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## On the fixed point sets of differentiable $G_2$ actions on a Euclidian space

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## ON THE FIXED POINT SETS OF DIFFERENTIABLE $G_2$ ACTIONS ON A EUCLIDEAN SPACE

KENJI HOKAMA

Recently, W. C. Hsiang and W. Y. Hsiang [5] investigated the fixed point sets of differentiable actions of compact simple Lie groups on Euclidean spaces and dealt with some cases such that the fixed point sets are non-empty. In this paper we deal with the case of the compact exceptional simple Lie group  $G_2$  of rank 2, which was left out in the above.

### 1. Subgroups of $G_2$

Let  $G$  be a compact connected Lie group and  $H$  a closed connected maximal rank subgroup of  $G$ . We denote the Weyl groups of  $G$  and  $H$  by  $W(G)$  and  $W(H)$  respectively. Then we have  $W(G) = N_T/T$  and  $W(H) = N_T \cap H/T$  where  $T$  is a maximal torus of  $H$  and  $N_T$  is the normalizer of  $T$  in  $G$ .

**Proposition 1.1.** *Let  $g$  be an element of  $G$ . If  $g \in N_T$  and  $(gT) W(H) (gT)^{-1} = W(H)$  then  $gHg^{-1} = H$ .*

*Proof.* Let  $\mathfrak{G}$ ,  $\mathfrak{H}$  and  $\mathfrak{T}$  be the Lie algebras of  $G$ ,  $H$  and  $T$  respectively and  $\mathfrak{G}^\mathbb{C}$ ,  $\mathfrak{H}^\mathbb{C}$  and  $\mathfrak{T}^\mathbb{C}$  their complexifications (i. e.  $\mathfrak{G}^\mathbb{C} = \mathfrak{G} + \sqrt{-1}\mathfrak{G}$  etc.). We denote the sets of non-zero roots of  $\mathfrak{G}^\mathbb{C}$  and  $\mathfrak{H}^\mathbb{C}$  with respect to  $\mathfrak{T}^\mathbb{C}$  by  $\Delta$  and  $\Delta'$  respectively. For any  $\alpha \in \Delta$  we define  $H_\alpha \in \mathfrak{T}^\mathbb{C}$  by the relation  $(H, H_\alpha) = \alpha(H)$  for all  $H \in \mathfrak{T}^\mathbb{C}$ , where the inner product is the Killing form of  $\mathfrak{G}^\mathbb{C}$ . Then,  $\sqrt{-1}H_\alpha$ ,  $\alpha \in \Delta'$  generate  $\mathfrak{T}$ . Let  $s_\alpha$  be the reflexion of  $\mathfrak{T}$  with respect to the hyperplane orthogonal to  $\sqrt{-1}H_\alpha$ . Then we can identify the Weyl group  $W(G)$  with the group generated by  $s_\alpha$ ,  $\alpha \in \Delta$  and similarly the Weyl group  $W(H)$  also has the same property. Now we consider the automorphism  $\text{Ad}_\mathfrak{G}(g)$  of  $\mathfrak{G}$ . By the assumption we have  $A = \text{Ad}_\mathfrak{G}(g)|_{\mathfrak{T}} \in W(G)$  and  $As_\alpha A^{-1} \in W(H)$ ,  $\alpha \in \Delta'$ . Since any reflexion of  $W(H)$  has a form of  $s_\alpha$ ,  $\alpha \in \Delta'$ , there exists  $\beta \in \Delta'$  such that  $As_\alpha A^{-1} = s_\beta$ . Let  $H$  ( $\in \mathfrak{T}$ ) be orthogonal to  $\sqrt{-1}H_\beta$ . Then  $s_\beta A^{-1}(H) = A^{-1}(H)$ , that is,  $A^{-1}(H)$  is orthogonal to  $\sqrt{-1}H_\alpha$ . Since  $A$  is an isometry, this implies that  $A(\sqrt{-1}H_\alpha)$  is orthogonal to  $H$  and hence  $A(\sqrt{-1}H_\alpha) = \sqrt{-1}cH_\beta$  for some real number  $c$ . Thus  $\alpha A^{-1} = c\beta$  and

since  $\alpha A^{-1}$  and  $\beta$  are roots in  $\mathcal{A}$  we have  $c = \pm 1$ . Then it follows that  $H$  is invariant under  $\text{Ad}_{\mathfrak{g}}(g)$  and hence  $gHg^{-1} = H$ . q. e. d.

For the later we note the following

**Proposition 1.2.** *Let  $G, H$  and  $T$  be as above. If  $K$  is the normalizer of  $W(H)$  in  $W(G)$ , then  $N_H/H \cong K/W(H)$  where  $N_H$  is the normalizer of  $H$  in  $G$ .*

*Proof.* Let  $\tilde{K} = \{g \in N_T | gHg^{-1} = H\}$ . There is an exact sequence:  $\{1\} \rightarrow \tilde{K} \cap H \rightarrow N_H/H \rightarrow \{1\}$ . Hence  $N_H/H \cong \tilde{K}/\tilde{K} \cap H \cong (K/T)/W(H)$ . Therefore it is sufficient to show that  $\tilde{K}/T = K$ . Clearly  $\tilde{K}/T \subseteq K$ . On the other hand  $g \in N_T$ ,  $(gT)W(H)(gT)^{-1} = W(H)$  for any  $gT \in K$ . Then  $gHg^{-1} = H$  by the proposition 1.1. Thus  $g \in \tilde{K}$ , that is  $\tilde{K}/T \supseteq K$ . q. e. d.

In the remainder of this section we denote the exceptional compact Lie group of rank 2 by  $G$ . The maximal subgroups of maximal rank of  $G$  are known to be isomorphic to  $SO(4)$  or  $SU(3)$  and the subgroups, which are isomorphic, are conjugate [2].

**Proposition 1.3.** (a) *Let  $L$  be the normalizer of  $SU(3)$  in  $G$ . Then  $L/SU(3) \cong Z_2$ .* (b) *The normalizer of  $SO(4)$  coincides with  $SO(4)$  itself.* (c) *Let  $H$  be a subgroup of  $G$  isomorphic to  $SU(2)$ . Then the normalizer of  $H$  is conjugate to  $SO(4)$ .*

*Proof.* The Weyl group  $W(G)$  is a dyhedral group of order 12, and  $W(SU(3))$  and  $W(SO(4))$  are isomorphic to the permutation group  $S_3$  of 3 letters and  $Z_2 \oplus Z_2$  respectively. Then  $W(SU(3))$  is a normal subgroup of  $W(G)$  and the normalizer of  $W(SO(4))$  in  $W(G)$  coincides with  $W(SO(4))$  itself. Hence (a) and (b) follow from the proposition 1.2.

Now we consider the case (c). Let  $a (\neq 1)$  be the element of the center of  $H$ . Then  $a^2 = 1$ . On the other hand the elements of order 2 in  $G$  are conjugate. In fact an element of order 2 is contained in a torus  $T$  ( $SU(3)$ ) and the elements of order 2 in  $SU(3)$  are conjugate. Then, since subgroups which are isomorphic to  $SU(3)$  are conjugate in  $G$ , it follows that the elements of order 2 in  $G$  are conjugate. Let  $K$  be the normalizer of  $a$ . Then clearly  $H \subseteq K$  and  $K \neq G$ , since  $G$  has no center. The center of  $SO(4)$  is  $Z_2 \oplus Z_2$  and hence  $K$  contains a subgroup isomorphic to  $SO(4)$ . Then it is clear that  $H$  is a normal subgroup of  $K$ , since  $H$  is isomorphic to  $SU(2)$ . Hence the normalizer of  $H$  in  $G$  is conjugate to  $SO(4)$ . q. e. d.

**Remark.** It is easy to see that the subgroups of  $G$  isomorphic to  $SU(2)$  are conjugate.

## 2. A property of a differentiable action

For the later we prove a theorem with respect to a differentiable action of a compact Lie group  $G$  of which a principal isotropy subgroup is a maximal torus of  $G$ .

**Proposition 2.1.** *Let  $G$  be a compact Lie group, and  $\varphi$  a real representation of  $G$ . If a principal isotropy subgroup of  $\varphi$  is a maximal torus of  $G$  and there is no exceptional orbit, then  $G$  is connected.*

*Proof.* Let  $G^0$  be the connected component of the identity of  $G$ . Then  $\varphi|G^0 = \text{Ad}_{G^0} \oplus \text{trivial part}$  [4]. Let  $V$  be a representation space of  $\varphi$  and  $T$  a maximal torus of  $G$ . We denote the set  $V - (\text{the singular set})$  by  $V_0$ . Since  $V_0$  is the principal orbit bundle of  $\varphi|G^0$  it is easily seen that  $V_0$  is  $G^0$ -equivariantly homeomorphic to  $G^0/T \times V_0/G^0$ .  $G/G^0$  acts naturally on  $V_0/G^0$ . Let  $g \in G/G^0$  be a prime order element. By the theorem of P. A. Smith in [1],  $g \in G/G^0$  has a fixed point in  $V_0/G^0$ , since  $V_0/G^0$  is homeomorphic to a Weyl chamber and hence  $V_0/G^0$  is contractible. Hence there is a point  $x \in V_0$  such that  $g \cdot G^0 x = G^0 x$ . Now by taking an element  $g_0 \in G^0$  satisfying  $gx = g_0 x$  we have  $g_0^{-1}g \in G_x$ . Since  $G_x$  is a maximal torus of  $G$  by the assumption we get  $g \in G^0$ . This is a contradiction. q. e. d.

**Theorem 2.2.** *Let  $\phi$  be a differentiable action of a compact connected Lie group  $G$  on a simply connected differentiable manifold  $M$ . If the connected component of the identity of a principal isotropy subgroup is a maximal torus of  $G$ , then the isotropy subgroups of  $\phi$  are connected.*

*Proof.* Let  $T$  be a maximal torus of  $G$ ,  $A = F(T, M)^{1)}$  and  $M_0 = M - (\text{the singular set})$ . Then the Weyl group  $W(G)$  acts on  $A$ . We easily see  $M_0 = G/T \times_{W(G)} (A \cap M_0)$  and  $\pi_1(M_0) = 0$ , since the singular set has at least codimension 3. By the homotopy exact sequence of a fibration:  $A \cap M_0 \rightarrow M_0 \rightarrow G/N_T$  we know that the number of the components of  $A \cap M_0$  is equal to order of  $W(G)$ . Then, since  $M_0$  is connected,  $W(G)$  acts simply transitively on the components of  $A \cap M_0$ . Hence  $\phi$  has no exceptional orbit and a principal isotropy subgroup is a maximal torus of  $G$ . Let  $x \in M$ . Then the slice representation at  $x$  of  $G_x$  satisfies the assumption of the proposition

1)  $F(T, M)$  is the set of fixed points of  $T$  in  $M$ .

2. 1 and hence  $G_x$  is connected.

q. e. d.

### 3. Weight systems

Let  $\phi$  be a differentiable  $G$  action on a Euclidean space and  $T$  be a maximal torus of  $G$ . Then by the theorem of P. A. Smith in [1] the local representation of  $T$  at a fixed point of  $T$  is well defined. The weight system of the local representation of  $T$  is defined to be the weight system of  $\phi$  and denoted by  $\Sigma\phi$ . We see in [5] that for each simple Lie group, weight systems of actions with a principal isotropy subgroup of a positive dimension are classified and also the fixed point sets are determined with few exceptions.

Now on we consider the case where  $G$  is the exceptional compact simple Lie group of rank 2. Then the non-zero root system of  $G$  is given by

$$\Delta'(G) = \{\pm\theta_i, \pm(\theta_i - \theta_j), i < j, i, j = 1, 2 \text{ and } 3\}.$$

Let  $\phi$  be a differentiable  $G$  action on a Euclidean space  $E^m$  with a principal isotropy subgroup  $H_\phi$  of positive dimension. Then it is known by [5] that

- (1)  $\Sigma'(\phi) = \Delta'(G)$  and  $H_\phi^0 =$  a maximal torus of  $G$ ,
- (2)  $\Sigma'(\phi) = \{\pm\theta_i, i = 1, 2 \text{ and } 3\}$  and  $H_\phi^0 = SU(3)$  or
- (3)  $\Sigma'(\phi) = \{\pm\theta_i, i = 1, 2 \text{ and } 3: \text{each weight has the multiplicity } 2\}$  and  $H_\phi^0 = SU(2)$ .

In the following sections we investigate the fixed point set for each of those cases.

### 4. Fixed point sets

First we consider the case where  $H_\phi^0$  is a maximal torus of  $G$ .

**Propositon 4.1.**  $F(G, E^m)$  is  $Z_p$ -acyclic for  $p=2$  and 3.

*Proof.* We suppose  $F(G, E^m)$  is empty and then show that we arrive at a contradiction. Since the isotropy subgroups of  $\phi$  are connected by the theorem 2. 1, the possible isotropy subgroups are maximal tori and subgroups which are isomorphic to  $U(2)$ ,  $SO(4)$  or  $SU(3)$ . Let  $T$  be a maximal torus of  $G$  and  $A = F(T, E^m)$ . Then the Weyl group  $W(G)$  acts on  $A$  and  $W(G)$  is a group of order 12 defined by the relations:  $t^6=1, s^2=1$  and  $sts=t^{-1}$ . Since  $W(G)_a = W(G_a)$  for any  $a \in A$ , the possible isotropy subgroups of  $W(G)$ -action on  $A$  are isomorphic to 1,  $Z_2, Z_2 \oplus Z_2$  or  $S_3$ . Because  $A$  is

$Z$ -acyclic, we see by the theorem of P. A. Smith that they are exactly isotropy subgroups. Let  $C = \{a \in A \mid W(G)_a \cong Z_2 \oplus Z_2\}$ . Then if  $c \in C$ ,  $G_c \cong SO(4)$  and the slice representation at  $c$  is  $\text{Ad}_{G_c} \oplus (m-14)\theta$ , where  $\theta$  is a trivial 1-dimensional representation of  $G_c$  [4]. Hence  $C$  is a submanifold of  $A$  of codimension 2. On the other hand there are three subgroups isomorphic to  $Z_2 \oplus Z_2$  and generated by  $\{s, st^3\}$ ,  $\{st, st^4\}$  and  $\{st^2, st^5\}$  respectively. Moreover they are conjugate. Therefore we see that  $C$  is the disjoint union of three  $Z_2$ -acyclic submanifolds of the same dimension. Also  $C$  is the fixed point set of  $t^3$  and hence  $Z_2$ -acyclic. This contradicts the above. Thus we see  $F(G, E^m)$  is non-empty.

Now let us take the subgroup  $Z_3$  of  $W(G)$  and consider a  $Z_3$ -acyclic submanifold  $F(Z_3, A)$ . Then the possible isotropy subgroups on  $F(Z_3, A)$  are  $SU(3)$  and  $G$ . Considering the slice representation of  $G$  we see that the fixed point set of  $G$  is open and closed in  $F(Z_3, A)$ . Thus, because of the connectedness of  $F(Z_3, A)$ , we see that  $SU(3)$  is not an isotropy subgroup. Hence  $F(G, E^m) = F(Z_3, A)$ . Similarly  $SO(4)$  is not an isotropy subgroup and hence we have  $F(G, E^m) = F(Z_2 \oplus Z_2, A)$  which is  $Z_2$ -acyclic. q. e. d.

Next we consider the case where  $H_\phi^0$  is isomorphic to  $SU(3)$ . Then  $\phi$  has a fixed point of  $G$ , since, if not,  $\phi$  has the uniform dimensional orbits, but this is impossible [3]. Let  $E_0 = E^m - (\text{the fixed set of } G)$  and  $F = F(SU(3), E^m) \cap E_0$ . Then we see  $E_0 = G/SU(3) \times_{L/SU(3)} F$ , where  $L$  is the normalizer of  $SU(3)$  and  $L/SU(3) \cong Z_2$  by the proposition 1.3.  $E_0$  admits a fibering  $F \rightarrow E_0 \rightarrow G/L$ . It is well known that  $G/SU(3) = S^8$  and hence  $G/L = P^8$  (real projective space). Then from the homotopy exact sequence of the fibering we know that  $F$  has 2 connected components, since  $E_0$  is simply connected. Thus  $Z_2$  acts simply transitively on the components of  $F$  and hence there is no exceptional orbit. Then as in the proof of the proposition 4.1 we have the following

**Proposition 4.2.**  $F(G, E^m)$  is  $Z_2$ -acyclic.

### 5. The case $H_\phi^0 = SU(2)$

Let  $G_x$ ,  $x \in E^m$  be an isotropy subgroup of the rank 2. Then the set of the complementary weights of  $G_x$  i. e.  $\lambda'(G) - \lambda'(G_x)$  is contained in  $\Sigma'(\phi)$ . Thus  $\lambda'(G) - \lambda'(G_x) \subseteq \{\pm \theta_i : i=1, 2 \text{ and } 3\}$  and hence  $\lambda'(G_x) \supseteq \{\pm(\theta_i - \theta_j) : i < j\}$ . Hence  $G_x$  has at least dimension 8. Therefore the possible isotropy subgroups of rank 2 of  $\phi$  are  $G$ ,  $L$  (=the normalizer of  $SU(3)$  and  $SU(3)$ ).

Let  $T$  be a maximal torus of  $SU(3) \subset G$ . Then we have

**Proposition 5.1.**  $F(SU(3), E^m)$  is identical with  $F(T, E^m)$ .

*Proof.* It is sufficient to show that  $G_a \cong SU(3)$  for any  $a \in F(T, E^m)$ . It is clear if  $G_a = G$ . Hence we may suppose that  $G_a^0 = gSU(3)g^{-1}$  for some  $g \in G$ . Since  $G_a^0 \cong T$  and  $W(SU(3))$  is normal in  $W(G)$  we can assume that  $g \in N_T$  and  $W(G_a^0)$  then equals  $(gT)W(SU(3))(gT)^{-1}$ . Hence by the proposition 1.2 we have  $G_a^0 = SU(3)$ .

From now on we assume  $F(G, E^m)$  is empty and show that we then arrive at a contradiction. Let us denote the singular set by  $E_s$  and put  $E_0 = E^m - E_s$ . Then we have  $E_s = G/SU(3) \times_{Z_2} F(T, E^m)$  by the proposition 5.1. Now we prove the following

**Proposition 5.2.**  $H_c^{p+m-12}(E_s; Z_2) = Z_2$  if  $0 \leq p \leq 6$ , and 0 otherwise<sup>2)</sup>.

*Proof.* Since  $E_s$  admits a fibering  $F(T, E^m) \rightarrow E_s \rightarrow G/L = P^6$ , there is a spectral sequence which converges to  $H_c^*(E_s; Z_2)$  and whose  $E_2$  terms are  $E_2^{p,q} (= H_c^p(P^6; H_c^q(F(T, E^m); Z_2)))$ .  $F(T, E^m)$  is an  $(m-12)$  dimensional acyclic manifold and hence  $H_c^q(F(T, E^m); Z_2) = 0$  if  $q \neq m-12$ , and  $Z_2$  if  $q = m-12$ . Thus  $E_2^{p, m-12} = Z_2$  for  $0 \leq p \leq 6$  and otherwise  $E_2^{p,q} = 0$ . This proves the proposition. q. e. d.

Then, by the exact sequence of the pair  $(E^m, E_s)$

$$\cdots \rightarrow H_c^i(E^m; Z_2) \rightarrow H_c^i(E_s; Z_2) \rightarrow H_c^{i+1}(E_0; Z_2) \rightarrow \cdots$$

we have

**Proposition 5.3.**  $H_c^i(E_0; Z_2) = Z_2$  if  $i = m$  or  $m-11 \leq i \leq m-5$ , and 0 otherwise.

Let us put  $A = F(SU(2), E^m)$  where  $SU(2)$  is considered as a subgroup of  $SU(3)$ . Then we have the following

**Proposition 5.4.**  $A \cap E_s = S^1 \times_{Z_2} F(T, E^m)$ , where  $Z_2$  acts on  $S^1$  antipodally.

*Proof.* Since  $E_s = G/SU(3) \times_{Z_2} F(T, E^m)$  it is clear that  $A \cap E_s = F(SU(2), G/SU(3)) \times_{Z_2} F(T, E^m)$ .  $G/SU(3) = S^6$  and  $G$  acts orthogonally on  $S^6$ . Then, since the isotropy representation of  $SU(3)$  is the standard representation  $\mu_3$  of  $SU(3)$ , it follows that  $F(SU(2), G/SU(3)) = S^1$ .

2) We use the Alexander-Spanier cohomology with compact supports.

Let  $S$  be a torus of  $SU(2)$ . Then  $A \subseteq F(S, E^m)$ . Since the local representation of  $SU(3)$  at a fixed point of  $SU(3)$  is  $2(\mu_3)_R \oplus (m-12)$  we have  $\dim A = \dim F(S, E^m) = m-8$ . Thus  $\bar{A} = F(S, E^m)$  since  $F(S, E^m)$  is connected. Hence  $A$  is acyclic.

Let  $A_0 = A - E_s$ . Then we have the following

**Proposition 5.5.**  $H_i^*(A_0; Z_2) = Z_2$  if  $i = m-8$ ,  $m-10$  or  $m-11$  and 0 otherwise.

*Proof.* As in the proposition 5.2 we have  $H_c^{p+m-12}(A \cap E_s; Z_2) = Z_2$  for  $p=0$  or 1. and 0 otherwise. Hence by the exact sequence of the pair  $(A, A \cap E_s)$  we get the proposition. q. e. d.

Now we show that our assumption i. e.  $F(G, E^m) \neq \emptyset$  leads to a contradiction. Let  $N$  be the normalizer of  $SU(2)$ . Then we have  $E_0 = G/SU(2) \times_{N/SU(2)} A_0$  and hence  $E_0$  admits a fibering  $A_0 \rightarrow E_0 \rightarrow G/N$ . Now consider the spectral sequence of the fibering whose  $E_2$  terms are  $E_2^{p,q} = H_c^p(G/N; H_c^q(A_0; Z_2))$  and that converges to  $H_c^{p+q}(E_0; Z_2)$ . Since  $N$  is isomorphic to  $SO(4)$  by the proposition 1.3 it is known that the Poincaré polynomial of mod 2 of  $G/N$  is  $1+t^2+t^3+t^4+t^5+t^6+t^8$ . Let us denote  $s = \max\{q \mid H_c^q(A_0; Z_2) \neq 0\}$ . Then  $E_2^{8,s} = H_c^8(G/N; H_c^s(A_0; Z_2)) \cong H_c^8(A_0; Z_2) \neq 0$ . We have  $H_c^s(A_0; Z_2) \cong H_c^{8+s}(E_0; Z_2)$ , since  $E_2^{8,s} (\cong E_3^{8,s} \cong \dots \cong E_\infty^{8,s})$  is the only non-zero term in degree  $8+s$ . Since  $8+s$  is the highest dimension with non zero cohomology, it must be  $8+s=m$  by the proposition 5.3 and  $H_c^{m-8}(\bar{A}_0; Z_2) = Z_2$ . Now we consider the differential  $d_2: E_2^{8,s} \rightarrow E_2^{8,s-1}$ . If  $E_2^{8,s-1} = 0$  then  $E_2^{6,s} \cong E_3^{6,s} \cong \dots \cong E_\infty^{6,s} \cong H_c^6(A_0; Z_2) \neq 0$  and this implies that  $H_c^{m-2}(E_0; Z_2) \neq 0$ . By the proposition 5.4 this is impossible and hence  $E_2^{8,s-1} \neq 0$ . Hence we have  $H_c^{m-9}(A_0; Z_2) \neq 0$ , which contradicts the proposition 5.5. We see from this that  $F(G, E^m)$  is not empty.

Next we prove the following

**Proposition 5.6.**  $F(G, E^m)$  is  $Z_2$ -acyclic.

*Proof.* Let  $T$  be a maximal torus of  $G$  and  $F = F(T, E^m)$ . Then the Weyl group  $W(G)$  acts on  $F$ . Take a subgroup  $Z_2 \oplus Z_2$  of  $W(G)$  and consider  $F(Z_2 \oplus Z_2, F)$ . Then the possible isotropy subgroups of  $\phi$  on  $F(Z_2 \oplus Z_2, F)$  are  $G$  and the normalizer  $L$  of  $SU(3)$ . Considering the slice representation of  $G$ , it is easily known that  $L$  is not an isotropy subgroup. Hence  $F(G, E^m) = F(Z_2 \oplus Z_2, F)$  and  $Z_2$ -acyclic. q. e. d.

We thus get our main theorem by summarizing the above as follows

**Theorem 5.7.** *Let  $G$  be the exceptional compact Lie group of rank 2 and  $\phi$  a differentiable  $G$  action on a Euclidean space with a principal isotropy subgroup  $H_\phi$  of a positive dimension. Then, if  $H_\phi^0$  is a maximal torus of  $G$ , the fixed point set of  $G$  is  $Z_p$ -acyclic for  $p=2$  and 3 and, if  $H_\phi^0$  is  $SU(3)$  or  $SU(2)$ , the fixed point set of  $G$  is  $Z_2$ -acyclic.*

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