Mathematical Journal of Okayama University

Volume 13, *Issue* 2 1967 *Article* 2

NOVEMBER 1968

On a result of Ribenboim

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ON A RESULT OF RIBENBOIM

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Throughout the present note, A will represent a ring with 1, B a subring of A with the same 1, σ a B-ring automorphism of A, and T the subset $\{y \in A\,;\, y\sigma \neq y\}$. If $\sigma \neq 1$ then it is easy to see that $A = T \cup \{T+T\} = B[T]$. The extension A/B is called left locally finite if $B[F]$ is left finite over B (finitely generated as a left B-module) for every finite subset F of A. In below, we shall give a generalization of the result cited in [1] with a notably short proof.

Proposition. *Let B be left Noetherian, and A/B a left locally finite extension that is not left finite. If $\sigma \neq 1$ then there exists a subset Q of A such that $Q \cap Q\sigma = \emptyset$ (empty set) and $B[Q, Q\sigma]$ is not left finite over B.*

Proof. Let Q' be an arbitrary finite subset of T with $Q' \cap Q'\sigma = \emptyset$, and set $B' = B[Q', Q'\sigma]$, $B^* = B[Q', Q'\sigma, Q'\sigma^{-1}]$, $T' = T \setminus B'$ (complement of B' in T) and $T^* = T' \setminus B^*$. Since $A = B[T] = B'[T']$, we readily see that $B[T']/B$ is not left finite. Accordingly, $B^*[T^*] = B^*[T'] \supset B[T']$ implies that $B[T^*]/B$ is not left finite. In particular T^* contains an element x. Since $x \notin B^*$, $B'\sigma^{-1} \subset B^*$ implies $x\sigma \notin B'$, and then we can easily see that $(\{x\} \cup Q') \cap (\{x\} \cup Q')\sigma = \emptyset$. Repeating the above argument, we obtain an infinite ascending chain $Q_1 \subsetneqq Q_2 \subsetneqq \cdots$ of finite subsets Q_n of T such that $Q_n \cap Q_n\sigma = \emptyset$ and $B \subsetneqq B[Q_1, Q_1\sigma] \subsetneqq B[Q_2, Q_2\sigma] \subsetneqq \cdots$. If we set $Q = \bigcup_{n=1}^{\infty} Q_n$ then $Q \cap Q\sigma = \emptyset$ and $B[Q, Q\sigma] = \bigcup_{n=1}^{\infty} B[Q_n, Q_n\sigma]$, that is not left finite over B.

REFERENCE

[1] P. RIBENBOIM: A property of automorphisms of an infinite Galois extension, Math. Annalen 166 (1966), 54—55.

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(*Received April 1, 1968*)