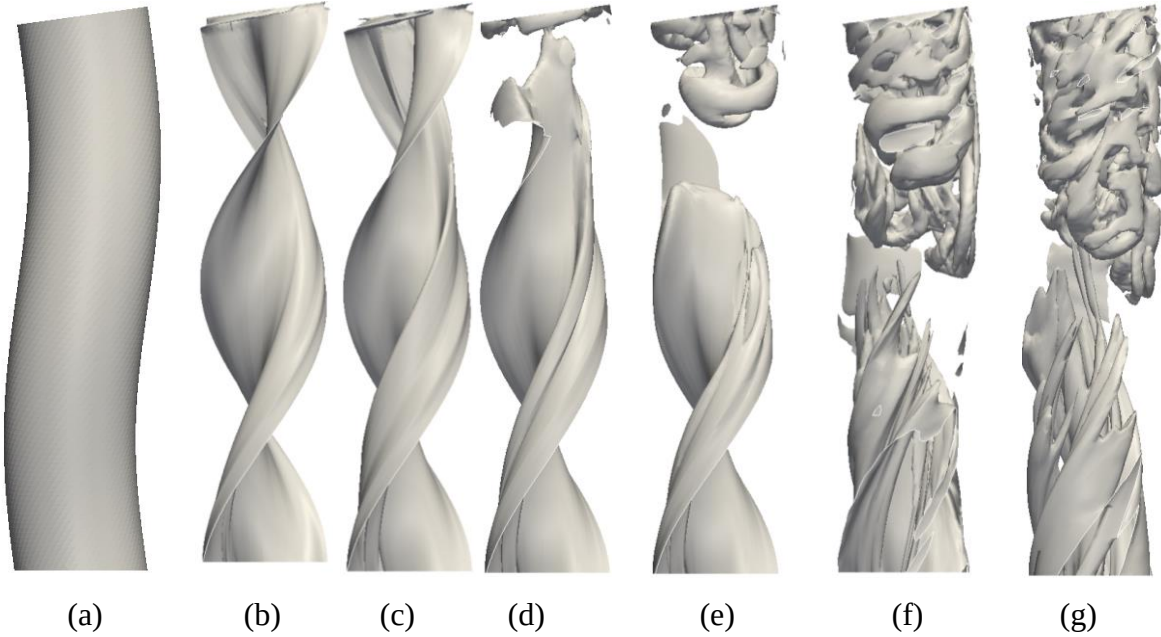


Fig. 4.16. Equi- Q value surface for $Re = 110$, $\beta = 1.6$ and $\delta = 0.1$ in the region $z = 0.42$ m-0.66 m. Figure 9 (a) is the pipe wall, (b) at $t = 2.0$ s, (c) 5.0 s, (d) 6.6 s, (e) 6.8 s, (f) 7.0 s and (g) 7.2 s.

4.4 Discussion and conclusion

In this study, we conducted 3D DNS of the viscous incompressible fluid flow through a helical pipe with circular cross section. We obtained steady solutions for $\delta = 0.4$ and compared them with those by the 2D study (Yamamoto *et al.* (1994)). We obtained the

critical Reynolds numbers of the laminar to turbulent transition by unsteady calculations for $\delta = 0.1$ and compared the results with the previous studies (Yamamoto *et al.* (1998) and Hayamizu *et al.* (2008)), where β was changed from 0.6 to 1.8.

In the steady calculations, we found the existence of the fully developed 2D flow region. Comparison of the present results on the cross section of the pipe in the fully developed 2D flow region with those by the 2D study (Yamamoto *et al.* (1994)) shows a very good agreement. In the present steady calculations, we observe that the secondary flow with two vortices exhibits a topological change into a single large vortex structure as β increases.

By conducting 3D unsteady calculations, we obtained the critical Reynolds numbers which give the critical states of the laminar to turbulent transition of the helical pipe flow. The results nearly agree with the preceding experimental data (Hayamizu *et al.* (2008)) as well as the 2D linear stability analysis Yamamoto *et al.* (1998) for $\beta \leq 1.4$. However, for $\beta \geq 1.5$, the critical Reynolds numbers obtained in the present study are much lower than those in the former studies.

It is found that an explosive 3D evolution of the disturbance occurs in the marginal instability state for $\beta = 1.6$, and the disturbance breaks down into turbulence in the downstream region as time proceeds. It is interesting that we found a vortical structure of the standing-wave disturbance and it is destroyed in the downstream region as time proceeds, which indicates that the standing-wave disturbance is a trigger of 3D instability at low Reynolds numbers. For $\beta \leq 1.4$, on the other hand, nearly 2D disturbances grow in the initial period, then become 3D gradually.

It is very interesting that visualization of the time evolution of disturbances shows the change as follows: nearly two-dimensional state $\rightarrow$ global three-dimensional modification $\rightarrow$ highly nonlinear behaviours $\rightarrow$ stationary nonperiodic oscillation. We also found that the flow oscillates between asymmetric two-, three-, four- and five- vortex solutions in the stationary nonperiodic oscillation. In that period, the region of the maximum axial velocity occurs at the left upper corner and the minimum at the right lower corner of the cross section.

Chapter 5

Effect of Torsion on the Friction Factor of Helical Pipe Flow

5.1 Objective and scope of this chapter

In this study, a comprehensive numerical study will be presented by 3D direct numerical simulations of a viscous incompressible fluid flow through a helical pipe with a circular cross section. The curvature, δ , torsion parameter, β , and the Reynolds number will be taken as various. The main objective of this study is to numerically investigate the influence of torsion, β , on the friction factor in laminar helical pipe based on the comparison with the experimental results of several helical circular pipes having different pitches and non-dimensional curvature. Friction factor of the fully developed region is important in many branches of engineer to determine the pumping power needed to overcome torsion- curvature induced pressure losses. The asymptotic behavior of the friction factor of helical pipe flow may be helpful to discuss a good physical insight and for engineering application. Finally, analysis of existing region of the global peak of the friction factor within the range of torsion parameter may give the advance study in engineering applications

The objectives of this study are:

- Develop a new model and method to investigate the 3D flow characteristics and friction factor through a helical pipe.
- Find the friction factor of the helical pipe flow compared with the experimental data.
- Expect a general scenario of the friction factor when the torsion parameter, curvature and the Reynolds number are changed.
- To clarify the effect of torsion on the friction factor of helical pipe flow.
- Play a pioneer role in the range of torsion parameter where the torsion has no remarkable effect on the friction factor of helical pipe flow over a wide range of the Reynolds number.
- Find a close relationship between the friction factor of curved pipe, helical pipe and straight pipe.

- Find the existing region of the global peak of the friction factor due to the effect of torsion for all the Reynolds numbers and curvatures.
- Help to understand the physics and mechanism of the friction factor behavior inside a helical pipe, which manifests the importance in engineering applications.

5.2 Results

In this section, the results of the 3D DNSs are presented. The friction factor obtained by DNSs are compared with those of an experimental study (Yamamoto et al. (1995)) of the helical pipe for a wide range of β . We examined two types of helical pipe with different curvature, for each of which we considered seven values of torsion (case I-VII), to compare the numerical results with the experimental data (Yamamoto et al. (1995)) . The curvature was around 0.1 for Type-1 and 0.05 for Type-2. The geometric parameters of the helical pipes are listed in Table 5.1.

Table 5.1: Characteristic lengths of the helical pipes

Type-1	Case	β	τ	$R[\text{mm}]$	$h [\text{mm}]$	$2a [\text{mm}]$	δ
	I	0.45	0.203	15.0	30.00	15.26	0.1017
	II	0.74	0.327	5.00	16.58	11.84	0.0987
	III	0.91	0.398	2.50	10.31	8.68	0.0964
	IV	1.11	0.501	3.00	14.70	15.35	0.1023
	V	1.17	0.510	1.50	8.08	8.53	0.0947
	VI	1.57	0.708	1.50	10.50	15.18	0.1012
	VII	1.70	0.756	1.00	7.68	11.81	0.0984
Type-2	Case	β	τ	$R[\text{mm}]$	$h [\text{mm}]$	$2a [\text{mm}]$	δ
	I	0.48	0.154	15.0	45.00	15.38	0.0513
	II	0.76	0.238	5.00	23.98	11.93	0.0497
	III	0.92	0.289	2.50	14.79	8.79	0.0488
	IV	1.11	0.354	3.00	21.00	15.16	0.0505
	V	1.18	0.364	1.50	11.52	8.54	0.0475
	VI	1.60	0.512	1.50	14.93	15.43	0.0514
	VII	1.71	0.537	1.00	10.91	11.81	0.0492

The computational mesh numbers were varied in order to check the numerical accuracy. We obtained the steady solutions at $Re = 200$ and $\delta = 0.1$ for $\beta = 1.2$ for five types of mesh, which are named M1, M2, M3, M4, and M5 as listed in Table 5.2. For each mesh, the volumetric flux through the pipe, Q , the maximum axial velocity, w_{max} , and the total number of grids are shown in Table 5.2. From Table 5.2, we concluded that M4 is appropriate for the computation in the present study.

Table 5.2. Grid sensitivity analysis for $Re = 200$ and $\delta = 0.1$ at $\beta = 1.2$

Mesh name	Mesh size			Q [m ³ /s]	w_{max} [m/s]	Total number of grids
	M	N	L			
M1	$8 \times 11 \times 360$			1.1908×10^{-4}	83.68	149760
M2	$10 \times 13 \times 360$			1.1907×10^{-4}	85.41	223200
M3	$12 \times 16 \times 360$			1.2039×10^{-4}	86.56	328300
M4	$14 \times 18 \times 360$			1.2042×10^{-4}	88.23	433440
M5	$16 \times 21 \times 360$			1.2041×10^{-4}	88.19	576000

In the confusing scenario of orthogonal and non-orthogonal helical coordinates, Germano (1982) found an advantage of the orthogonal system of helical coordinates that it can be easily generalized to a generic system of coordinates in the normal plane. On the basis of the orthogonal helical coordinate system, Yamamoto et al. (1994) numerically investigated helical pipe flow using the spectral method assuming that the flow field is uniform in the axial direction of the helical pipe.

In the present study, an appropriate mesh system with 3D orthogonal helical coordinates was successfully created to conduct accurate DNSs of helical pipe flow using an FVM-based open-source computational fluid dynamics package (OpenFOAM). To evaluate numerical accuracy, we compared the contour plot of the axial flow in the fully developed flow region obtained by the present 3D DNS with that obtained in the spectral study of Yamamoto et al. (1994), as shown in Fig. 5.1. By observing the maximum axial velocity and the shape of the contour pattern in Fig. 5.1, we found good agreement between the present DNS and the spectral result. The contour lines are drawn at increments of 20 in the axial velocity from the

pipe wall for both the present results and the spectral study, where the axial velocity is nondimensionalized by dividing by $(a/\nu)\sqrt{2\delta}$.

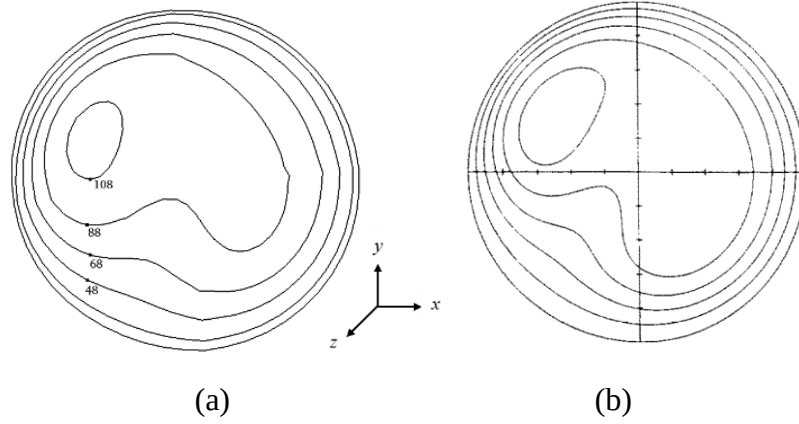


Fig. 5.1. Contours of the axial velocity for $Re = 145$ and $\delta = 0.4$. The present DNS results are the axial velocity on a cross section in the fully developed region. (a) $\beta = 0.8$ at $z = 0.36$ m. (b) corresponding results of the spectral study Yamamoto et al. (1994). The increment of the contours is 20 and the outer wall is on the right side.

5.2.1 Comparison with experimental data for Type-1

A helical pipe becomes a toroidal pipe for $\beta = 0$ and tends to a straight pipe as $\beta \rightarrow \infty$. The secondary flow caused by the centrifugal force due to curvature is affected by torsion, which results in different flow patterns from those in a toroidal pipe. It is interesting to note that when β is very small, a symmetric vortex pair of secondary flows similar to that is observed in a toroidal pipe,^{6,11)} and when β increases, the secondary flow pattern changes into that of one large vortex.

In order to compare the numerical results with the experimental data (Yamamoto et al. (1995)), we conducted numerical calculations for $\beta = 0.45, 0.91, 1.17, 1.57$, and 1.70 without the turbulence model. Here, we used $\delta = 0.1$ in the numerical calculations because the experimental curvature is very close to 0.1 as listed in Table 5.1. In the present DNS calculation, the friction factor of the pipe, f , is defined by

$$f = \frac{\Delta p}{L} \frac{2a}{\rho \bar{w}^2}, \quad (5.1)$$

where Δp is the pressure difference between two positions of the pipe in the region where the flow is well developed and axially invariant, and the distance between the two positions is L .

We also compared the present DNS results with Eq. (5.2) (1959).

$$f_I = \frac{1376\sqrt{\delta}}{(1.56 + \log_{10} Re\sqrt{\delta})^{5.73}} \quad (5.2)$$

Note that Eq. (5.2) is valid for $\beta = 0$ or $\beta \ll 1$. For comparison, we plot the present DNS results ($\circ$), the experimental data of Yamamoto et al. (1995) ($\square$), and Eq. (5.2) (dotted line) in Fig. 3. Equation (5.3) (dash-dotted line) represents f_s , the friction factor for a laminar flow in a straight pipe.

$$f_s = \frac{64}{Re} \quad (6)$$

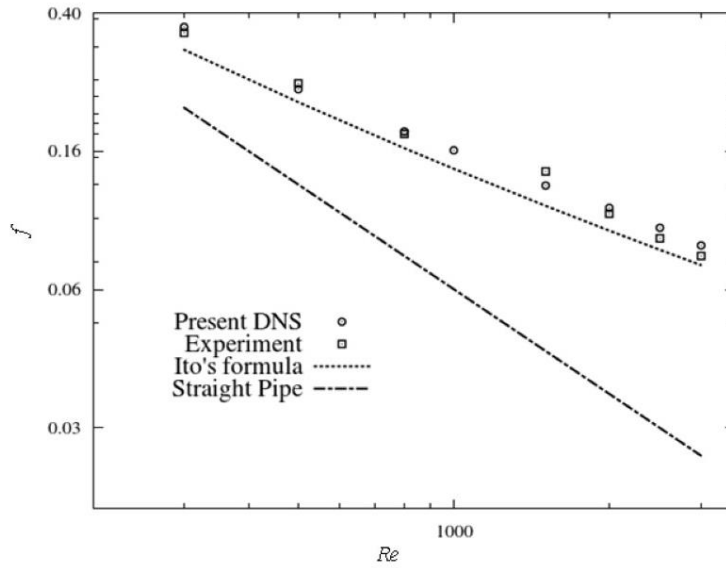


Fig. 5.2. Friction factor of the helical pipe for $\beta = 0.45$ and $\delta = 0.1$.

It was found that the DNS results are very close to the experimental data and reasonably close to Eq. (5.2) over a wide range of the Reynolds number, $300 \leq Re \leq 3000$. Note that Hayamizu et al. (2008) experimentally found the critical Reynolds number of the laminar to turbulent transition to be $Re_c = 2200$ for $\beta = 0.45$ and $\delta = 0.1$ by observing the unsteady behavior of the secondary and axial flows. We found that the friction factor obtained by DNS is in good agreement with the experimental data for $Re \leq 1500$; for $Re \geq 1500$, the

agreement is only reasonably good, probably due to the turbulence effect. It was found that the friction factor of the helical pipe is larger than that of a toroidal pipe for small β . In Fig. 5.2, it can be seen that the results of both the DNS and the experiment lie above the curve of Eq. (5.2). It is well known that the friction factor of a straight pipe is smaller than that of a toroidal pipe; thus, the friction factor curve for a straight pipe lies below than that of Eq. (5.2), as shown by Eq. (5.3) in Fig. 5.2.

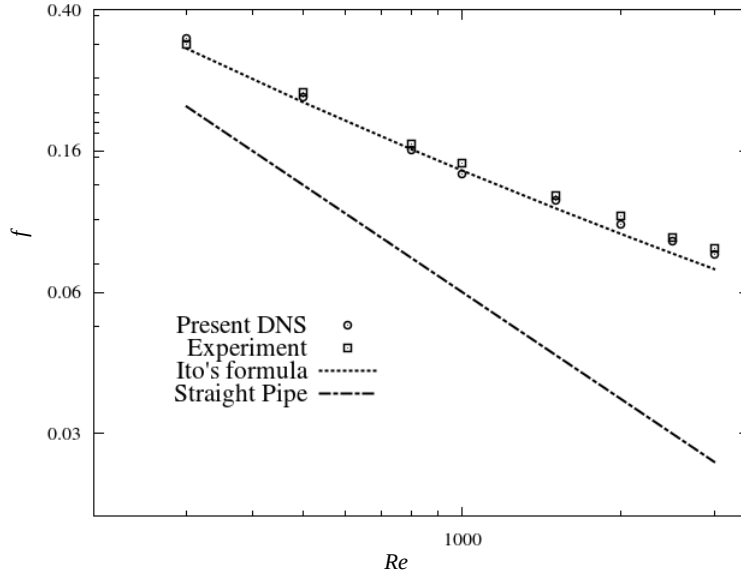


Fig. 5.3. Friction factor of the helical pipe flow at $\delta = 0.1$ for $\beta = 0.91$.

The friction factor of the helical pipe obtained by the present DNS without turbulence models, which is very close to the experimental data, decreases as β increases and is in fairly good agreement with that of a toroidal pipe (Eq. (5.2)) for $\beta = 0.91$ as shown in Fig. 5.3.

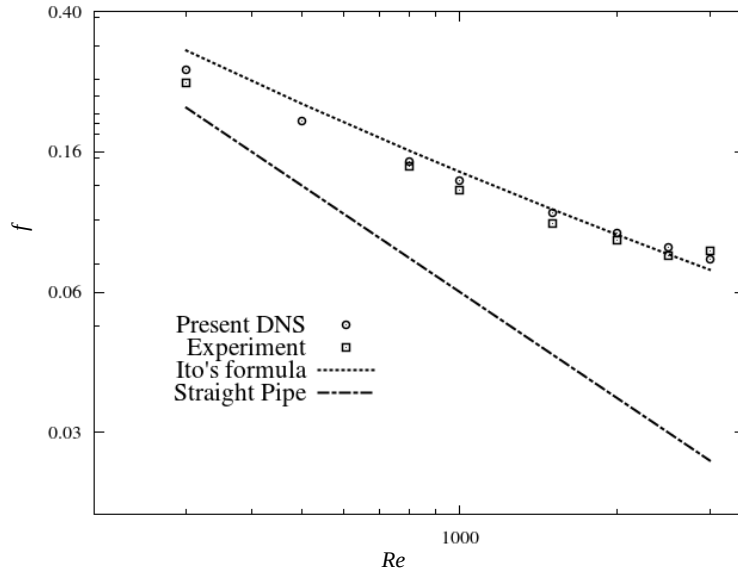


Fig. 5.4. Friction factor of the helical pipe flow at $\delta = 0.1$ for $\beta = 1.17$.

In Fig. 5.4, we find that the DNS results of the friction factor are in good agreement with the experimental data. It is interesting that the friction factor decreases with increasing Re , and appears below the curve for a toroidal pipe given by Eq. (5.2) but above the curve of Eq. (5.3) for $\beta = 1.17$.

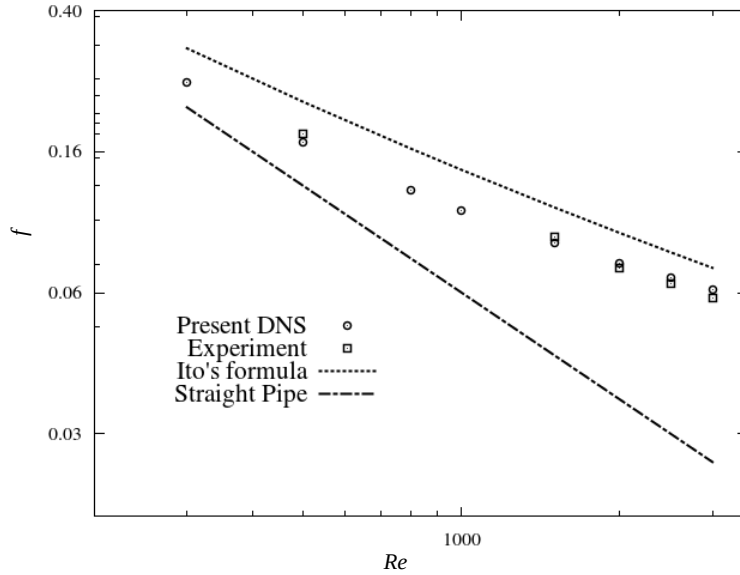


Fig. 5.5. Friction factor of the helical pipe flow at $\delta = 0.1$ for $\beta = 1.57$.

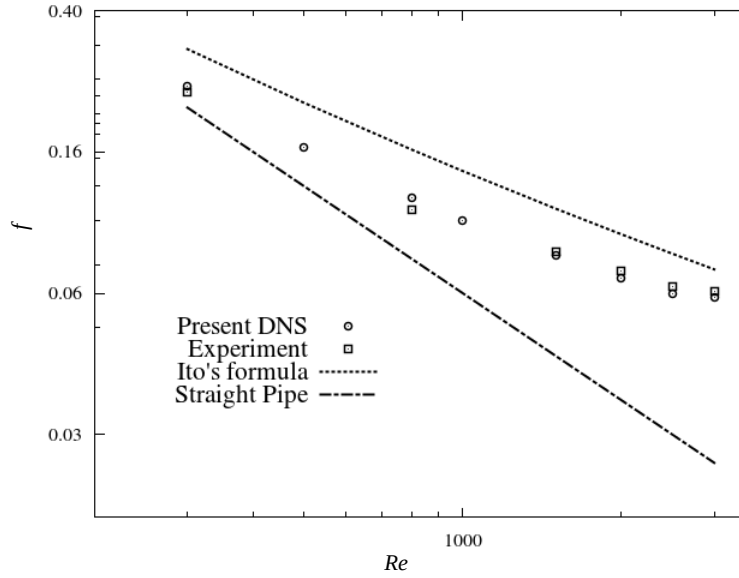


Fig. 5.6. Friction factor of the helical pipe flow at $\delta = 0.1$ for $\beta = 1.70$.

Figures 5.5 and 5.6 show the friction factor of the helical pipe for $\beta = 1.57$ and 1.70, respectively, where the present DNS results are close to the experimental data. It was found that the friction factor decreases as β increases further. The friction factor approaches that of a straight pipe for low Reynolds numbers.

Finally, from Figs. 5.1-5.6, we observed that the friction factor of the helical pipe is greater than that of a toroidal pipe for small β , and the friction factor gradually decreases and approaches that for a straight pipe as β increases to over 1.0. Therefore, the effect of torsion on the friction factor of the helical pipe becomes marked if β exceeds about 1.0, which is in good agreement with the experimental results (Yamamoto et al. (1995)).

5.3 Axial flow distribution for Type-1

In this subsection, we study the axial flow distribution in the well-developed axially invariant region. On the cross section, a high axial flow velocity yields a centrifugal force toward the outside, so that a fluid particle with a high axial velocity subjected to a large centrifugal force is moved to the outer region.

In the present DNS, we obtain the axial velocity distribution along the x -axis for $Re = 1000$ and $\delta = 0.1$ over a wide range of β as shown in Fig. 5.7, where x is nondimensionalized by dividing a and the axial velocity w is nondimensionalized by dividing by $\bar{w}$.

In Fig. 5.7, we find that a position with the maximum velocity exists near the outer wall of the pipe for $\beta = 0.45$. If β increases, however, the position of the maximum axial velocity moves to the inner side and lies near the inner wall for $\beta = 0.91$. When β becomes still larger, the position of the maximum axial velocity returns to the center of the cross section and the axial velocity distribution approaches that of the Hagen-Poiseuille flow.

In Fig. 5.7, we see that the maximum axial velocity gradually increases for $\beta \leq 1.17$ as β increases, and it becomes maximum for $\beta \cong 1.17$. For $\beta > 1.17$, the maximum axial velocity decreases gradually. Note that on the cross section, the pressure curve slowly increases from the inner to outer wall along the x -axis and the maximum pressure lies on the outer wall as shown in Fig. 5.8. Here, the pressure, p , is nondimensionalized by dividing by $(a/\nu)^2 \frac{1}{\rho}$.

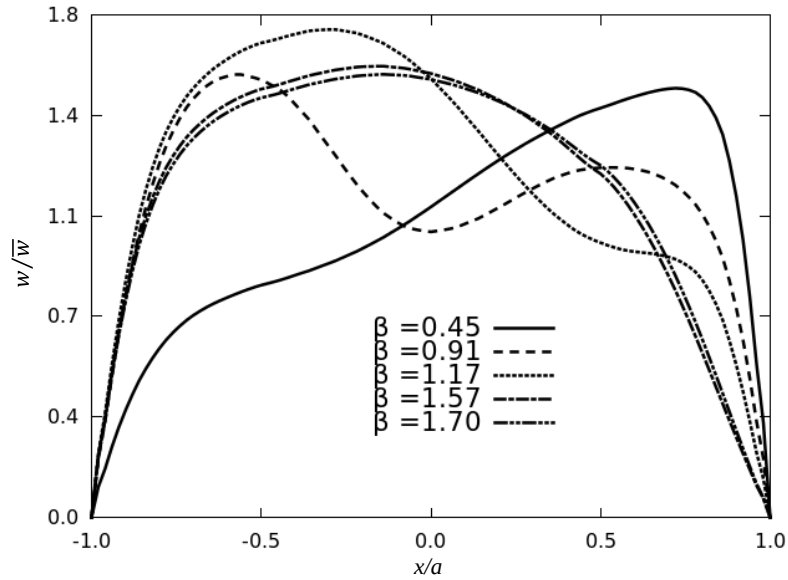


Fig. 5.7. Distribution of the axial velocity for $0.45 \leq \beta \leq 1.70$ and $Re = 1000$ at $\delta = 0.1$ on the center line ($y = 0$).

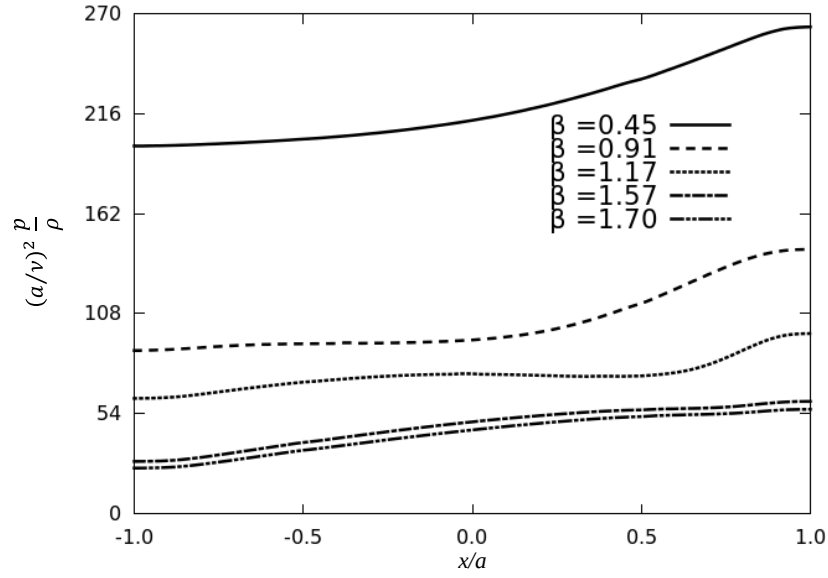


Fig. 5.8. Distribution of pressure for $0.45 \leq \beta \leq 1.70$ and $Re = 1000$ at $\delta = 0.1$ on the center line ($y = 0$).

5.4 Friction factor over a wide range of β

The characteristics of the friction factor of the helical pipe have been described so far. In order to show them comprehensively, we plot the values of the friction factor obtained from the present DNS over the wide range of $0.02 \leq \beta \leq 2.8$ for three Reynolds numbers ($Re = 80, 300$, and 1000). Figures 5.9-5.11 and Figs. 5.12-5.14 show the friction factor of the helical pipe for $\delta = 0.1$ and 0.05 , respectively.

In Fig. 5.9-5.14, the friction factor of the helical pipe experimentally obtained by Yamamoto et al. (1995)) is plotted as squares (■) and that obtained by the present DNS is plotted as circle (●). Equations (5) and (6) are also plotted in the figures. Note that no experimental data exists for $Re = 80$ in Figs. 5.9 and 5.12.

5.4.1 Friction factor for Type-1 ($\delta = 0.1$)

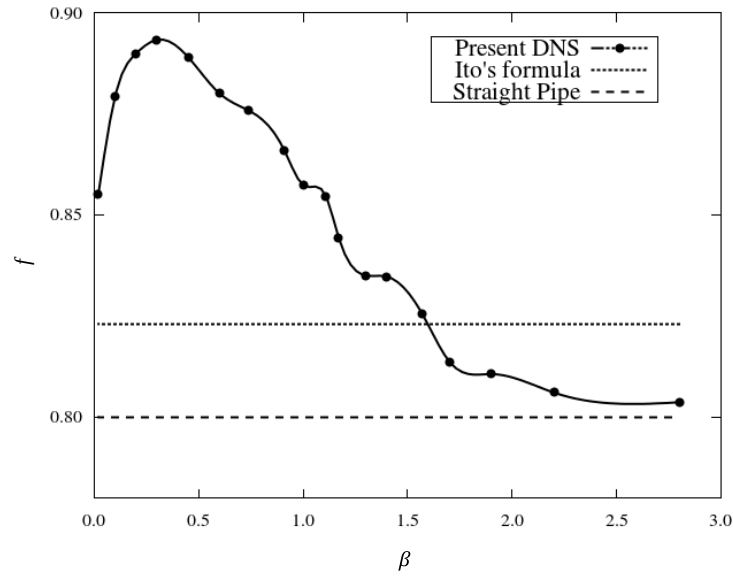


Fig. 5.9. Friction factor of the helical pipe for $Re = 80$ at $\delta = 0.1$.

In Figs. 5.9-5.11, it is shown that the friction factor first increases from that of a toroidal pipe and then almost monotonically decreases to that of a straight pipe as β increases from zero. The maximum of the friction factor occurs at $\beta \cong 0.3$ for $Re = 80$ and 300 and at $\beta \cong 0.2$ for $Re = 1000$. The present DNS results agree reasonably well with the experimental data for $Re = 300$ and 1000, which supports the accuracy of the present calculations.

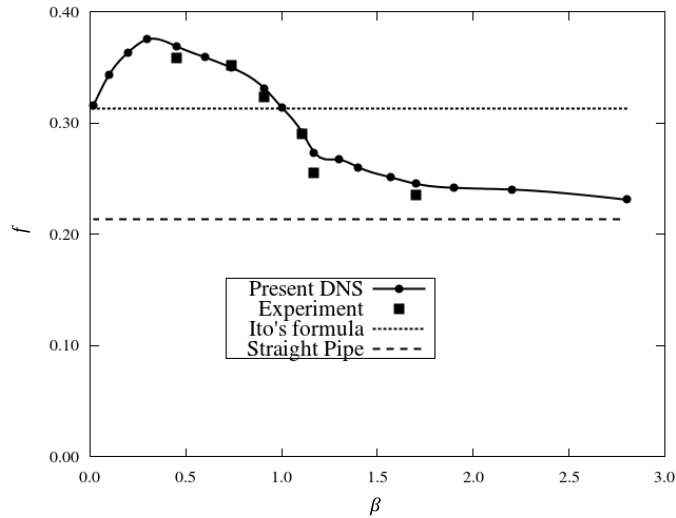


Fig. 5.10. Friction factor of the helical pipe for $Re = 300$ at $\delta = 0.1$.

Although the results of Liu and Masliyah (1993) for $Dn = 300$ and $\delta = 0.16$ (Fig. 19(a) in their paper), which correspond to $Re \cong 250$ since $Dn = Re\sqrt{\delta}$, qualitatively agree with the present results for $Re = 300$, the relative increase in the peak value from that for a toroidal pipe is slightly different. We found that the relative increase in the present DNS is 7% larger than that of Liu and Masliyah (1993), and regarding the approach to the value for a straight pipe, the present value approaches much more slowly than that observed by Liu and Masliyah (1993). It is noteworthy that for a low Reynolds number ($Re = 80$), the friction factor has no peak in the result of Liu and Masliyah (1993) (Fig. 19(b) in their paper), where their value of $Dn = 20$ corresponds to $Re = 80$ and $\delta = 0.0625$. Because the present DNS results show good agreement with the experimental data and the present DNS is 3D (two-dimensional (2D) in Liu and Masliyah (1993)), it is very plausible that our results are more accurate than those of Liu and Masliyah (1993).

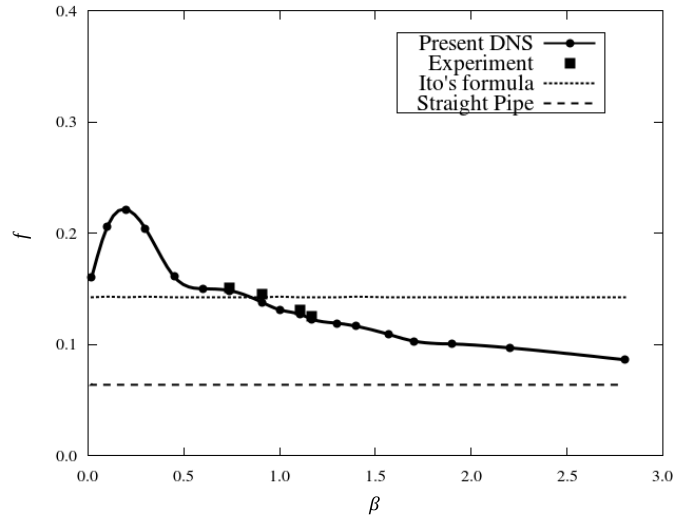


Fig. 5.11. Friction factor of the helical pipe for $Re = 1000$ at $\delta = 0.1$.

In Fig. 5.11, we found that the friction factor rapidly decreases from the maximum value and slowly tends to that of a straight pipe as β increases for $Re = 1000$. Note that in the region $0.8 \leq \beta \leq 1.8$, Hayamizu et al. (2008) found that the steady solution is linearly unstable and that unsteady solutions are expected to appear. Although, we did not calculate the time average of the friction factor for unsteady solutions in the unstable region of Fig. 5.11, it is interesting that the results of the present steady calculations are in good agreement with the experiment data.

5.4.2 Friction factor for Type-2 ($\delta = 0.05$)

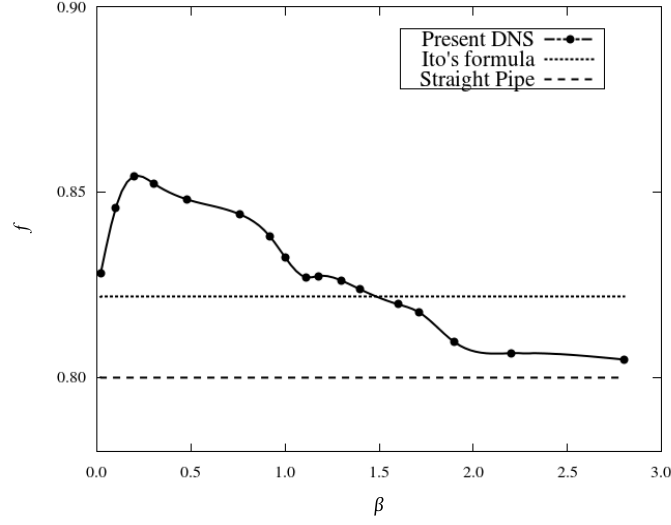


Fig. 5.12. Friction factor of the helical pipe for $Re = 80$ at $\delta = 0.05$.

It is shown in Figs. 5.12-5.14 that the friction factor first increases then decreases as β increases. The friction factor monotonically decreases toward that of a straight pipe as β further increases. The maximum of the friction factor occurs at $\beta \cong 0.3$ for $Re = 80$ and 300, as respectively shown in Figs. 5.12 and 5.13, and at $\beta \cong 0.2$ for $Re = 1000$ as shown in Fig. 5.14. For the Reynolds numbers $Re = 300$ and 1000, the value of β giving the maximum friction factor is the same at $\delta = 0.1$ and 0.05.

Although we have no experimental data available for the small Reynolds number of $Re = 80$, the experimental data for $Re = 300$ and 1000 are in good agreement with the DNS results as shown in Figs. 5.13 and 5.14. The present DNS results of the friction factor for $Re = 300$ and 1000 at $\delta = 0.05$ are qualitatively similar to the results at $\delta = 0.1$, but the values of the maximum the friction factor are smaller than those for $\delta = 0.1$.

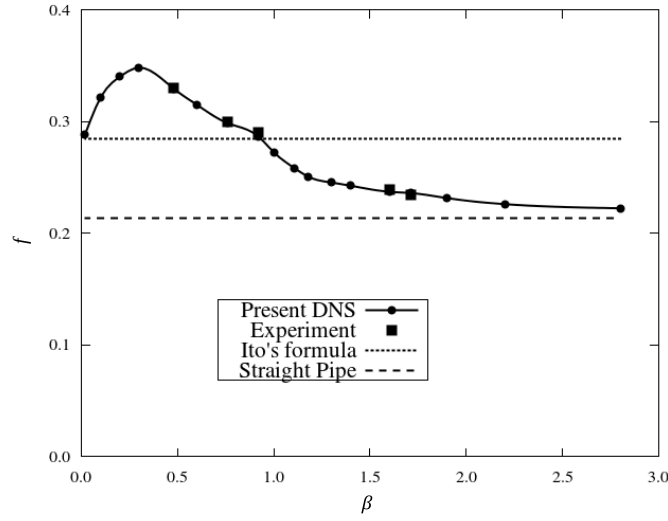


Fig. 5.13. Friction factor of the helical pipe for $Re = 300$ at $\delta = 0.05$.

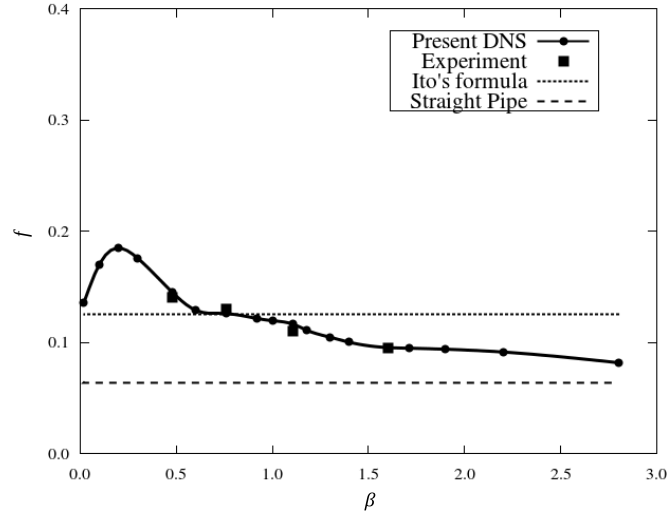


Fig. 5.14. Friction factor of the helical pipe for $Re = 1000$ at $\delta = 0.05$.

From the discussion on Figs. 5.9 -5.14, we can conclude that the formula for the friction factor of a toroidal pipe is not applicable to a helical pipe for $\beta \geq 0.1$ at $\delta = 0.1$ as illustrated by the cases studied in the present paper. We found that the friction factor of the helical pipe approaches that of a straight pipe for $\beta > 2.5$ at low Reynolds numbers (Figs. 5.9-5.10 and 5.12-5.13) of $Re = 80$ and 300 . For the higher Reynolds number (Figs. 5.11 and 5.14) of $Re = 1000$, the peak of the friction factor is sharp and the approach the value for a straight pipe is slower than that at low Reynolds numbers. We list the calculated values of the friction factor in Table 5.3.

Table 5.3: Friction factors for three Reynolds numbers at $\delta = 0.1$ and 0.05 .

$\delta = 0.1$					$\delta = 0.05$			
Case	f			Case	f			
β	$Re = 80$	$Re = 300$	$Re = 1000$	β	$Re = 80$	$Re = 300$	$Re = 1000$	
0.02	0.855023	0.316098	0.160373	0.02	0.828002	0.288205	0.135716	
0.1	0.879217	0.343315	0.205309	0.1	0.845671	0.321137	0.169997	
0.2	0.889863	0.363433	0.221278	0.2	0.854284	0.339921	0.184821	
0.3	0.893296	0.375112	0.204154	0.3	0.852379	0.348112	0.175538	
0.45	0.889213	0.368951	0.160806	0.48	0.848215	0.329384	0.144926	
0.6	0.880045	0.359121	0.150001	0.6	0.846299	0.315156	0.129231	
0.74	0.875791	0.349575	0.148332	0.74	0.844001	0.298492	0.125773	
0.91	0.865968	0.331143	0.137681	0.92	0.838005	0.286531	0.121474	
1.0	0.857328	0.314151	0.131084	1.0	0.832425	0.272459	0.119494	
1.11	0.854577	0.290768	0.126872	1.11	0.827025	0.257889	0.11625	
1.17	0.844429	0.273529	0.122608	1.18	0.827358	0.250459	0.110837	
1.3	0.835134	0.267219	0.119010	1.3	0.826123	0.245237	0.104619	
1.4	0.834749	0.260123	0.116265	1.4	0.823737	0.242384	0.100113	
1.57	0.825541	0.251055	0.108944	1.60	0.819731	0.236993	0.095283	
1.70	0.813712	0.245279	0.102894	1.71	0.817634	0.235875	0.094712	
1.9	0.810629	0.241623	0.100317	1.9	0.809522	0.231359	0.093813	
2.2	0.806061	0.240062	0.096842	2.2	0.806522	0.225904	0.091279	
2.8	0.803658	0.230966	0.086121	2.8	0.804825	0.222186	0.081712	

5. 5. Discussions and conclusion

A numerical study involving 3D DNSs of the flows through helical pipes has been conducted to obtain the friction factor over a wide range of the torsion parameter, β , for three Reynolds numbers, and two curvatures, The obtained results were compared with experimental data and also the results of a previous study. For small β , we found that the friction factor of the helical pipe becomes larger than that of a toroidal pipe. When β further increases, the friction factor tends to decrease from that of a toroidal pipe after taking a maximum and approaches that of a straight pipe. Therefore, the effect of torsion on the friction factor of the helical pipe becomes large when β exceeds about 1.0, which is in good agreement with the experimental results. We also found that the magnitude of the axial velocity is smaller than that of a toroidal pipe but larger than that of a straight pipe for $\beta \geq 1.17$, which is consistent with the effect of torsion on the friction factor.

The results of the present DNSs were in reasonably good agreement with the experimental data¹⁾ for $Re = 300$ and 1000 , while no experimental data was available for $Re = 80$. The numerical results obtained by Liu and Masliyah (1993) were qualitatively similar to the present results for $Re = 300$ and 1000 , but the relative peak value of the friction factor was larger and the approach to the asymptotic value for a straight pipe was slower in the present results than in the work of Liu and Masliyah (1993). In addition, although their friction factor curve²⁾ had no peak for a low Reynolds number of $Re = 80$, ours had a peak at nearly the same value of β as that for $Re = 300$ and 1000 . Because the present DNS results were 3D and supported by the experimental data, while the simulation by Liu and Masliyah (1993) was 2D, our results are thought to be superior to those of Liu and Masliyah (1993). It is noteworthy that we found a global peak of the friction factor over a wide range of β for low to high Reynolds numbers, where the value of the peak decreases as the Reynolds number increases and the curvature decreases. It is interesting that the peak existed in the range of $0.2 \leq \beta \leq 0.3$ for all values of the Reynolds number and curvature studied in the present paper; thus, the torsion parameter was found to play an important role in the helical pipe flow. It is also interesting that the friction factor formula for a toroidal pipe, was not applicable for

$\beta \geq 0.1$ as illustrated by the cases studied in the present paper, and that the friction factor of a helical pipe gradually approached that of a straight pipe for $\beta > 2.5$.

Chapter 6

Thermal flow through a helical pipe with circular cross section

6.1 Objectives and scope of this chapter

The study of heat transfer in a helical pipe has been performed extensively, both experimentally and numerically, because of a lot of engineering applications and scientific interest. Due to higher heat transfer efficiency compared to straight pipe configuration, heat transfer in a helical pipe is frequently used in power generation, nuclear industries, refrigeration and air-conditioning systems, heat recovery systems, food and dairy processes and other thermal processing plants. A main flow feature in a helical pipe is the presence of the secondary flow motion generated by the combined effects of centrifugal force and pitch-induced torsion. The secondary vortices promote fluid mixing and in turn enhance heat transfer performance resulting in larger pressure drop in the flow passage.

In this study, Laminar forced convective heat transfer in a helical pipe with circular cross section subjected to wall heating is presented numerically by 3D DNS comparing with the experimental data. Numerical calculations will be carried out over the wide range of torsion parameter, β , Reynolds number, Re , for three types of the Prandtl number. The main objective of this study is to numerically investigate the effect of torsion on the averaged Nusselt number (averaged over the peripheral of the pipe cross section) at the well-developed axially invariant flow region supported by the experimental data. In this study, we will investigate the heat transfer flow characteristics such as secondary flow, fluid trajectory of the fluid particle and temperature distribution due to the effect of torsion.

The objectives of this study are:

- Develop a new model and method to investigate the 3D forced convective heat transfer in a helical pipe.
- A way to find the appearance of the well-developed axially invariant flow region.
- Find the effect of the heated wall and non-heated wall on the well-developed axially invariant flow region.

- To find the stabilizing or destabilizing effect of torsion on the secondary flow in a helical pipe.
- Find the general scenario of the averaged Nusselt number due to the effect of torsion and the Prandtl number over the wide range of Reynolds number and comparison with the experimental data.
- Expect a general scenario of the averaged Nusselt number due to the effect of the Reynolds number while the torsion is changed
- Find the secondary flow, fluid trajectory of the fluid particle and temperature distribution on helical pipes over the wide range of torsion and the Reynolds number.
- Play a pioneer role in finding the local Nusselt number over the circle at the well-developed axially invariant flow region.
- Find a close relationship between the local Nusselt number and the vortical structure of the secondary flow with heated wall in helical pipe flow.
- To find the existence of the global minimum and global maximum of the averaged Nusselt number.
- Help to understand the physics and mechanism of the flow behavior with heat transfer inside a helical pipe.

6.2 Results

In this section, the results of DNS are presented. In the first part, steady solutions obtained by DNS are compared with the experimental data (Shatat (2010)) in the fully developed region of the helical pipe for $\beta = 0.0012$ and $\delta = 0.018$. In the second part, we obtained the Nusselt number averaged over the pipe circumference in consideration of the effect of torsion. The computational mesh numbers have been varied in order to check the numerical accuracy, where the number of mesh in the radial direction is M , and N in the circumferential direction. The uniform mesh was generated in the z -direction, where the number of mesh is L . We obtained the steady solutions at $Re = 1633$ and $\delta = 0.018$ for $\beta = 0.0012$ and $Pr = 8.5$ for four types of the mesh, which are named as M1, M2, M3 and M4 as listed in Table 6.1. For each mesh, the maximum axial velocity, w_{max} , the averaged Nusselt number, $\overline{Nu}$ over the circumference of the pipe wall at a fully developed region and the total number of

grids are shown in Table 6.1. Based on Table 6.1, we concluded that M3 is sufficient for the computation of the force convective heat transfer in a helical pipe.

Table 6.1: Grid sensitivity analysis for $Re = 1633$ and $\delta = 0.018$ at $\beta = 0.0012$

Mesh name	Mesh size			$\overline{Nu}$	w_{max} [m/s]	Total number of grid
	M	N	L			
M1	8	11	360	15.38938	1.66007	249600
M2	10	13	360	16.53598	1.67090	372000
M3	12	16	360	17.82339	1.68025	527400
M4	14	18	360	17.84031	1.68158	722400

6.2.1 Nusselt number

The Nusselt number, Nu , defined in Eq. (7), is used as an index of the heat transfer of forced convection from the uniformly heated wall. It should be pointed out that since the experiment was performed for $Pr = 8.5$, we compared the present DNS results with the experimental data for this case. To validate the present DNS, we obtained Nu for two cases of Pr . They are plotted with the symbol $\bullet$ for $Pr = 8.5$ and $\blacktriangle$ for $Pr = 7.5$ as a function of the Reynolds number in Fig. 4, where Manlapaz and Churchill's (1981) correlation formula

$$\overline{Nu} = \left[\left(\frac{48}{11} + \frac{51/11}{[1+1342/Pr Dn^2]^2} \right)^3 + 1.816 \left(\frac{Dn}{1+1.15/Pr} \right)^{3/2} \right]^{1/3} \quad (6.1)$$

is also shown with the solid line and the experimental data with the symbol $\blacksquare$. It should be remarked that Eq. (6.1) is valid for negligible torsion as $\beta \ll 1$.

As shown in Fig. 6.1, the averaged Nusselt number increases as Re increases. It is found that the present DNS results for $Pr = 8.5$ is very close to the experimental data, and Eq. (6.1) over the wide range of the Reynolds number, $1000 \leq Re \leq 4100$. It should be pointed out that Yamamoto et al. (1995) experimentally found the critical Reynolds number of the laminar to turbulent transition, $Re_c = 2600$, for $\beta = 0.002$ and $\delta = 0.01$ with no thermal effect, by observing the unsteady behavior of secondary and axial flow. Since the velocity flow field is hardly affected by the temperature field in the case of forced convection, the critical Reynolds number may be nearly the same for the present forced convection.

For $Pr = 8.5$, we found that for higher Reynolds numbers ($Re > 2600$), the Nusselt numbers obtained by the present DNS are a little higher than the values obtained experimentally, while the Nusselt numbers by DNS are in an excellent agreement with the experimental data for $Re < 2600$. The small differences for $Re \geq 2600$ are probably due to the turbulence effect. We also found that Nu slightly decreases as the Pr decreases, as shown in Fig. 6.1 for $Pr = 7.5$. For $Pr = 7.5$, Nu is in fairly good agreement with Eq. (6.1) although the curve of Eq. (6.1) for $Pr = 7.5$ is not shown in Fig. 6.1.

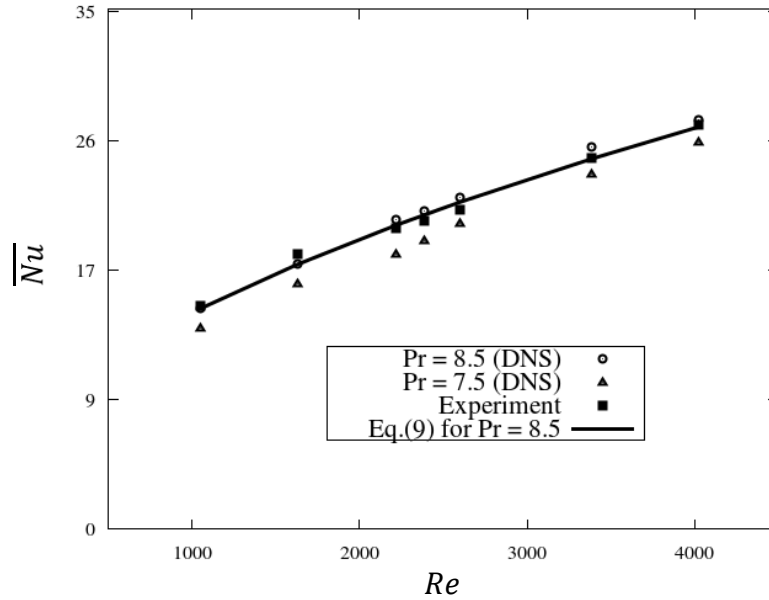


Fig. 6.1. The Nusselt number for $\beta = 0.0012$ and $\delta = 0.018$. The symbols $\bullet$ and $\blacktriangle$ represents the present DNS results for $Pr = 8.5$ and 7.5 , respectively, and the experimental data by Shatat (2010) with the symbol $\blacksquare$ for $Pr = 8.5$.

6.3 Torsion effect on the Nusselt number

In order to study the effect of torsion on the Nusselt number over the range of Reynolds number, $1000 \leq Re \leq 4100$, we conducted numerical calculations for $\beta = 0.10, 0.30, 0.45$ and 0.80 at $\delta = 0.018$ and $Pr = 4.02$ without turbulence model. It should be remarked that no experimental data nor numerical result is available for higher torsion case to the author's knowledge of literature. Since the present DNS results agree with the experimental data pretty well for $\beta = 0.0012$ as shown in Fig. 6.1, the accuracy of the present calculations is assured.

It should be remarked that Shatat (2010) experimentally found that the temperature averaged over the peripheral circle of the pipe wall varies linearly from the inlet to the outlet of the helical pipe, which is in good agreement with the present DNS result. Shatat (2010) found that the fully developed region exists in one turn after two turns from the inlet of the helical pipe except for the region near the outlet of the helical pipe. On the other hand, we numerically found the fully developed region in two turns after three turns from the inlet of the helical pipe. We compared the experimental results with DNS data in the fully developed region for each study.

In Fig. 6.2, we show DNS results for five values of β , where Nu decreases as β increases from zero for $1000 \leq Re \leq 4100$, and a minimum of Nu occurs at $\beta = 0.1$. It is interesting that Nu tends to increase from a minimum as β increases for $\beta > 0.1$. For a wide range of β , it is found that a maximum of Nu occurs at $\beta = 0.45$. If β further increases, Nu decreases again.

Yang *et al.* (1995) found that Nu significantly decreases as β increases from zero (a toroidal pipe) by the 2D numerical simulation. The present 3D DNS result is in good agreement with the result of Yang *et al.* (1995) for $\beta \leq 0.1$.

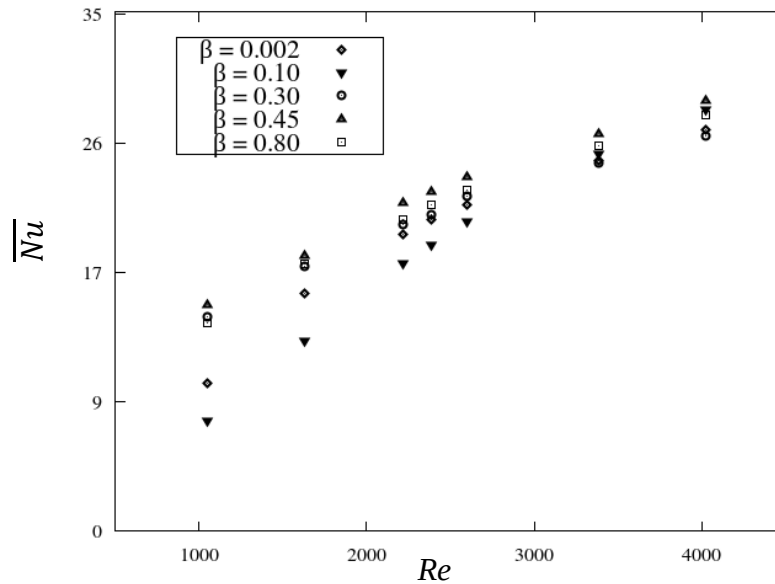


Fig. 6.2. The Nusselt number for $\delta = 0.018$ at $Pr = 4.02$. The symbol $\blacktriangledown$ represents the Nu for $\beta = 0.1$, $\circ$ for $\beta = 0.3$, $\blacktriangle$ for $\beta = 0.45$ and $\square$ for $\beta = 0.8$.

The variation of Nu as a function of β is shown in Fig.6.3 for three Reynolds numbers ($Re = 1047, 1633$ and 2600) at $Pr = 4.02$ and $\delta = 0.018$. It should be remarked that turbulence may affect the value of Nu for $Re > 2600$ in experiments (Yamamoto et al. (1995)). In Fig. 6.3, it is clear that Nu increases as Re increases. It is found in Fig. 6.3 that each curve of Nu has two local minimums and two local maximums, where the global minimum exists at $\beta \cong 0.1$ and the global maximum at $\beta \cong 0.55$ for any value of Re .

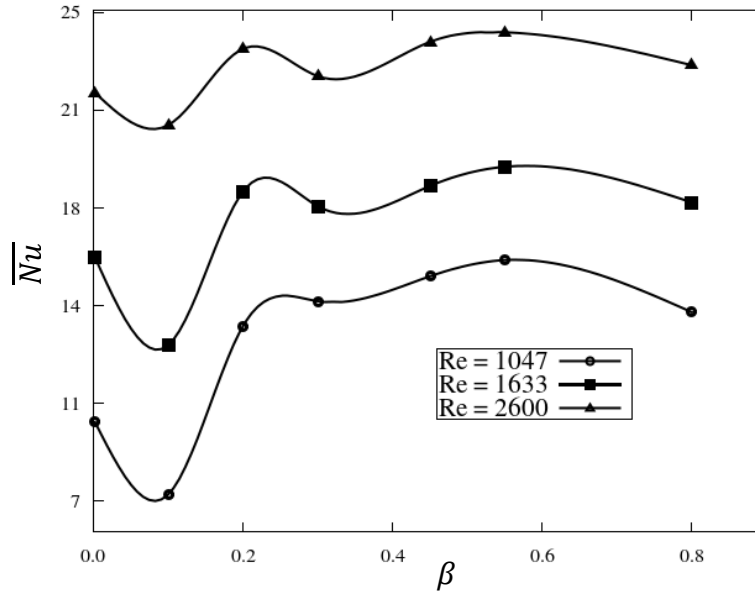


Fig. 6.3 The Nusselt number as a function of β for $\delta = 0.018$ at $Pr = 4.02$. The line of $\circ$ for $Re = 1047$, $\square$ for $Re = 1633$ and $\blacktriangle$ for $Re = 2600$.

6.4 Torsion effect on the secondary flow and temperature field

In the present 3D steady calculations, we found the fully developed flow region, where the flow and temperature profiles do not change appreciably with the pressure gradient $\partial p / \partial z$ being nearly constant in the z -direction. This region lies in the part of the helical pipe after three turns from the inlet of the helical pipe except for the region near the outlet. It should be noted that forced convection in a helical pipe is affected by the secondary flow in the cross section very much since the temperature field is strongly dependent on it. Therefore, it is important to examine the secondary flow pattern in the forced convection.

A helical pipe is characterized by the value of β . When β is zero, the helical pipe reduces to a toroidal pipe, and if β tends to infinity, it approaches a straight pipe. One of the important differences between a laminar flow in a toroidal pipe and that in a helical pipe is that two symmetrical vortices appear in the cross section for the toroidal pipe, while one dominates another for the helical pipe due to the effect of the finite value of β .

In order to study the cases of the minimal and maximal averaged Nusselt number, we show the vector plots of the secondary flow for $\beta = 0.002$ (small or negligible torsion case), 0.1 (minimal averaged Nusselt number), 0.55 (maximal averaged Nusselt number) at the fully developed region for $Re = 1047$, $\delta = 0.018$ and $Pr = 4.02$ in Fig. 6.4. We also show the cut section of the contours of the Q -value (the second invariant of the velocity gradient tensor) surface (see Lesieur *et al.* (2005)) on the same figure.

For the small torsion case ($\beta = 0.002$), the axial flow yields the centrifugal force toward outside of the pipe, and the fluid is moved from the inner to outer side. Because of mass conservation, the fluid returns to the inner side around the upper and lower region of the cross section. As a result, secondary flow is formed in the cross section exhibiting two symmetric circulating cells shown in Fig. 6.4 (a). If torsion effect begins the work, the symmetric two-cell pattern changes into the asymmetric two-cell pattern, as shown in Fig. 6.4 (b).

In Fig. 6.4 (b), we observe asymmetric secondary flow with a slightly weak anti-clockwise vortex in the upper semicircle and a slightly strong clockwise vortex in the lower semicircle for $\beta = 0.1$. For $\beta = 0.55$ in Fig. 6.4 (c), a large and strong vortex is produced in the lower part of the cross section and the vortex becomes small and weak in the upper part. This shows that, torsion of the pipe increases the strength of the vortex rotating in the same direction as torsion. It should be noted that since torsion works as a rotational force, its effect is to induce a circulating motion in the fluid. It is seen in Fig. 6.4 (c) that for $\beta = 0.55$, the secondary flow approximately consists of one large vortex, the center of which lies in the middle of third and fourth quadrant of the cross section.

To supply additional insight into the vector plot of secondary flow, we show the 2D stream lines, the projection of the trajectory of fluid particle on the cross section for several values of β as shown in Fig. 6.5, where the vortical structure is seen more clearly.

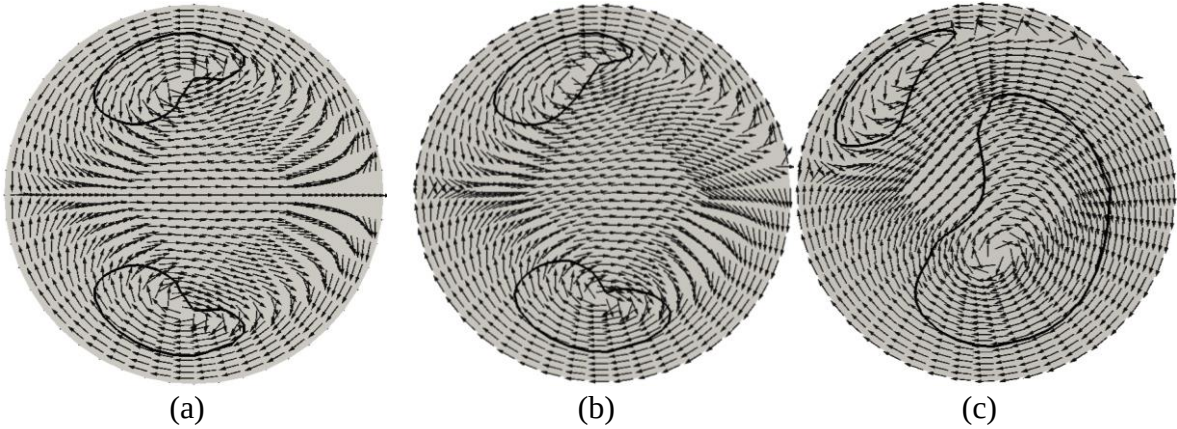


Fig. 6.4. Vector plots of the secondary flow and contours of the Q -value for $Re = 1047$, $\delta = 0.018$ and $Pr = 4.02$, where an outer wall is on the right-hand side of each figure. The thick lines denote the contour of the Q -value, where $Q = 1.0$ for all figures. Figure 6.4 (a) is for $\beta = 0.002$ at $z = 0.16$ m, (b) $\beta = 0.1$ at $z = 5.95$ m and (c) $\beta = 0.55$ at $z = 2.01$ m.

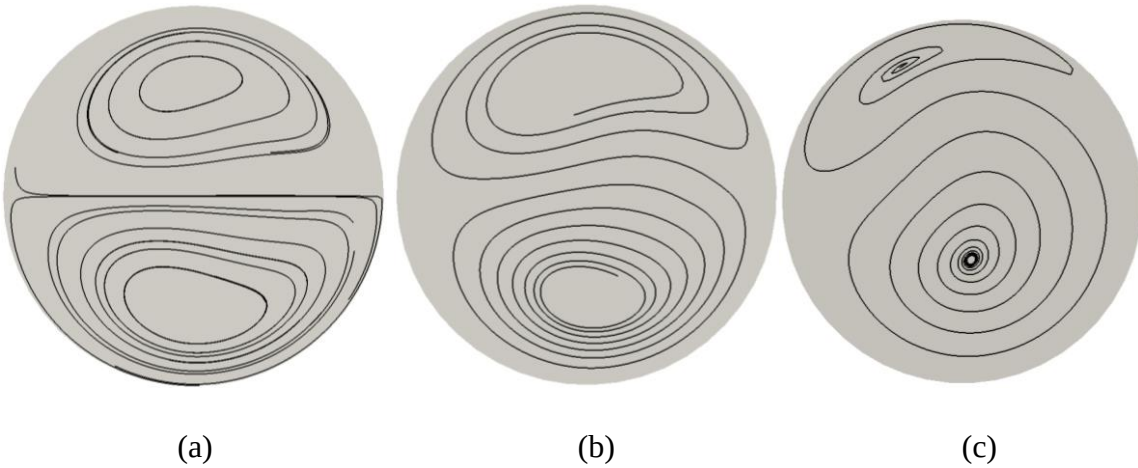


Fig. 6.5. 2D stream lines of the secondary flow for $Re = 1047$, $\delta = 0.018$ and $Pr = 4.02$, where an outer wall is on the right-hand side of each figure. Figure 6.5 (a) is for $\beta = 0.002$ at $z = 0.16$ m, (b) $\beta = 0.1$ at $z = 5.95$ m and (c) $\beta = 0.55$ at $z = 2.01$ m.

Then, we study the effect of torsion on the temperature field, where the temperature is non-dimensionalized by $\frac{T-T_w}{T_w-T_b}$. Figure 6.6 displays the temperature field in the cross section of the helical pipe obtained by DNS at the fully developed flow region for $\beta = 0.002, 0.1$ and 0.55 at $Re = 1047$, $\delta = 0.018$ and $Pr = 4.02$, where the wall of the helical pipe is homogenously heated. The contour lines are drawn at every -10 of the temperature profile for $\beta = 0.002$ and 0.1 and every -20 of the temperature profile for $\beta = 0.55$.

It should be noted that the minimum temperature region exists inside the pipe because fluid particles gain heat on the pipe wall and move into the inner pipe region convected by the axial and secondary flow. It is found that a contour of the minimum temperature lies in the middle of the first and second quadrant on the upper semi-circle for $\beta = 0.002$ and the second minimum lies in the middle of the third and fourth quadrant, as shown in Fig. 6.6 (a). In Fig. 6.6 (b) for $\beta = 0.1$, we notice that the temperature field becomes slightly asymmetric similarly as the secondary flow in Fig. 6.4(b). As β increases, the asymmetry of the temperature field becomes conspicuous as shown in Fig. 6.6(c) for $\beta = 0.55$. We find the minimum temperature region lies in the second quadrant on the upper semi-circle of the cross section.

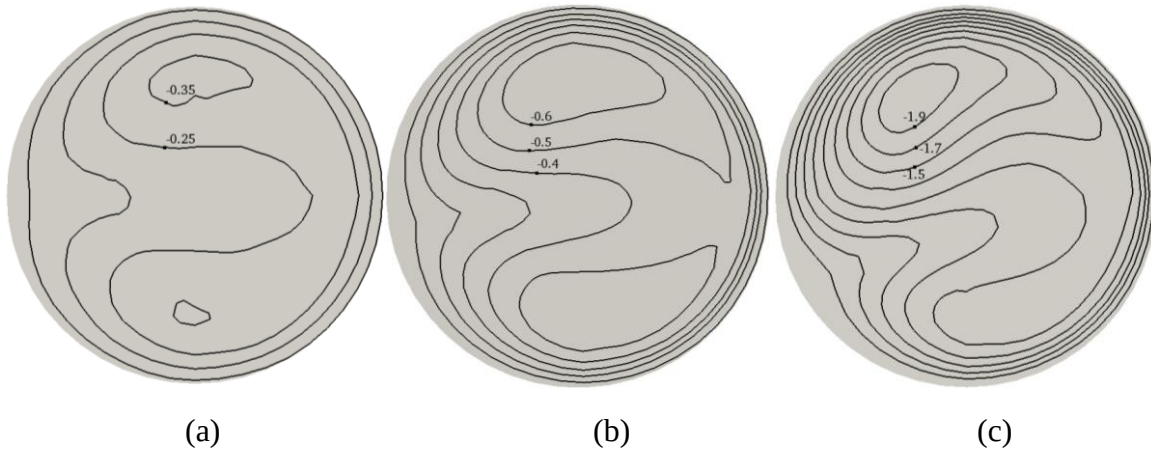


Fig. 6.6. Contours of the temperature field for $Re = 1047$, $\delta = 0.018$ and $Pr = 4.02$, where the outer wall is on the right-hand side of each figure. Figure 6.6 (a) is for $\beta = 0.002$ at $z =$

0.16 m, (b) $\beta = 0.1$ at $z = 5.95$ m and (c) $\beta = 0.55$ at $z = 2.01$ m. The increment of the temperature is -10 for $\beta = 0.002$ and 0.1 and -20 for $\beta = 0.55$.

6.5 Local Nusselt number

Figure 6.7 shows the distribution of the local Nusselt number along the peripheral circle over the cross section at fully developed flow region for $Re = 1047$, $\delta = 0.018$ and $Pr = 4.02$. The torsion parameter, β , which controls the flow characteristics by the rotational force varies from 0.002 to 0.55. The abscissa in Fig. 6.7 is the angle along the peripheral circle of the cross section and ordinate the local Nusselt number at different positions on the circle.

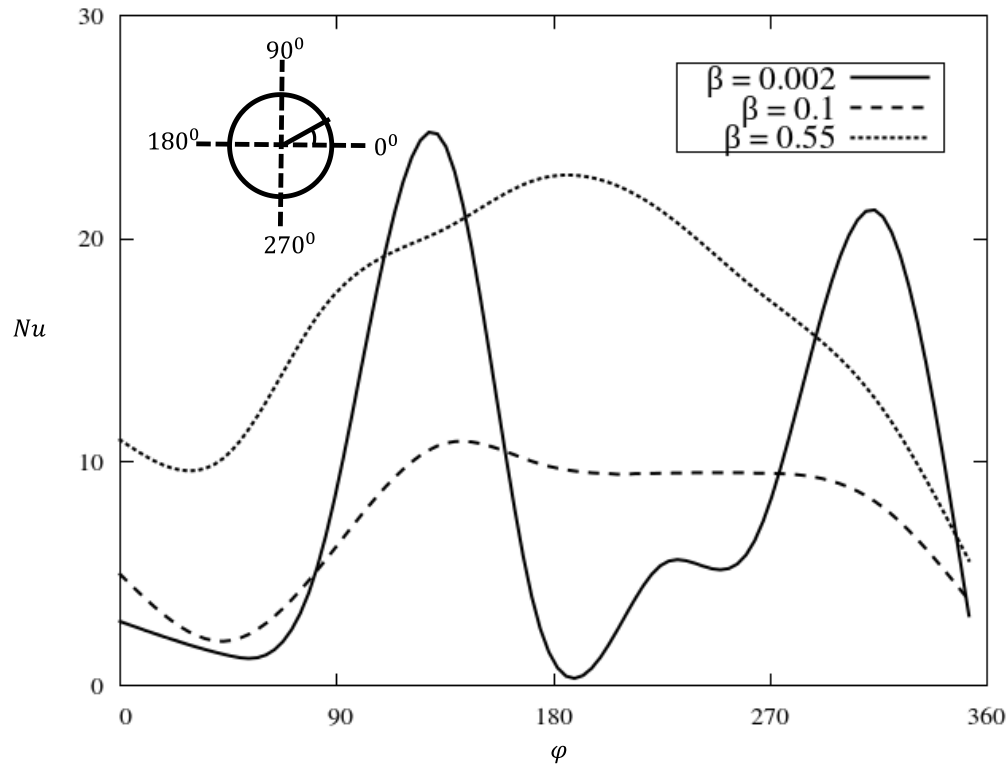


Fig. 6.7. Local Nusselt number distribution for $Re = 1047$, $\delta = 0.018$ and $Pr = 4.02$. The solid is for $\beta = 0.002$ at $z = 0.16$ m, a dashed line for $\beta = 0.1$ at $z = 5.95$ m and a dotted line for $\beta = 0.55$ at $z = 2.01$ m.

For the small torsion case ($\beta = 0.002$), we found the wavy behavior of the local Nusselt number on the circle as shown with a solid line in Fig. 6.7. This result is similar to the previous study of Yang et al. (1995). In its wavy behavior, it is clearly seen that two maximal

peaks of the local Nusselt number exist in the upper and lower semicircles and the minimum value of the local Nusselt number is located on the inside wall. We notice that the peak of the local Nusselt number is slightly higher at the upper wall than at the lower wall. It is found that the main contribution to the averaged Nusselt number comes from these two peaks.

It is interesting that for $\beta = 0.1$, any peak of the local Nusselt number disappears and the distribution of the local Nusselt number becomes rather flat as shown by a broken line in Fig. 10. The rotational force due to increase of torsion parameter weakened the two-cell pattern of the secondary flow and consequently wiped out the peaks of the local Nusselt number. As a result the averaged Nusselt number takes a bottom at $\beta \cong 0.1$.

However, if β further increases and becomes 0.55, the distribution of the local Nusselt number goes upward as shown by a dotted line in Fig. 6.7, so that the averaged Nusselt number recovers its value and becomes greater than that for $\beta = 0.002$. It is caused by a strong swirling motion of the secondary flow at large torsion parameter. The moderate peak of the local Nusselt number lies around the inside wall.

In summary, for small torsion parameter, heat transfer from the pipe wall to fluid occurs at two local regions on the wall due to the two-cell pattern of the secondary flow. For large torsion parameter, on the other hand, heat transfer occurs globally on the wall due to the large one cell pattern of the secondary flow. For the intermediate torsion parameter, total heat transfer becomes minimum because both the local and global heat transfer are not effective.

6.6 Discussion and conclusion

3D DNS study of the flow through helical pipes has been conducted and the averaged Nusselt number was obtained for the wide range of torsion parameter, β , three Reynolds numbers, $Re = 1047, 1633$ and 2600 , three Prandtl numbers, $Pr = 8.5, 7$ and 4.02 and curvature, $\delta = 0.018$. Obtained results were compared with the experimental data (Shatat [1]) and also the previous study (Yang et al. [3]).

In 3D steady calculations with homogeneously heated wall temperature, we found the appearance of the fully developed flow region, where we compared the averaged Nusselt number obtained by DNS with the experimental data and the formula by Manlapaz and Churchill's [12]. For low torsion parameter, $\beta = 0.0012$, we found the Nusselt number of the present DNS was in good agreement with the experimental data for $Pr = 8.5$. If the Prandtl number decreases, we found the Nusselt number slightly decreases at low torsion case, which is in good agreement with the results by Yang et al. [3].

If β increases from zero (a toroidal pipe), then we found the averaged Nusselt number of the helical pipe begins to decrease than that of a toroidal pipe, and increases after taking the global minimum at $\beta \cong 0.1$. If β further increases, we found a local maximum of the averaged Nusselt number, which decreases and then takes the global maximum at $\beta \cong 0.55$ for any value of the Reynolds number. For the wide range β , we found that the torsion effect is remarkable on the averaged Nusselt number for any value of the Prandtl Number, and torsion parameter was found to play an important role in the forced convection of helical pipe flow.

In order to obtain physical insight into the reason of variation of the averaged Nusselt number as β changes, we plotted the secondary flow, 2D stream lines and the temperature field on the cross section for several values of β . It was found that the secondary flow exhibits nearly symmetric two-cell pattern for small β case, then becomes asymmetric and finally changes into one-cell pattern as β increases. The temperature field changed from a slightly asymmetric form to a strongly asymmetric form via an asymmetric form. We also plotted the local Nusselt number over the peripheral circle and studied the topological change of the temperature field as β increases. It was found that the minimum of the averaged Nusselt number occurred when both the effect of the secondary flow with symmetric two-cell pattern and that with one-cell pattern become feeble for $\beta \cong 0.1$. We hope the present result may contribute the improvement of heat exchanger using helical pipes.

Chapter 7

Turbulent flow through a helical pipe with circular cross section

7.1 Objective and scope of this chapter

Fluid flows in a helical pipe has been attracting considerable attention because of abundant applications not only to engineering but also to medical sciences. Flows in a helical pipe are often in a turbulent state at high Reynolds numbers. Pressure gradient is needed to drive turbulent flows in a pipe may be much larger than that for laminar flow in the presence of turbulent eddy viscosity. Therefore, it is necessary to study turbulent helical pipe flow in order to develop efficient transportation pipe system for liquids and gases. Due to the combined effects of centrifugal force and pitch-induced torsion, the fluid flow in a helical pipe differs from those in straight or toroidal (a helical pipe with a zero pitch) pipes. The curved pipe inevitably has a finite pitch if it has several turns, and hence the pipe is more or less in a helically coiled shape. The secondary flow caused by the centrifugal force due to curvature is affected by torsion, which creates different flow patterns from those in a toroidal pipe. The existence of complex secondary flows in the cross section is one of the remarkable features of the fluid flow in a helical pipe.

In this study, a comprehensive numerical study will be presented for a viscous incompressible turbulent fluid flow through a helical pipe with a circular cross section. Numerical calculations are carried out for two helical circular pipes having different pitches and the same non-dimensional curvature, δ , with the wide range of the torsion parameter, β , and the Reynolds number. The main objective of this study is to numerically estimate the critical Reynolds number of transition turbulent to fully develop turbulent by observing the axial flow distribution supported by the experimental data, and to find the best turbulence models that can good predict the characteristics of turbulent flow behavior. In addition to that we investigated the turbulent flow characteristics (such as the secondary flow, the axial flow and the intensity of the turbulent kinetic energy) in a helical pipe by use of three turbulence models incorporated in OpenFOAM comparing with the experimental data.

The objectives of this study are:

- To investigate the turbulent flow characteristics through a helical pipe by different turbulence models.
- In 3D ensemble average steady calculations with several turns of a helical pipe, find the appearance of the fully developed flow region.
- Estimate the critical Reynolds number of the transition from transitional to fully developed turbulent flow in a helical pipe by observing the axial flow distribution based on the experimental data.
- To clarify the consistency of the estimated critical Reynolds number in present study.
- Find a general scenario of the behavior of the turbulent flow characteristics with the ensemble-averaged secondary flow, the axial flow and the intensity of the turbulent kinetic energy.
- Find the best turbulence model that can predict the scenario of turbulent flow characteristics.
- To clarify, whether the momentum transfer due to turbulence dominates or not on the secondary flow pattern of the turbulent helical pipe flow.
- To conclude that the torsion effect is more remarkable on turbulent flows or laminar flows by comparing the flow behavior of the turbulent and laminar flows,
- Help to understand the physics and mechanism of the turbulent flow behavior inside a helical pipe.

7.2. Results

Hayamizu et al. (2008) obtained the critical Reynolds numbers of laminar to turbulence transition through a helical pipe with circular cross section in the $Re-\beta$ plane. They also found the transitional and supercritical behaviors of disturbances over a wide range of torsion parameter $0.02 \leq \beta \leq 1.8$. The critical Reynolds numbers are 1097, 717, 480, 318, 228, 106 and 554 for $\beta = 0.6, 0.8, 1.0, 1.2, 1.4, 1.6$ and 1.8, respectively.

In this section, the results of the numerical simulations are presented. We first investigate the transition to fully developed turbulence by comparing the experimental data with numerical results. Then, we discuss the fully developed turbulent flow characteristics. We show the results for the Reynolds number ranging from 3000 to 21000 with different pitches, where the cases for torsion parameter $\beta = 0.02$ and 0.45 are addressed in section 7.1 and 7.2, respectively. We confirmed that the flow attains a sufficiently developed equilibrium state in the middle part of the helical pipe and the state continues up to near the outlet of the pipe except for the region very close to the outlet.

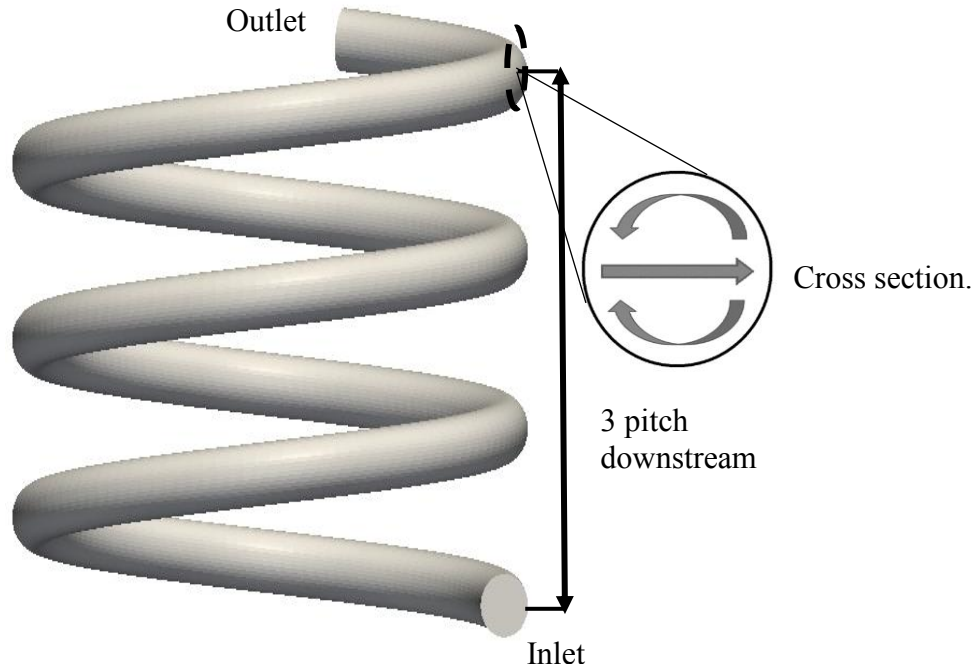


Figure 7.1. A view of the 3.2 pitch helical pipe showing the three-pitch downstream cross section for $\beta = 0.02$ and $\delta = 0.1$.

The computational mesh numbers have been varied in order to check the numerical accuracy. In Fig. 7.2, we show the axial velocity distribution along the x-axis using RNG $k - \varepsilon$ model at $Re = 5320$ for $\beta = 0.02$ and $\delta = 0.1$ for four types of the mesh, which are named as M1, M2, M3 and M4 as listed in Table 7.1. For each mesh, the center line turbulent kinetic energy, $k_c = (R_{xx} + R_{yy} + R_{zz})/2$, the maximum axial velocity, $\bar{w}_{max}$, and the total number of grids are listed in Table 6.1. Based on Table 7.1 and Fig. 7.2, we concluded that M3 is sufficient for the computation of the turbulent helical pipe flow.

Table 7.1: Grid sensitivity analysis for $Re = 5320$ for $\beta = 0.02$ at $\delta = 0.1$.

Mesh name	Mesh size			k_c [m^2/s^2]	$\bar{w}_{max}$ [m/s]	Total number of grid
	M	N	L			
M1	8	11	360	8.0626×10^{-3}	1.3339	159744
M2	10	13	360	12.0239×10^{-3}	1.3479	238080
M3	12	16	360	15.5679×10^{-3}	1.3633	350208
M4	14	18	360	15.5317×10^{-3}	1.3653	462336

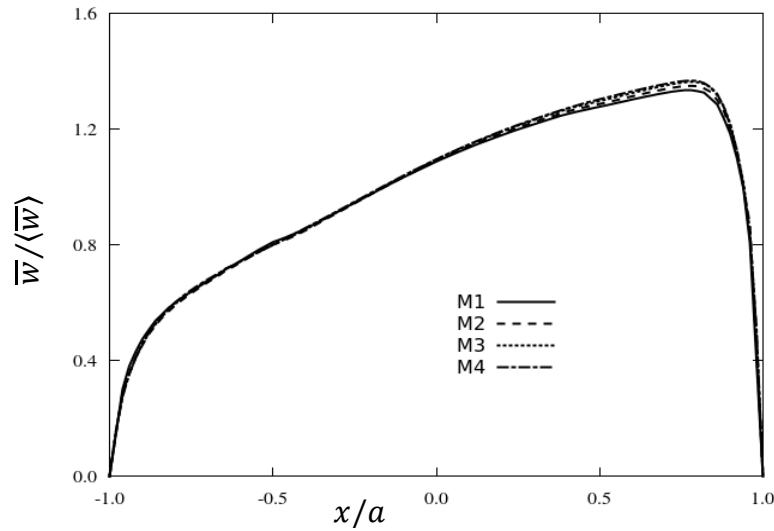


Figure 7.2. Axial flow distribution on the x-axis for $\beta = 0.02$ and $\delta = 0.1$.

7.3 Results of the case for $\beta = 0.02$

(A) Axial velocity distribution

In this subsection, we study the axial velocity distribution of the developing and fully developed turbulence. The development of the boundary layer near the outer wall is more delayed compared to that near the inner wall since the high axial flow region is moved to the outer wall by the secondary flow. Hence the boundary layer thickness on the outer wall is much smaller than that on the inner wall. In the present paper, we numerically obtained the axial velocity distribution along the x -axis and y -axis on the cross section of three-pitch downstream cross section (see Fig. 7.1) at $\beta = 0.02$ and $\delta = 0.1$ for the Reynolds number ranging from 5000 to 21000, as shown in Fig. 7.3 and 7.4, respectively. Here, the radial and vertical distance x and y is non-dimensionalized by a and the axial velocity $\bar{w}$ by $\langle \bar{w} \rangle$.

As shown in Fig. 7.3, it is clear that for low Reynolds numbers ($Re = 5320$ and 6880), any turbulence model cannot predict well the axial velocity profile of the experimental results especially in the inner region of the pipe (small x). It is interesting that for higher Reynolds numbers ($Re = 12140$ and 20850), on the other hand, the axial velocity profile of the experimental results is pretty well predicted by turbulence models although Launder-Sharma $k - \varepsilon$ model cannot predict well for $Re = 20850$ in the outer region (large x). The fact that two turbulence models can predict the velocity profile of the experimental results for $Re = 12140$ and 20850 suggests that the flow attains the fully developed turbulent state in these cases, and that the transition to fully developed turbulence occurs between $Re = 12140$ and 20850 .

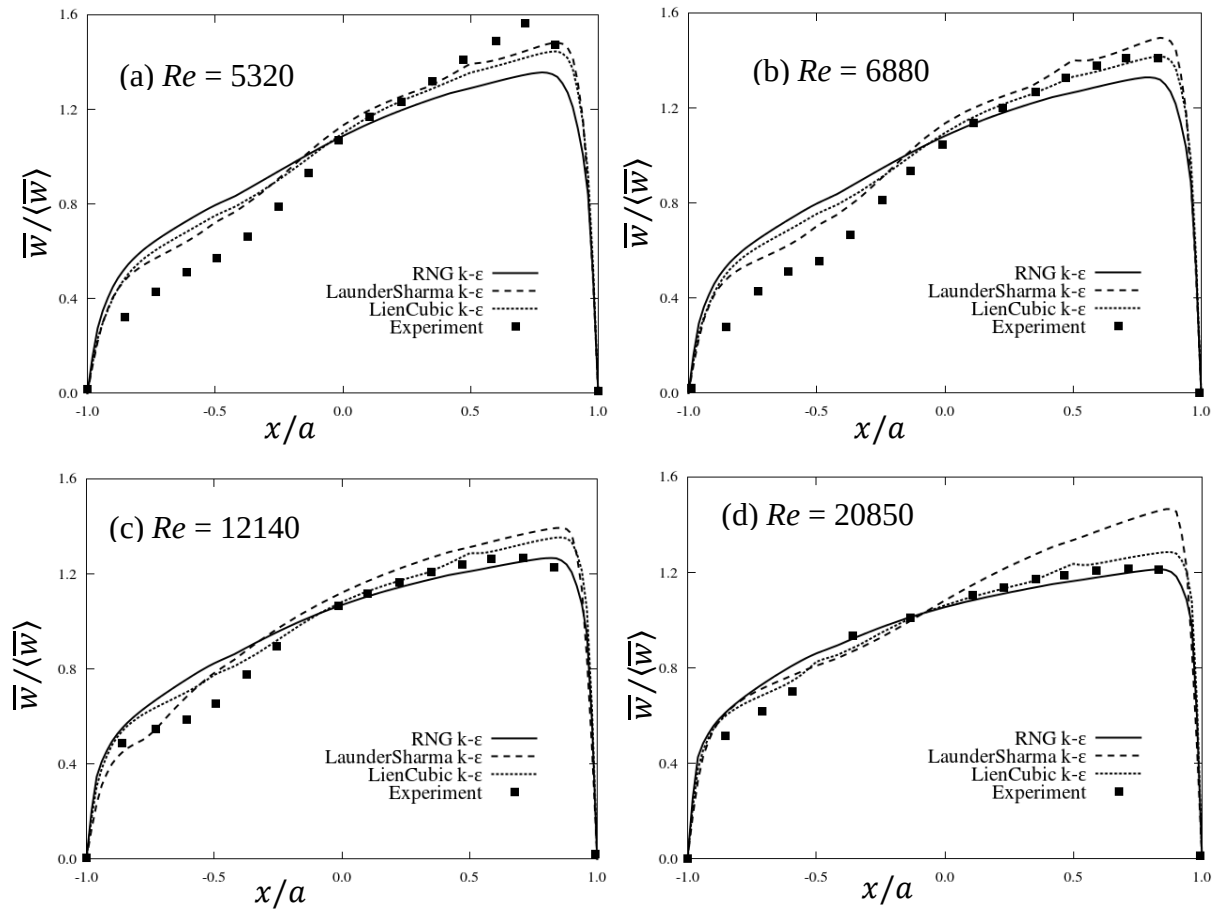


Fig. 7.3. Axial velocity distribution on the x -axis for $\beta = 0.02$ at $\delta = 0.1$. The outer wall is on the right-hand side.

Figure 6.4 shows vertical cuts of the axial velocity at three-pitch downstream (see Fig. 7.1) from the entrance for four Reynolds numbers. In Fig. 7.4, we observe that the axial velocity profile has local maxima near the upper and lower wall, and that no turbulent model can predict the axial velocity profile of the experimental data in low Reynolds numbers ($Re = 5320$ and 6880). For high Reynolds numbers ($Re = 12140$ and 20850), however, the predictions of two turbulence models show a pretty good agreement with the experimental data. For the largest Reynolds number ($Re = 20850$), the agreement is excellent. The reason why Launder-Sharma $k - \varepsilon$ model cannot give a good prediction is due to the fact that it is an isotropic turbulence model.

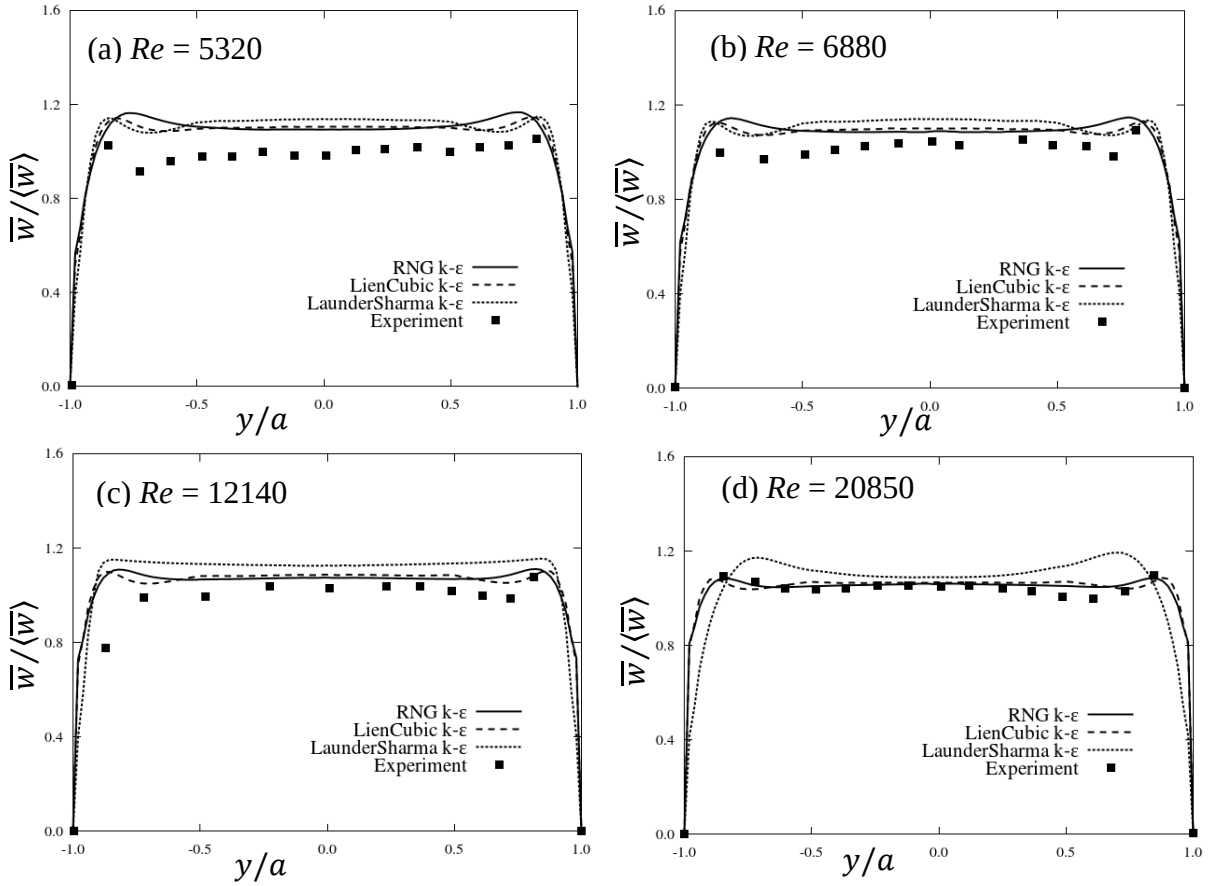


Fig. 7.4. Axial velocity distribution on the y -axis for $\beta = 0.02$ at $\delta = 0.1$. The upper wall is on the right-hand side.

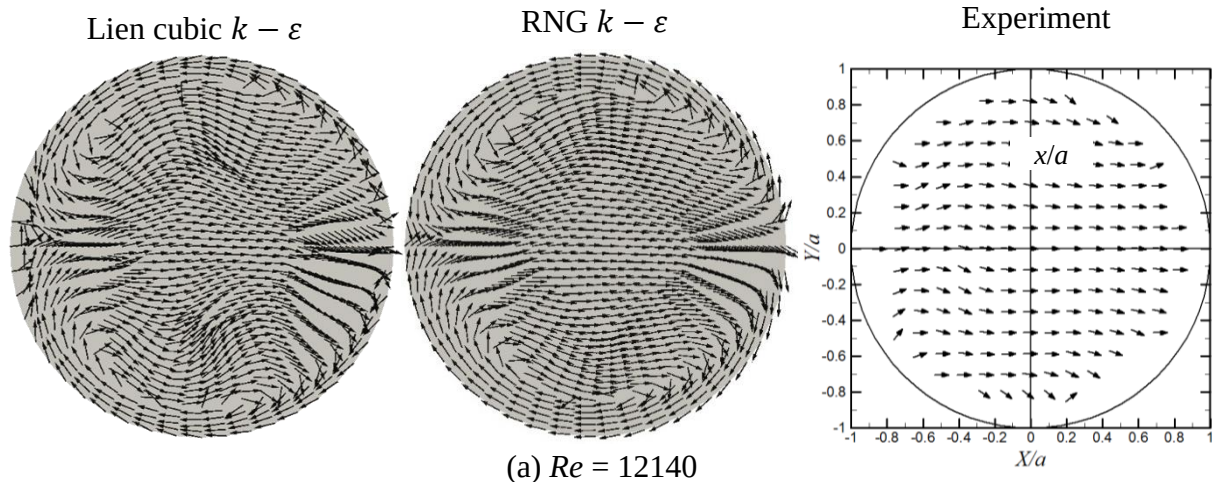
(B) Secondary flow pattern on the cross section of the helical pipe

A helical pipe is characterized by finite torsion. When torsion is zero, the helical pipe reduces to a toroidal pipe, and if torsion tends to infinity, it reduces to a straight pipe. One of the important differences between a laminar flow in a toroidal pipe and that in a helical pipe is there appear two symmetrical vortices in the cross section for the toroidal pipe, while these vortices are not symmetrical for the helical pipe due to the effect of non-zero torsion.

Figure 7.5 shows the projection of the fluid velocity (secondary flow) on the cross section of the helical pipe at the three-pitch downstream for $\beta = 0.02$ and $\delta = 0.1$. Numerical simulations were performed by use of Lien-cubic $k - \varepsilon$ and RNG $k - \varepsilon$ turbulence models.

Experimental results of the secondary flow for $Re = 12140$ were given by Wada (2010) and those for $Re = 20850$ by Yamamoto et al. (2005). The reason why we consider the three-pitch downstream cross section of the 3.2 pitch helical pipe both numerically and experimentally is due to the fact that the experimental data were taken there.

Now, we investigate the patterns of the secondary flow for high Reynolds numbers ($Re = 12140$ and 20850) because turbulence models predict the experimental data fairly well for high Reynolds numbers as discussed in section 7.1(A). In Fig. 7.5, it is found that the ensemble-averaged (numerical simulation) or time-averaged (experiments) secondary flow consists of uniform flow directed from the inner to outer sides of the helical pipe over almost all the cross section both for $Re = 12140$ and 20850 . The arrows in the figure show the secondary velocity field. Recirculating regions associated with the back flow from the outer to inner sides are very small both in the upper and lower semicircles. Flow structures are similar for two turbulence models, Lien cubic $k - \varepsilon$ and RNG $k - \varepsilon$ turbulence models, and experiments. It should be remarked that recirculating regions are hardly discerned in the experimental data probably because the experimental resolution near the pipe wall is not sufficient although some sign of the recirculating flow is seen in the upper and lower near wall regions. It is interesting to note that two vortices associated with the recirculating flow are clearly seen in the upper and lower hemisphere for $\beta = 0.02$ if the flow is laminar ($Re = 200$) as shown in Fig. 7.6, which was numerically obtained by use of OpenFOAM without any turbulence model. Therefore, the shrinking of these vortical regions may be due to the turbulence effect of the enhancement of momentum transfer on the cross section.



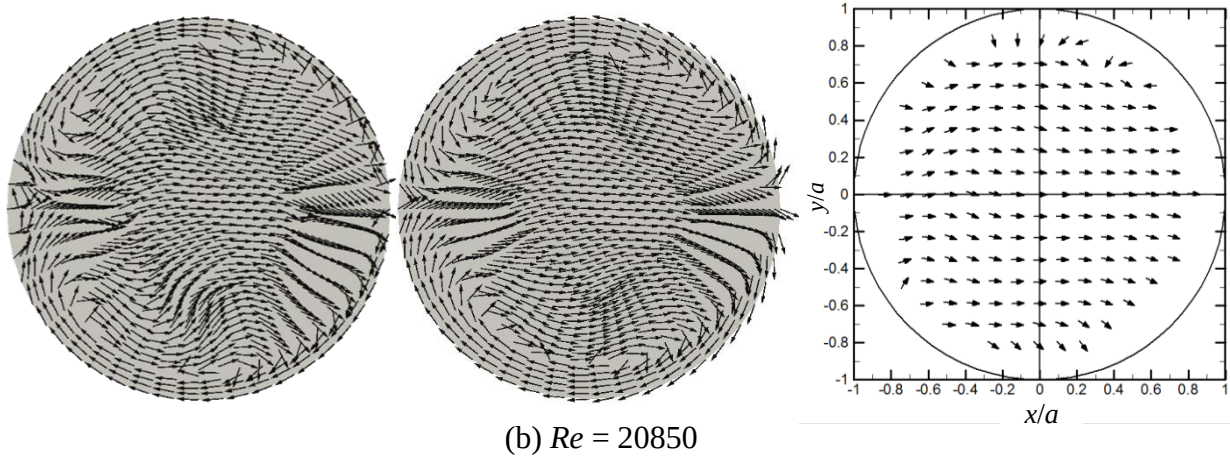


Fig. 7.5. Vector plots of the secondary flow for $\beta = 0.02$ and $\delta = 0.1$. Two figures on the left-hand side are the results of numerical simulations and one on the right-hand side is the experimental result. (a) is for $Re = 12140$ and (b) for $Re = 20850$. The outer wall is on the right-hand side.

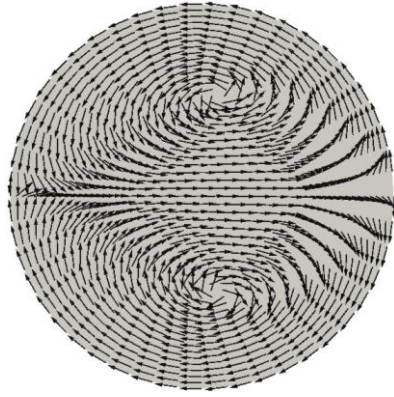


Fig. 7.6. Numerical result of vector plots of the secondary flow for $\beta = 0.02$ and $\delta = 0.1$ at $Re = 200$.

(C) Axial velocity pattern on the cross section

In this subsection, we study the axial velocity distribution on the cross section numerically and experimentally for $\beta = 0.02$ and $\delta = 0.1$ in Fig. 7.7(a-b). To compare with the experimental results, we performed the numerical simulations with Lien-cubic $k - \varepsilon$ and RNG $k - \varepsilon$ turbulence models. Experimental results of the axial flow for $Re = 12140$ were

given by Wada (2010) and those for $Re = 20850$ by Yamamoto et al. (2005). The contour lines were drawn at every 0.2 of the axial velocity from the pipe wall, where the axial velocity is non-dimensionalized by the mean axial velocity $\langle \overline{w} \rangle$.

In the contours of the axial velocity, we observe that the axial velocity is greatly influenced by the outer flux of the secondary flow, since the maximum region of the axial velocity lies in the outer side of the cross section. In general, the agreement between numerical and experimental results is very good even if we observe the level of the contours. Especially, the agreement of RNG $k - \varepsilon$ is noteworthy.

For $Re = 12140$ shown in Fig. 7.7(a), it is noticed that the contour of high axial velocity region is shifted to the outer side of the cross section for both the results of the turbulence model and the experimental data due to a large centrifugal force. The maximum value of the non-dimensional axial velocity is 1.36 for Lien-cubic $k - \varepsilon$ model and 1.26 for RNG $k - \varepsilon$ model. It should be noted that the maximum axial velocity is 1.27 in the experiment, which shows that RNG $k - \varepsilon$ model is in a excellent agreement with the experimental results. The symbol * represents the position of the maximum axial velocity.

For $Re = 20850$ shown in Fig. 7.7(b), a high axial velocity region moves further to the outer wall direction. It is found that the maximum axial velocity is 1.29 for Lien-cubic $k - \varepsilon$ model and 1.21 for RNG $k - \varepsilon$ model. The maximum axial velocity of the experiment is 1.21, which shows again excellence of RNG $k - \varepsilon$ model. In the fully developed turbulent flow, RNG $k - \varepsilon$ model is found to predict the axial velocity flow pattern very well with comparison to the experimental results.

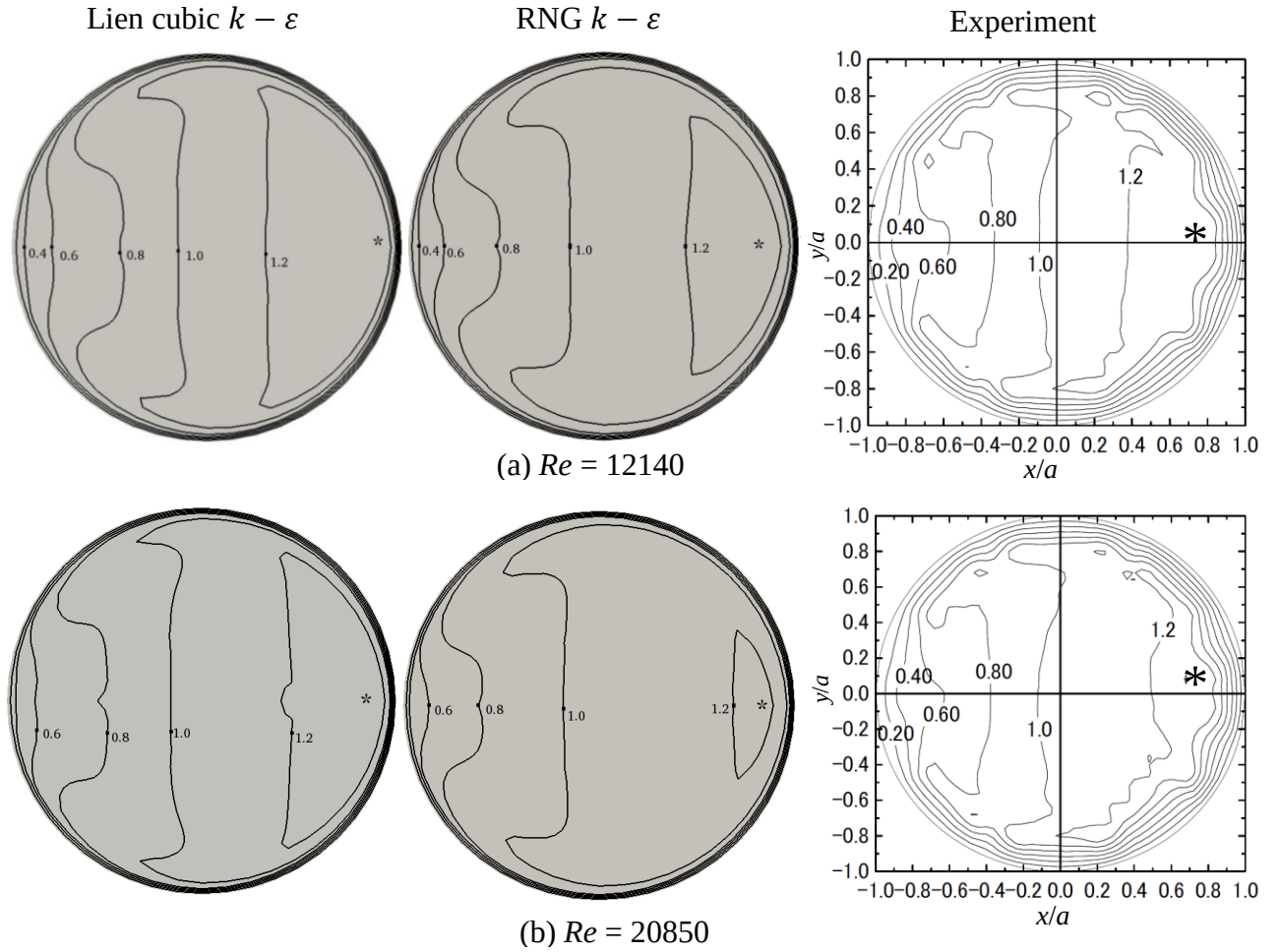


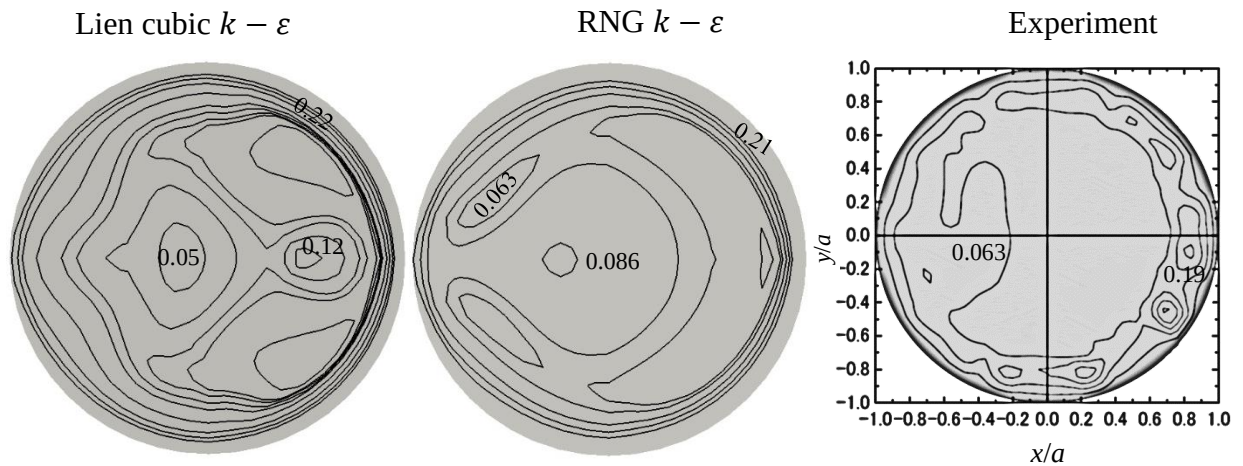
Fig. 7.7. Contours of the axial velocity for $\beta = 0.02$ and $\delta = 0.1$. Two figures on the left-hand side are the results of numerical simulations and one on the right-hand side is the experimental result. (a) is for $Re = 12140$ and (b) for $Re = 20850$.

(D) Turbulent Intensity

The intensity of turbulence of a helical pipe obtained in the present numerical simulations is shown in Fig. 7.8 for higher Reynolds numbers ($Re = 12140$ and 20850). Numerical calculations were performed by Lien-cubic $k - \varepsilon$ model, RNG $k - \varepsilon$ model. We study the turbulent intensity by view of the local peak level and contour patterns. Figure 6.8 shows the contours of the non-dimensional turbulent intensity, $(2k)^{1/2}/\langle \bar{w} \rangle$, at three pitch downstream cross section from the inlet, where k is the intensity of the turbulent kinetic energy. Experimental results of contours of the turbulent intensity were obtained by Yamamoto et al.

(2005). Figure 7.8(a) shows the contour of turbulent intensity for $Re = 12140$. It is found that the peak of the turbulent intensity on the cross section lies near the outer wall by Lien-cubic $k - \varepsilon$ model and RNG $k - \varepsilon$ model, where the peak level is 0.22 for Lien-cubic $k - \varepsilon$ and 0.21 for RNG $k - \varepsilon$ model. It should be remarked that the experimental peak level is 0.19 near the outer wall. The agreement between numerical and experimental results is pretty good as for the peak level. As for the shape of contours, we find that Lien-cubic $k - \varepsilon$ model agrees with the experimental results pretty well but RNG $k - \varepsilon$ model does not predict well for $Re = 12140$.

For a higher Reynolds number, $Re = 20850$, Fig. 7.8 (b) shows that the turbulent intensity is relatively suppressed at the inner side and somewhat enhanced at the outer side of the helical pipe. The peak level of the turbulent intensity is 0.18 for Lien-cubic $k - \varepsilon$ model and 0.17 for RNG $k - \varepsilon$ model. Since the experimental maximum value is 0.16, the agreement is also good for this case. As for the shape of contours, Lien-cubic $k - \varepsilon$ model agrees pretty well with the experimental results, although both turbulence models give nearly the same quality of prediction.



(a) $Re = 12140$

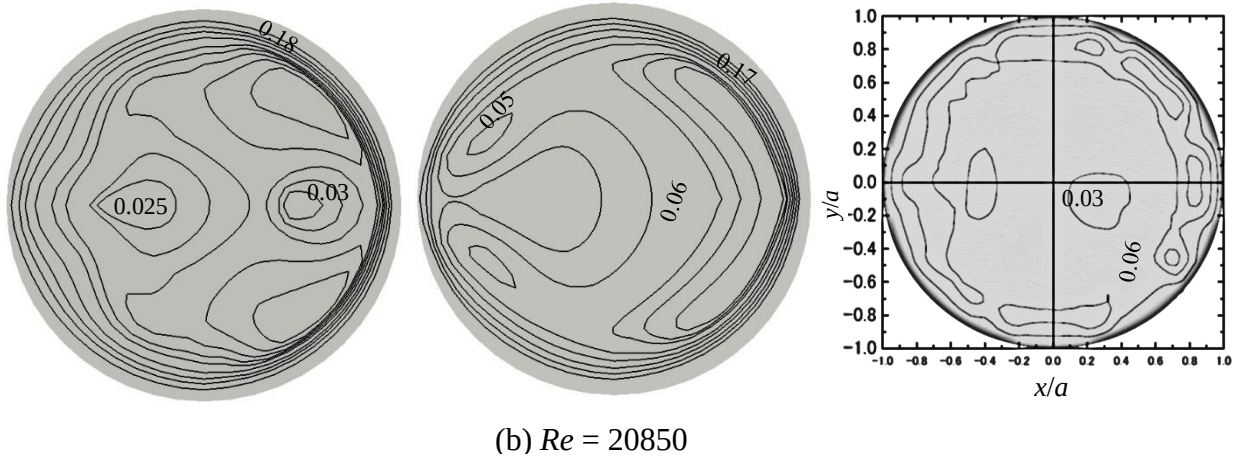


Fig. 7.8. Turbulent intensity, $(2k)^{1/2}/\langle \bar{w} \rangle$ for $\beta = 0.02$ and $\delta = 0.1$ at $Re = 12140$ and 20850 , where k is the intensity of turbulent kinetic energy and $\langle \bar{w} \rangle$ the mean axial velocity. Two figures on the left-hand side are the results of numerical simulations and one on the right-hand side is the experimental result. (a) is for $Re = 12140$ and (b) for $Re = 20850$.

7.4 Results of the case for $\beta = 0.45$

(A) Axial velocity distribution

We first compared the numerical results of three RANS models with the experimental data of the mean axial velocity profile for $\beta = 0.45$ and $\delta = 0.1$, as shown in Figs. 7.9 and 6.10. We considered the nine-pitch helical pipe for the comparison with the experimental results of Yamamoto et al. (2005) and Hayamizu et al. (2008). We studied turbulence on the cross section at the 8.3-pitch downstream of the helical pipe both numerically and experimentally. Numerical and experimental results of the axial velocity profile is normalized by the $\langle \bar{w} \rangle$. We plot the velocity profile for a low Reynolds number, $Re = 3060$, a middle Reynolds number, $Re = 5550$, and high Reynolds numbers, $Re = 9340$ and 19910 , along the radial x and vertical y direction.

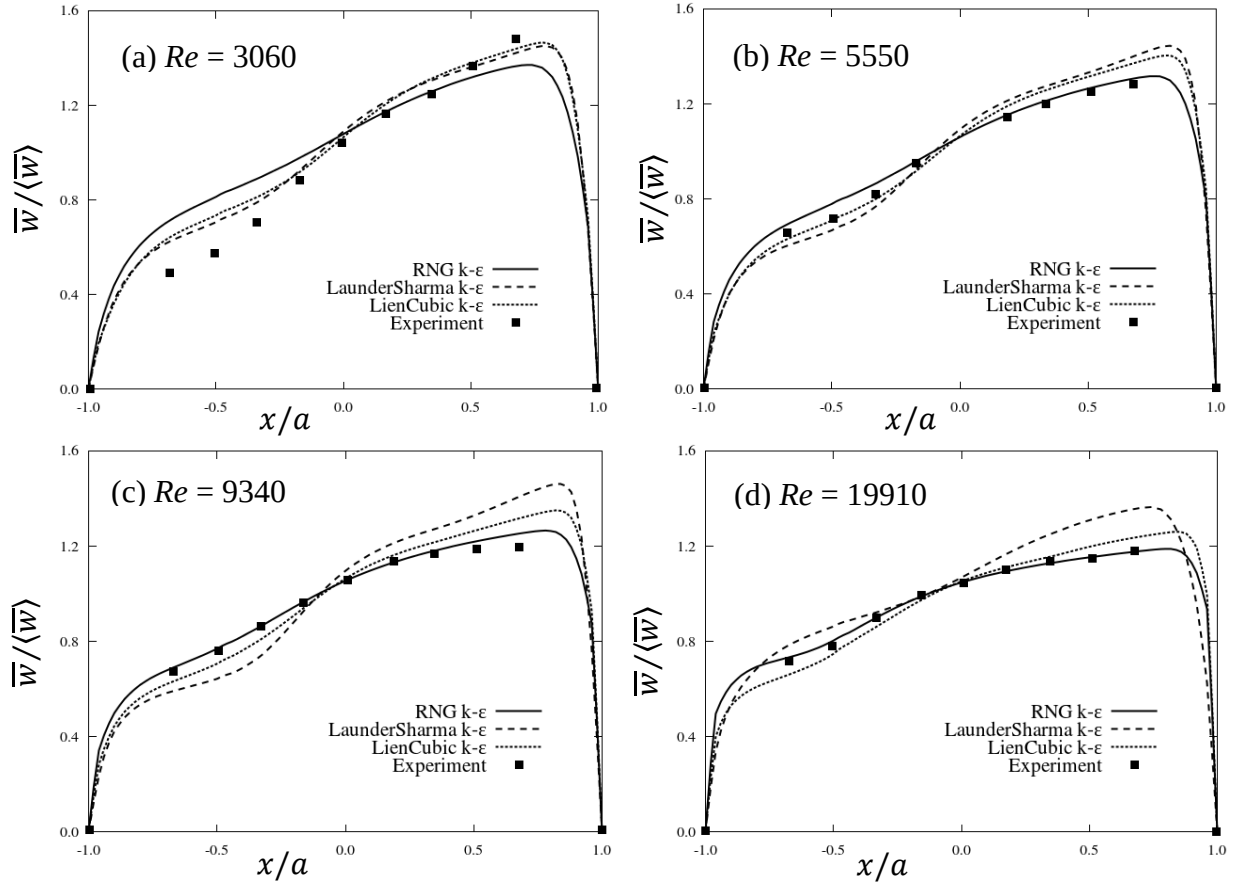


Fig. 7.9. Axial velocity distribution on the x -axis for $\beta = 0.45$ at $\delta = 0.1$. The outer wall is on the right-hand side.

In Fig. 7.9, it is clear that any turbulence model cannot predict well the experimental data for the low Reynolds number case, $Re = 3060$, especially in the inner region of the helical pipe. In the case of $Re = 5550$, RNG $k - \varepsilon$ model and Lien-cubic $k - \varepsilon$ model can predict the experimental data in the positive and negative x region, respectively. It is remarkable that the prediction of the axial velocity profile by RNG $k - \varepsilon$ model is excellent for $Re = 9340$ and 19910. It should be remarked that for low torsion parameter ($\beta = 0.02$), the flow attains the fully developed states around $Re = 12140$, and that, for high torsion parameter ($\beta = 0.45$), the flow seems to attain the fully developed state between $Re = 5550$ and 9340, which may be slightly larger than 5550. Therefore, the torsion effect decreases the critical Reynolds number of the transition to fully developed turbulence as it decreases the critical Reynolds number of laminar to turbulent transition (Yamamoto et al. (1998)).

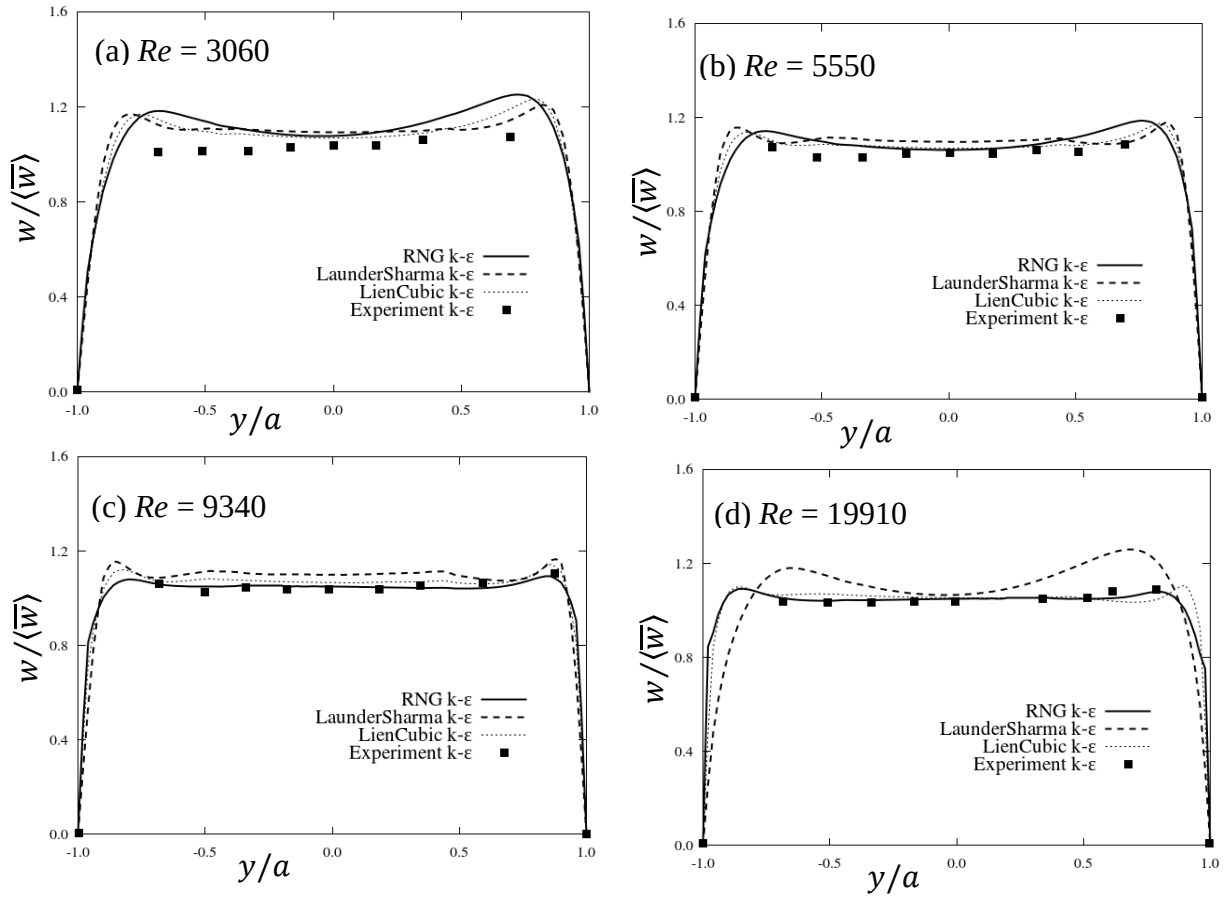


Fig. 7.10. Axial velocity distribution on the y -axis for $\beta = 0.45$ at $\delta = 0.1$. The upper wall is on the right-hand side.

Figure 7.10 displays a comparison of the numerical results of the axial velocity along y -axis for the three turbulence models with the experimental data by Yamamoto et al. (2005). In Fig. 6.10, we observe that the axial velocity profile has two local maxima near the upper and lower wall on the cross section similarly as $\beta = 0.02$. We also observe that no turbulence model can predict the axial velocity profile of the experimental data for $Re = 3060$, while the prediction is fairly good for $Re = 5550$. For higher Reynolds number ($Re = 9340$ and 19910), the prediction of RNG $k - \epsilon$ turbulence model shows a very good agreement with the experimental data.

(B) *Axial velocity pattern on the cross section*

Figure 7.11 shows the contours of the axial velocity at 8.3-pitch downstream cross section from the inlet of the helical pipe obtained numerically and experimentally for $\beta = 0.45$ and $\delta = 0.1$. Numerical calculations with Lien-cubic $k - \varepsilon$ and RNG $k - \varepsilon$ turbulence models were performed for comparison with the experimental data of the axial flow for $Re = 9340$ obtained by Wada (2010) and those for $Re = 19910$ by Yamamoto et al. (2005). The contour lines were drawn at every 0.2 of the axial velocity from the pipe wall, where the axial velocity is non-dimensionalized by the mean axial velocity $\langle \bar{w} \rangle$. The peak of the axial velocity is shifted towards the outer side by the effect of curvature. The symbol * represents the position of the maximum axial velocity.

The difference from the case for $\beta = 0.02$ is that the high velocity region is less concentrated for the case of $\beta = 0.45$. This may be due to the large clockwise circulation caused by large torsion. When $Re = 9340$ shown in Fig. 7.11(a), it is clearly seen that the contour pattern of the axial velocity is predicted fairly well by both turbulence models. The peak level of the axial velocity is 1.35 for Lien-cubic $k - \varepsilon$ model and 1.26 for RNG $k - \varepsilon$ model, while it is 1.19 by the experimental data. Therefore, RNG $k - \varepsilon$ model prediction is closer to the experimental data. In Fig. 7.11(b), numerical and experimental results are shown for $Re = 19910$. It is found that the peak level of the axial velocity is 1.27 for Lien-cubic $k - \varepsilon$ model and 1.18 for RNG $k - \varepsilon$ model, while it is 1.17 by the experimental results. Therefore, RNG $k - \varepsilon$ model's prediction is excellent. The axial velocity pattern by both turbulence models is similar to the experimental results.

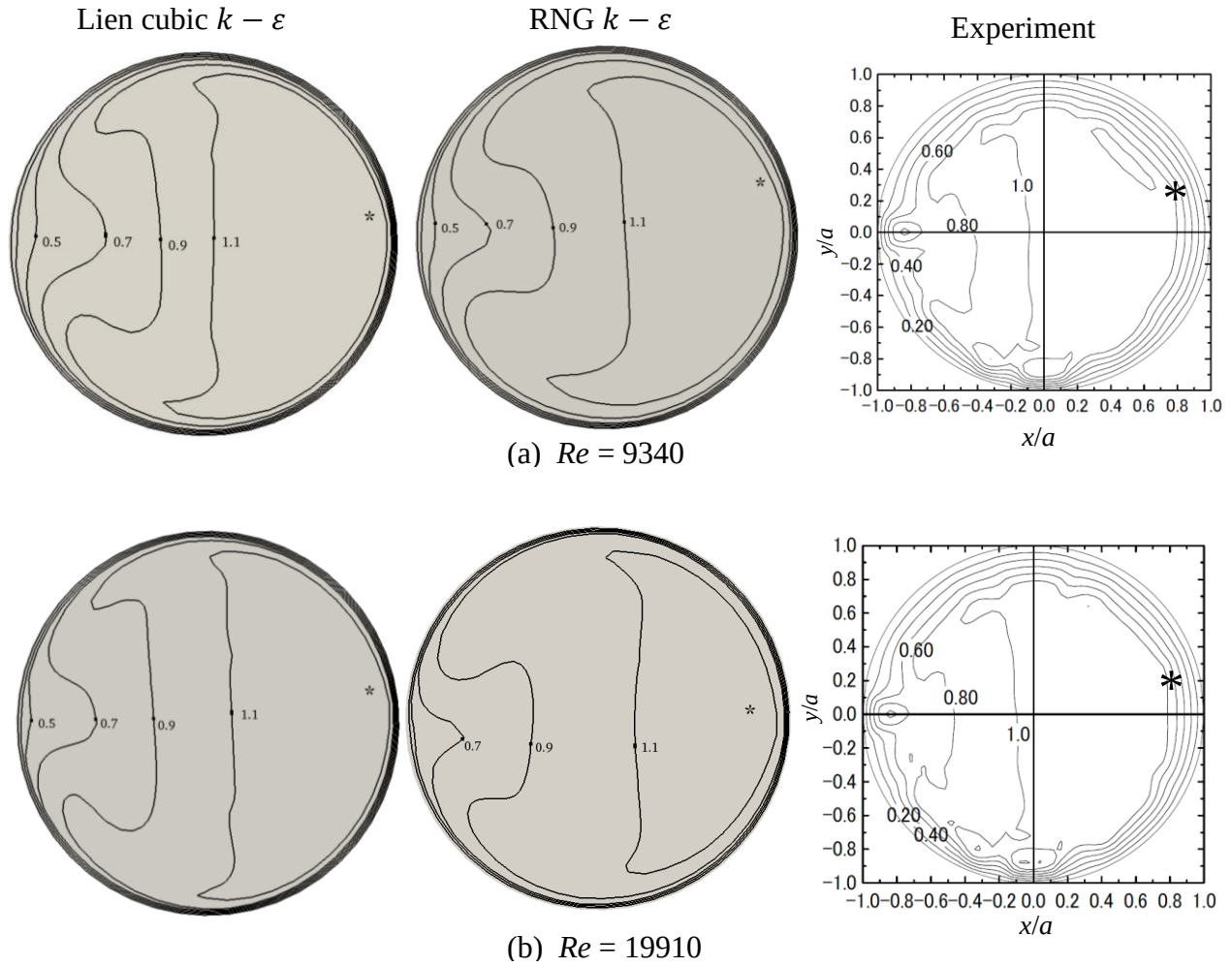


Fig. 7.11. Contours of the axial velocity for $\beta = 0.45$ and $\delta = 0.1$. Two figures on the left-hand side are the results of numerical simulations and one on the right-hand side is the experimental result. (a) is for $Re = 9340$ and (b) for $Re = 19910$.

(C) Turbulent Intensity

In Fig. 7.12, we show the intensity of turbulent kinetic energy of the helical pipe by the numerical simulations with Lien-cubic $k - \varepsilon$ model and RNG $k - \varepsilon$ model as well as the experimental results for higher Reynolds numbers ($Re = 9340$ and 19910). The turbulent intensity was calculated as $(2k)^{1/2}/\langle \overline{w} \rangle$ at 8.3-pitch downstream cross section from the inlet. Both numerical and experimental results show that high intensity region lies close to the outer wall for $Re = 9340$ and 19910 .

Contours of the turbulent intensity in Fig. 7.12(a) for $Re = 9340$ show that the peak level is 0.26 for Lien-cubic $k - \varepsilon$ and 0.238 for RNG $k - \varepsilon$ model, while it is 0.19 by the experimental results. The agreement between numerical and experimental results is fairly good as for the peak level. Contours of the turbulent intensity in Fig. 7.12(b) for $Re = 19910$ show that the peak level is 0.20 for Lien-cubic $k - \varepsilon$ model and 0.174 for RNG $k - \varepsilon$ model, while it is 0.17 by the experimental results. As for the contour pattern, Lien-cubic $k - \varepsilon$ model can express the asymmetry with respect to $y = 0$ of the experimental results, while RNG $k - \varepsilon$ model cannot for $Re = 9340$ and 19910.

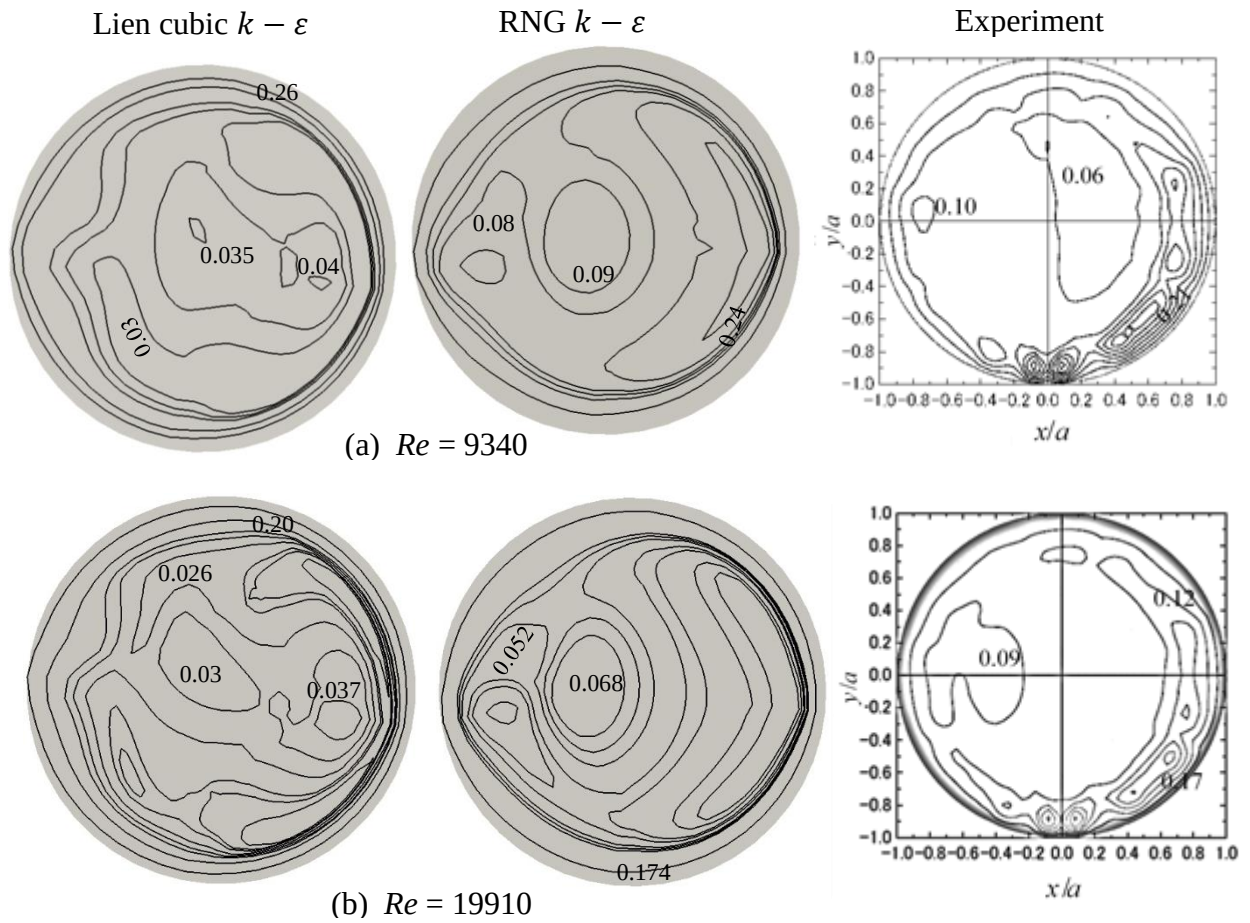


Fig. 7.12. Turbulence intensity, $(2k)^{1/2}/\langle \bar{w} \rangle$, for $\beta = 0.45$ and $\delta = 0.1$ at $Re = 9340$ and 19910, where k is the intensity of turbulent kinetic energy and $\langle \bar{w} \rangle$ the mean axial velocity. Two figures on the left-hand side are the results of numerical simulations and one on the right-hand side is the experimental result. (a) is for $Re = 9340$ and (b) for $Re = 19910$.

In order to compare with the experimental data by Yamamoto et al.(1995), we conducted numerical calculations for $\beta = 0.45$ and $\delta = 0.1$ by use of RNG $k - \varepsilon$ turbulence model. We also compared the numerical results with Ito's formula (1959).

It should be remarked that Ito's formula (1959) holds for $\beta = 0$ or $\beta \ll 1$. In Fig. 7.13, we show the present numerical results ($\circ$), the experimental data by Yamamoto et al. (1995) ($\square$) and Ito's formula (a solid line). Figure 7.13 shows a good agreement between the present numerical results and the experimental data, and a good agreement with Ito's formula for large Reynolds numbers.

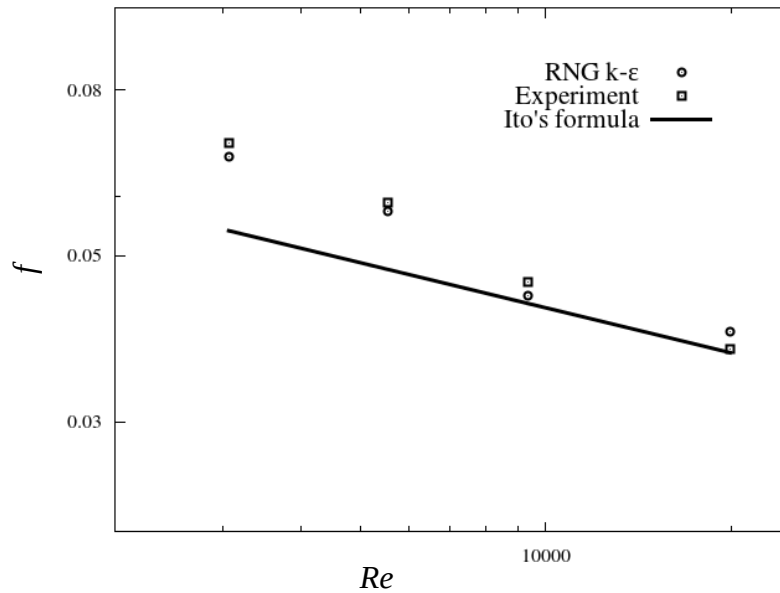


Fig. 7.13. Friction factor of the helical pipe for $\beta = 0.45$ and $\delta = 0.1$.

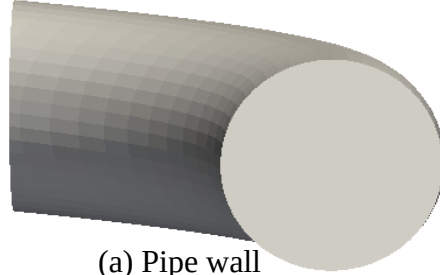
7.5 Discussion and conclusion

The secondary flow pattern in the cross section of the helical pipe is affected by two geometrical factors, curvature and torsion. In the present paper, the curvature effect is expressed by the non-dimensional curvature δ , and torsion effect by the torsion parameter β . When $\delta > 0$, two recirculating vortices develop even if δ is very small for the laminar flow. If β increases from zero, one large circulating vortex dominates for the laminar flow. It should be noted that since torsion works as a rotational force, its effect is to induce a rotating motion in the fluid. It is well known that torsion has a destabilizing effect on the laminar helical pipe flow (Yamamoto et al. (1994)). Here, we study the difference of flow structures

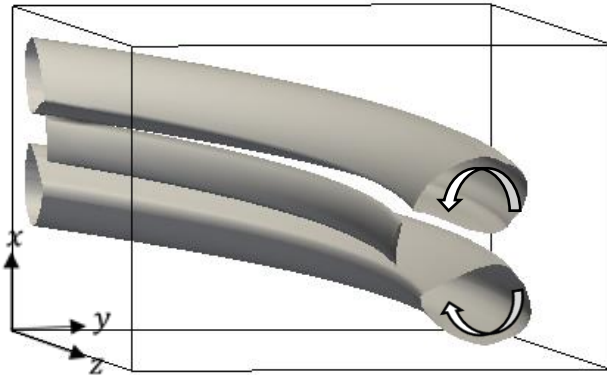
between laminar and turbulent flows under the combined effects of torsion and curvature. Actually, in the case of turbulent helical pipe flow, we observe a different flow patterns from the laminar case.

We calculated the Q -value (Lesieur et al. (2005)), the second invariant of the velocity gradient tensor, in order to catch the vortical structures for both laminar and turbulent flows. We plot equi- Q -value surfaces at the region of the helical pipe in Fig. 7.14 for $\beta = 0.02$ and in Fig. 7.15 for $\beta = 0.45$.

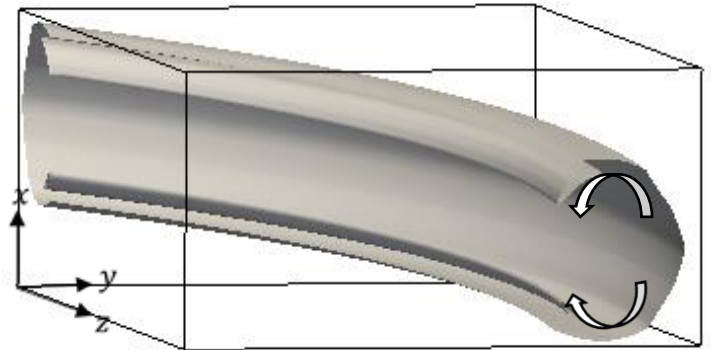
First, the small β case ($\beta = 0.02$) is discussed. Figures 7.14 (b) and (c) show the contours of equi- Q value surface for $Re = 200$ and 20850, respectively. It is interesting that secondary vortices become weaker, the smaller anticlockwise vortices shrink and the larger clockwise vortices become faint if the flow becomes turbulent, which has been already pointed out in Figs. 6.5 and 6.6. This phenomena were caused by the enhancement of momentum transfer of turbulence in the cross section. In the turbulent flow, turbulent fluctuations are dominant and break the recirculating vortices.



(a) Pipe wall



(b) $Re = 200$



(c) $Re = 20850$

Fig. 7.14. Equi- Q value surface for $\beta = 0.02$ and $\delta = 0.1$. (a) shows the pipe wall, (b) the case for $Re = 200$, where $Q = 0.5$ and (c) the case for $Re = 20850$, where $Q = 70$.

Then, the large β case ($\beta = 0.45$), is discussed. Figures 7.15 (b) and (c) show the contours of equi- Q value surface for $Re = 200$ and 19910, respectively. It is interesting that the size of the large clockwise secondary vortex survives with its size increasing, while the small anticlockwise secondary vortex disappears if the flow becomes turbulent. Therefore, the large clockwise vortex is found to be less affected by turbulence, although the rotational force by torsion is stronger than the case for $\beta = 0.02$.

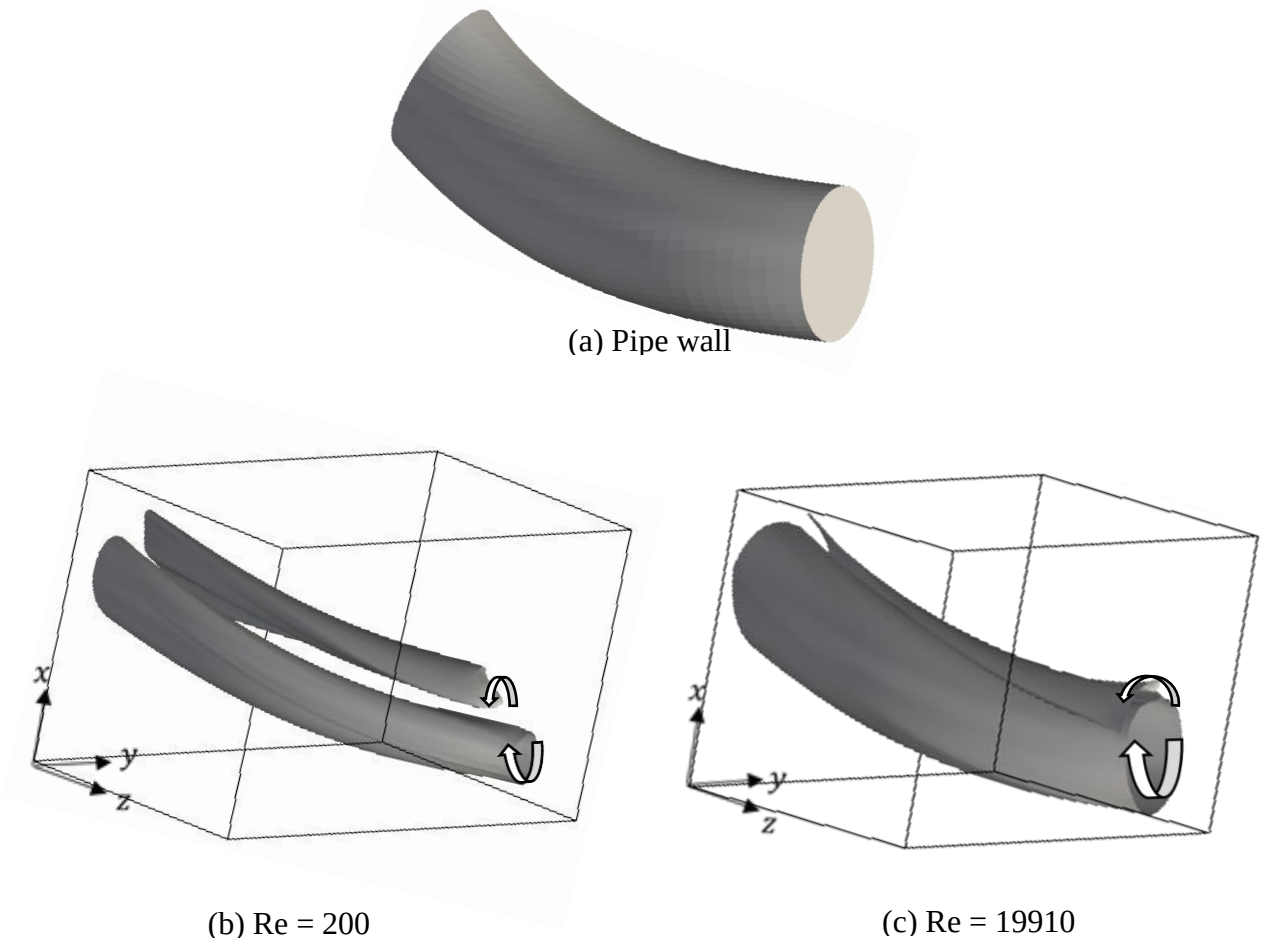


Fig. 7.15 . Equi- Q - value surface for $\beta = 0.45$ and $\delta = 0.1$. (a) shows the pipe wall, (b) the case for $Re = 200$, where $Q = 0.5$ and (c) the case for $Re = 19910$, where $Q = 70$.

A numerical study of the transitional and fully developed turbulent flows through a helical pipe with circular cross section was conducted by use of OpenFOAM. Numerical calculations were performed with three turbulence models. The effects of the Reynolds number and torsion parameter on the nature of the turbulent flow were studied. We investigated the cases of $Re = 5320, 6880, 12140$ and 20850 for $\beta = 0.02$ and $Re = 3060, 5550, 9340$ and 19910 for $\beta = 0.45$, at $\delta = 0.1$.

The critical Reynolds number exhibiting the change from transitional turbulence to fully-developed turbulence was estimated by the comparison of the present numerical results with the experimental data. It is found that the transition to fully developed turbulence occurs around $Re = 12140$ for $\beta = 0.02$ and slightly larger than $Re = 5550$ for $\beta = 0.45$. This tendency of the decrease of the critical Reynolds number as β increase is similar to the laminar to turbulent transition.

For relatively low Reynolds numbers when turbulence is transitional, three turbulence models cannot predict well the experimental results of the mean axial velocity distribution. If the Reynolds number increases when the flow becomes fully developed turbulence, RNG $k - \varepsilon$ model shows an excellent agreement with the experimental data of the axial velocity distribution for both $\beta = 0.02$ and 0.45 .

A larger and stronger vortex is produced in the lower semicircle and a smaller vortex in the upper semicircle tends to diminish as torsion increases for the helical pipe whether the flow is laminar or turbulent. However, it is interesting that this change occurs more drastically for the turbulent flow than the laminar flow.

The experimental results of the intensity of the turbulent kinetic energy are well predicted by Lien cubic $k - \varepsilon$ model and RNG $k - \varepsilon$ model for $\beta = 0.02$ and for $\beta = 0.45$. It should be noted that the peak level of the turbulent intensity occurs near the outer wall, which is in a good agreement with the experimental results.

Chapter 8

General conclusions

In this dissertation, we have conducted 3D DNS of viscous incompressible fluid flow through a helical pipe with circular cross section. Note that we also used three different turbulent models based on the Reynold averaged Navier-Stokes equation (RANS) for turbulent flow through a helical pipe. Flow characteristics are studied over the wide range of Dean number, Dn , curvature, δ , torsion parameter, β , and Reynolds number, Re , for both the laminar and turbulent flow cases. OpenFOAM is applied as a main tool for the numerical approach. because the boundary condition of the helical pipe is complex and it is necessary to set inlet and outlet boundary conditions to compare the experimental results.

In the confusing scenario of orthogonal and non-orthogonal helical coordinates, Germano (1982) found an advantage of the orthogonal system of helical coordinates that it can be easily generalized to a generic system of coordinates in the normal plane. On the basis of the orthogonal helical coordinate system, Yamamoto et al. (1994, 1998) numerically investigated helical pipe flow using the spectral method assuming that the flow field is uniform in the axial direction of the helical pipe and found this agreement may seem to be a fortunate incidence because the theoretical study assumed two-dimensionality in the pipe axial direction, whereas the experimentally observed instability was inevitably 3D. Therefore a coordinate system, such as a cylindrical or helical coordinate system, is not essential for OpenFOAM, but how the grid system is generated dominates the accuracy of the numerical solution. For generate a suitable mesh in the flow domain, an appropriate mesh system with 3D orthogonal helical coordinates was successfully created to conduct accurate DNSs and turbulence models of helical pipe flow using an FVM-based open-source computational fluid dynamics package (OpenFOAM).

The results, obtained by the present numerical simulation for both the laminar and turbulent flows, have been discussed in the respective chapters (**Chapter 4 to 7**) in detail. Since few experimental data and 2D spectral results are available for both the laminar and turbulent flows cases and comparison of our results with those by the experimental data and 2D spectral

study. It is found that the comparison of the present numerical calculations with the experimental results available. Yamamoto *et al.* (1995) experimentally obtained the instability of the helical pipe flows and found that the torsion has a destabilizing effect and the curvature has a stabilizing effect on the flow. Also, they found that the friction factor monotonically decreases as torsion increases from the value for the minimum torsion. Yamamoto *et al.* (2005), Hayamizu *et al.* (2008) and wada (2010) found the turbulent flow characteristics through a helical pipe by experimentally. In addition to that recently Shatat (2010) experimentally investigated heat transfer characteristics through a helical pipe where the pipe wall is more heated from the inlet of pipe. Here, we compare the present numerical results by 3D DNSs and turbulence models with the recent experimental data. Note that the judgement of the stability and instability was done by observing the magnitude of the disturbances using the flow visualization at the outlet of the helical pipe for the experiment study. On the other hand, our 3D calculations show that the disturbance grows toward the downstream and its magnitude attains maximum at the outlet for the unstable case, while the disturbance does not grow for the stable case.

In the steady calculations, we found the existence of the well-developed axially invariant 2D flow region, where we calculated the laminar flow characteristics with the effect of heat transfer and the nature of ensemble-averaged turbulent flows. Obtained results of flow characteristics comparison with the time-averaged experimental data and previous studies show a very good agreement. As for the nature of the flow characteristics, we observe that the secondary flow with two vortices exhibits a topological change into a single large vortex structure as β increases.

In the 3D unsteady calculations, we obtained the neutral curve of the critical Reynolds numbers which gives the critical states of the laminar to turbulent transition of the helical pipe flow by observing the unsteady behaviors instead of linear stability analysis. By comparing obtained the present results by different turbulent models with the experimental data of axial flow distribution, we estimated the critical Reynolds number of transition turbulence to fully-developed turbulent. The tendency of the critical Reynolds number from

the transition turbulent to fully developed turbulent decreases as β increase, which is qualitatively similar to the tendency of the neutral curve of the laminar to turbulent transition.

It is interesting that we found an explosive 3D evolution of the disturbances and vortical strictures of standing-wave disturbance in time evolution calculation at the marginal instability state of laminar to turbulence transition in helical pipes, where the disturbance breaks down into turbulence in the downstream region as time proceeds.

Note that we found the friction factor of the helical pipe becomes larger than that of a toroidal pipe for small β . When β further increases, the friction factor tends to decrease from that of a toroidal pipe after taking a maximum and approaches that of a straight pipe. Therefore, the effect of torsion on the friction factor of the helical pipe becomes large when β exceeds about 1.0, which is in good agreement with the experimental results. It is interesting that the peak of the friction factor existed in the range of $0.2 \leq \beta \leq 0.3$ for all values of the Reynolds number and curvature; thus, the torsion parameter was found to play an important role in the helical pipe flow. Additionally the friction factor formula for a toroidal pipe in the previous study was not applicable for $\beta \geq 0.1$, and that the friction factor of a helical pipe gradually approached that of a straight pipe for $\beta > 2.5$.

For the heat transfer case, regarding the averaged Nusselt number obtained for certain values of low torsion parameter, we found it slightly decreases as the Prandtl number decreases. The averaged Nusselt number of the helical pipe begins to decrease than that of a toroidal pipe if β increases from zero (a toroidal pipe), and increases after taking the global minimum at $\beta \cong 0.1$. If β further increases, we found a local maximum of the averaged Nusselt number, which decreases and then takes the global maximum at $\beta \cong 0.55$ for any value of the Reynolds number. For a wide range of β , the wavy behavior of the averaged Nusselt number was obtained. Note that the torsion remarkably affects on the averaged Nusselt number for any value of the Prandtl Number, and torsion parameter was found to play an important role in the forced convection of helical pipe flow. It was found that the minimum of the averaged Nusselt number occurred when both the effect of the secondary flow with symmetric two-cell pattern and that with one-cell pattern become feeble for $\beta \cong 0.1$. We hope the present results may contribute the improvement of heat exchanger using helical pipes.

It should be pointed out that we found the RNG $k - \varepsilon$ model is the best turbulent model by comparing with the experimental data, and it can predict the turbulent flow characteristics when the flow is fully developed turbulence. For the ensemble-averaged secondary flow case, we found a larger and stronger vortex is produced in the lower semicircle and a smaller vortex in the upper semicircle tends to diminish as torsion increases for the helical pipe whether the flow is laminar or turbulent. However, it is interesting that this change occurs more drastically for the turbulent flow than the laminar flow.

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