3-D eddy current analysis in moving conductor of permanent magnet type of retarder using moving coordinate system

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Abstract: A 3-D dc steady state eddy current analysis of a permanent magnet type of retarder which rotates at high speed is carried out using a moving coordinate system. The method of dc steady state analysis using a moving coordinate system is described. The calculated braking torque is compared with measured one.

Keywords: eddy current, moving conductor, permanent magnet, retarder, finite element method

I. INTRODUCTION

In heavy vehicles, auxiliary braking systems such as permanent magnet type of retarders, which produce a braking torque by fluxes and eddy currents, are used sometimes [1]. In order to improve the performance of the retarder, the dc steady state flux and eddy current distributions should be analyzed taking into account the rotation with high speed. However, the results obtained are oscillating due to the high Peclet number [2] when the ordinary Galerkin FEM using a fixed coordinate system is used. On the other hand, it has been already shown that the above-mentioned problem can be avoided in the analysis using "upwind" technique [1,2] or a moving coordinate system [3].

In this paper, the method of dc steady state eddy current analysis using a moving coordinate system, in which a complicated method for determining the parameter of "upwind" is not required, is described. The 3-D eddy current analysis of the permanent magnet type of retarder at high speed is carried out. The calculated braking torque of retarder is compared with measured one.

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II. MODEL OF RETARDER

Fig. 1 shows a model of permanent magnet type of retarder. Only 1/24 of the whole region is shown due to symmetry. The outer rotor rotates with a constant speed from 1,000 to 5,000rpm. The outer rotor, pole piece and yoke are made of carbon steel (rotor and pole piece: S10C, yoke: S15C) and the nonlinearity is taken into account. Fig. 2 shows the B-H curve. The permanent magnet (Sm_2Co_{17}) is assumed to be magnetized in parallel direction and the remanent flux density (magnetization) Br is 1T. In the dc steady state, eddy currents flow only in the outer rotor. The conductivity of outer rotor is 7×10^6S/m.

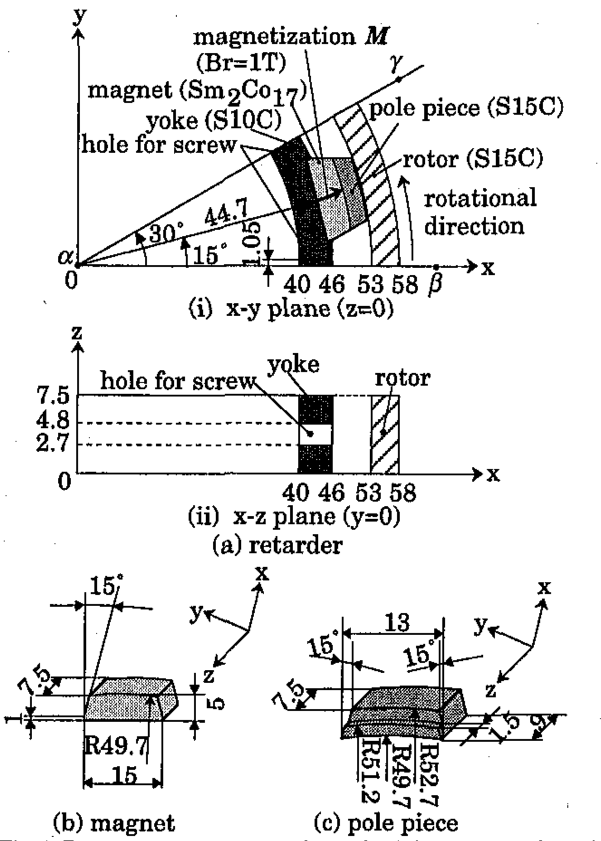
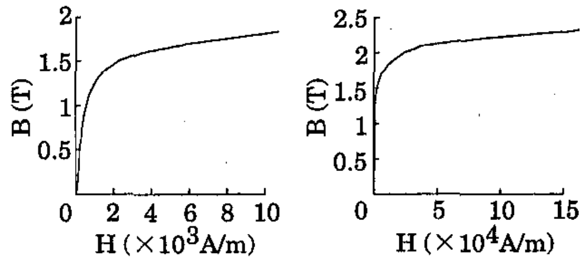


Fig. 1. Permanent magnet type of retarder (1/24 region, unit:mm).



(a) $0 \leqq H \leqq 10000$ (b) $0 \leqq H \leqq 150000$

Fig. 2. B-H curve (S15C).

III. ANALYSIS

A. Fundamental Equations

In the moving conductor region of the retarder, eddy currents are induced. The fundamental equations of the A–ϕ method (A: magnetic vector potential, ϕ: electric scalar potential) using a moving coordinate system [3] are given by

$$\mathrm{rot}(\nu\,\mathrm{rot}\boldsymbol{A}) = -\sigma\left(\frac{\partial \boldsymbol{A}}{\partial t} + \mathrm{grad}\phi\right) \quad (1)$$

$$\mathrm{div}\left\{-\sigma\left(\frac{\partial \boldsymbol{A}}{\partial t} + \mathrm{grad}\phi\right)\right\} = 0 \quad (2)$$

where ν and σ are the reluctivity and the conductivity, respectively. In the permanent magnetic region (standstill) without eddy currents, the fundamental equation is given by

$$\mathrm{rot}(\nu_o\,\mathrm{rot}\boldsymbol{A}) = \nu_o\,\mathrm{rot}\boldsymbol{M} \quad (3)$$

where $\boldsymbol{M}$ is the magnetization vector. ν_o is the reluctivity in vacuum.

B. Discretization of Eddy Current Term

The eddy current term $\boldsymbol{J}_e(=-\sigma\,(\partial \boldsymbol{A}/\partial t+\mathrm{grad}\phi)$ in (1) and (2) can be discretized by the backward difference method. Then, $\boldsymbol{J}_e\,(\mathrm{p}_2)^{t+\Delta t}$ at a point p_2 at the instant $t+\Delta t$ is represented as follows [3]:

$$\boldsymbol{J}_e(\mathrm{p}_2)^{t+\Delta t} = -\sigma\left\{\frac{\boldsymbol{A}^*(\mathrm{p}_2)^{t+\Delta t} - \boldsymbol{A}(\mathrm{p}_1)^t}{\Delta t} + \mathrm{grad}\phi^*(\mathrm{p}_2)^{t+\Delta t}\right\} \quad (4)$$

where it is assumed that the point p_1 is moved to the point p_2 during the time interval Δt as shown in Fig. 3. The superscript (*) indicates the unknown variable. $\boldsymbol{A}(\mathrm{p}_1)$ is interpolated using the potential at each node in an element e which contains the point p_1 as follows:

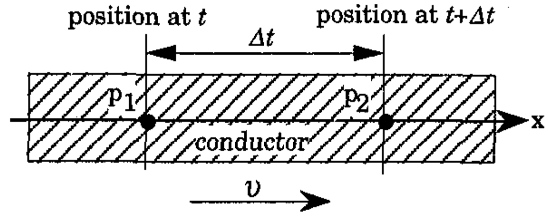


Fig. 3. Moving conductor with constant velocity.

$$\boldsymbol{A}(\mathrm{p}_1) = \sum_{i=1}^{n^{(e)}} N_i^{(e)} \boldsymbol{A}_i^{(e)} \quad (5)$$

where $n^{(e)}$ is the number of nodes in the element e, $N_i^{(e)}$ is the interpolation function.

In order to calculate the dc steady state using (4), the time iteration is required until the distribution of $\boldsymbol{A}$ does not change with time as follows:

$$\boldsymbol{A}(\mathrm{p}_1)^t = \boldsymbol{A}(\mathrm{p}_1)^{t+\Delta t} \quad (6)$$

The following equation is obtained by substituting (6) into (4):

$$\boldsymbol{J}_e(\mathrm{p}_2) = -\sigma\left\{\frac{\boldsymbol{A}^*(\mathrm{p}_2) - \boldsymbol{A}^*(\mathrm{p}_1)}{\Delta t} + \mathrm{grad}\phi^*(\mathrm{p}_2)\right\} \quad (7)$$

In the above equation, superscript $t+\Delta t$ is omitted. If both $\boldsymbol{A}(\mathrm{p}_1)$ and $\boldsymbol{A}(\mathrm{p}_2)$ are treated as the unknown variables, the dc steady state flux and eddy current distributions can be obtained without time iteration. In this case, the coefficient matrix becomes unsymmetric. The ILUBCGSTAB method [4] is used to solve the linear equations. In the case of standstill, the ICCG method is used, because the finite element matrix becomes symmetric.

C. Boundary Condition and Mesh

The analyzed region can be redused to 1/2 of the whole region by the boundary condition A_x=A_y=0 on the x-y plane (z=0). Moreover, in order to reduce the analyzed region to 1/24 shown in Fig. 1, the periodic boundary condition [5] should be investigated. Fig. 4 shows the relationships of flux densities and eddy current densities on the boundary surfaces α-β and α-γ. The directions of the z-component of flux densities $\boldsymbol{B}_{\alpha-\beta}$ and $\boldsymbol{B}_{\alpha-\gamma}$ on the surfaces α-β and α-γ become opposite. As the relationship of the magnetic vector potentials $\boldsymbol{A}$ on surfaces α-β and α-γ is same with that of flux

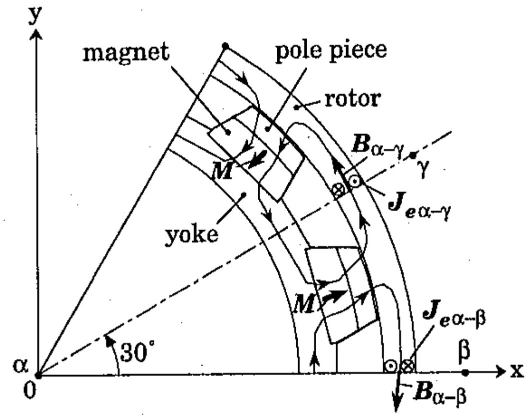


Fig. 4. Relationship of flux densities and eddy current densities on boundary surfaces α–β and α–γ.

densities, the following periodic boundary condition of $\boldsymbol{A}$ is applied:

$$A_{x\alpha-\beta} = -A_{x\alpha-\gamma}\cos 30° - A_{y\alpha-\gamma}\sin 30° \quad (8)$$

$$A_{y\alpha-\beta} = A_{x\alpha-\gamma}\sin 30° - A_{y\alpha-\gamma}\cos 30° \quad (9)$$

$$A_{z\alpha-\beta} = -A_{z\alpha-\gamma} \quad (10)$$

where, for example, $A_{x\alpha-\beta}$ is the x-component of the magnetic vector potential $\boldsymbol{A}$ on the boundary surface α–β. As the directions of the eddy current densities $\boldsymbol{J}_{e\alpha-\beta}$ and $\boldsymbol{J}_{e\alpha-\gamma}$ on the surfaces α-β and α-γ become opposite each other as shown in Fig. 4, the following periodic boundary condition of ϕ is applied:

$$\phi_{\alpha-\beta} = -\phi_{\alpha-\gamma} \quad (11)$$

Fig. 5 shows the finite element subdivision using 1-st order hexahedral nodal elements. In the analysis using the mesh, the maximum value of the Peclet number ($Pe=\mu\sigma vL/2$, v: velocity in the rotating direction, L: length of element in the rotating direction) of the outer rotor is about 80. In this case, the relative permeability and the rotational speed are assumed to be 720 and 5,000 rpm, respectively. The time interval Δt is chosen such that the rotor rotates at 0.1deg during the period of Δt.

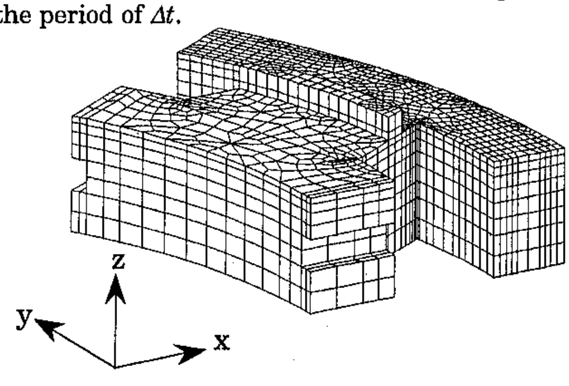


Fig. 5. Mesh without air region.

IV. RESULTS AND DISCUSSION

A. Flux Density

Fig. 6 shows the flux distributions. The flux distributions in the rotor change with the rotational speed. Stable flux distributions without spurious oscillations can be obtained even at high Peclet number (nearly 80 at 5,000rpm). The flux flows near inner and outer surfaces. This is because the flux can also flow near the upper and outer surfaces of rotor by 3-D effect as shown in Fig. 7.

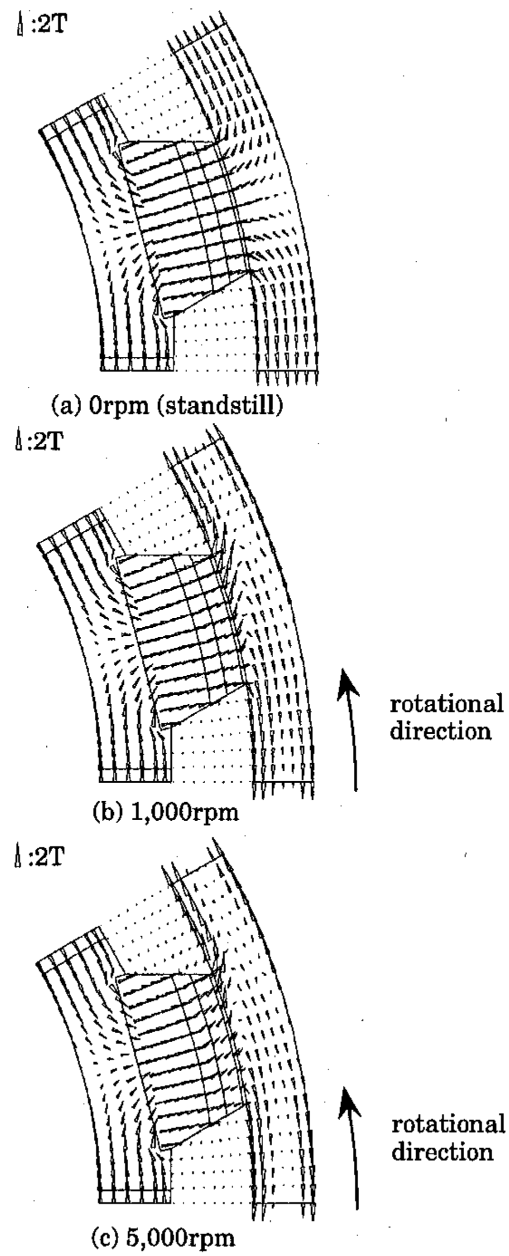
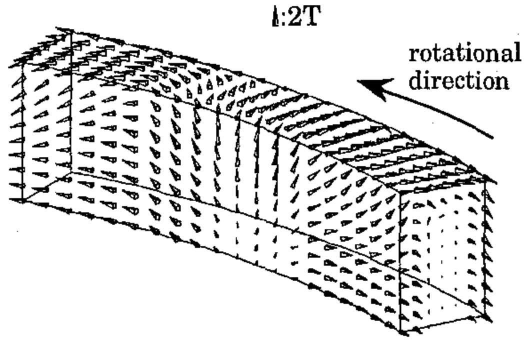
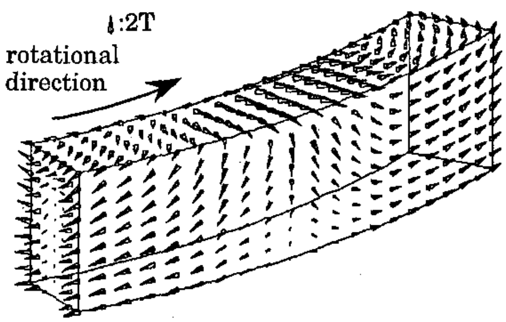


Fig. 6. Flux distributions (x-y plane, z=0).

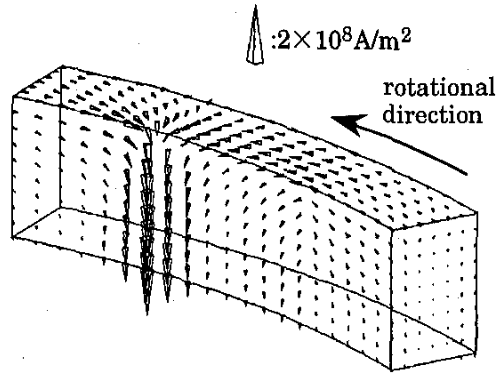


(a) upper and inner surfaces of rotor

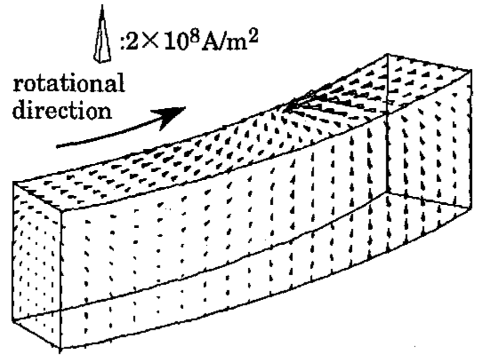


(b) upper and outer surfaces of rotor

Fig. 7. Pass of flux (5,000rpm).



(a) upper and inner surfaces of rotor



(b) upper and outer surfaces of rotor

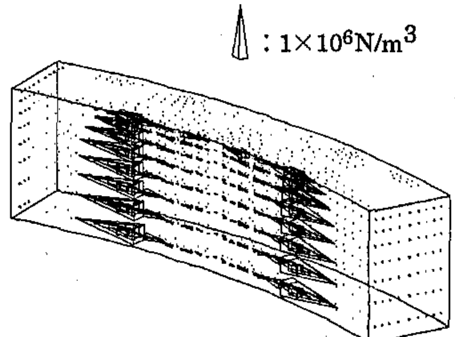
Fig. 8. Eddy current distribution (5,000rpm).

B. Eddy Current Density

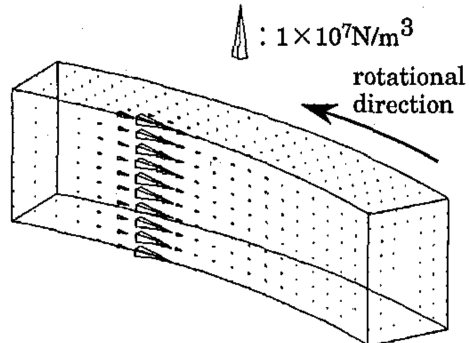
Fig. 8 shows the eddy current distributions. The eddy current flows in the direction to cancel the flux generated from the magnet. The large eddy currents flow in the inner part of rotor.

C. Braking Torque

The braking torque is calculated using the nodal force method [6]. The distributions of the tangential component F_t of electromagnetic force are shown in Fig. 9. In the case of standstill, the braking torque which is the summation of the $F_t \cdot r$ (r: radius) becomes zero. The large F_t related to braking torque occurs in the inner part of rotor at high speed.



(a) 0rpm (standstill)



(b) 5,000rpm

Fig. 9. Tangential component of electromagnetic force.

D. Memory Requirements and CPU Time

Table I shows the discretization data. The CPU time for the analysis at high speeds are about 15 times that at standstill. The memory requirements at high speeds is increased to about 3 times that at standstill, because the matrix is unsymmetric and ϕ is added as unknown variables. The CPU time is increased when the speed becomes high, because the number of iterations for ILUBCG method is increased due to the ill-condition.

Table I Discretization data and CPU time

speed (rpm)	0 (standstill)	1,000	3,000	5,000
number of elements	18,564			
number of nodes	20,706			
number of unknowns	52,614	58,806		
number of non-zeros	2,020,743	8,153,600		
memory requirements (MB)	47	139		
number of nonlinear iterations	5	9	9	9
total CPU time (h)	1.0	13.0	14.8	16.1

Convergence criterion for Newton–Raphson method : 0.01T
Convergence criterion for ICCG method (standstill) : 10^{-7}
Convergence criterion for ILUBCGSTAB method (rotation) : 10^{-7}
Computer used : IBM 3AT workstation (49.7 MFLOPS)

V. EXPERIMENTS

The radial component B_r of flux density in the air gap at standstill is measured by using a Hall probe. The discrepancy of flux densities in the air gap between measurement and calculation is about 5% as shown in Fig. 10. In this comparison, the inner radius of the rotor is increased to 55mm in order to insert a Hall probe in the air gap.

The braking torque is measured using the measurement system shown in Fig. 11. In this measurement, the stator (magnet part) is rotated by the motor. The torque meter is set between the motor and the stator. Fig. 11(b) shows the situation when that the rotor is elevated. Fig. 12 shows the comparison between the calculated and measured braking torques. The calculated braking torque is fairly good agreement with measured one.

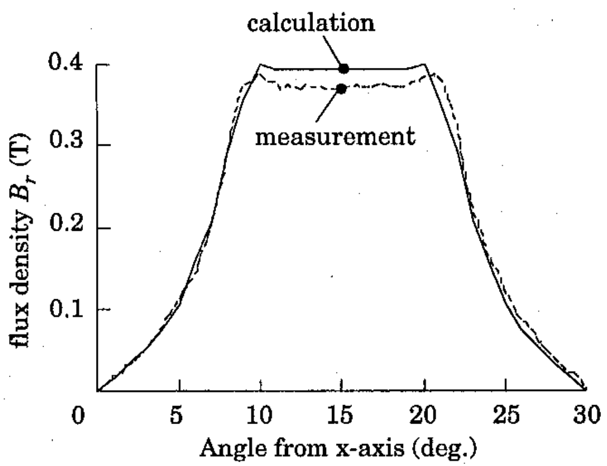
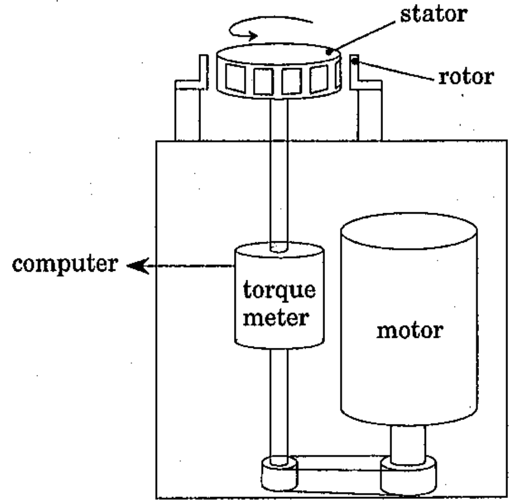
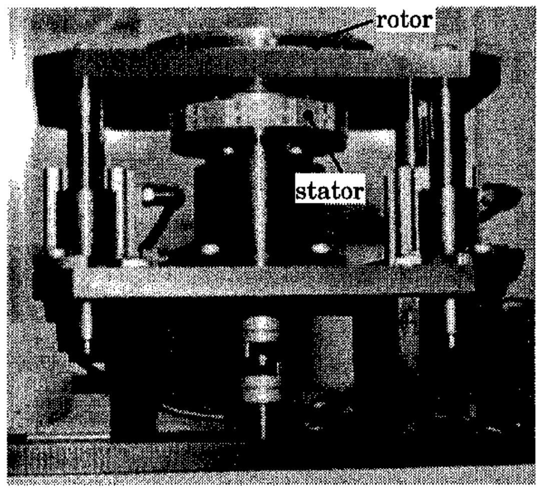


Fig. 10. Flux distribution in gap (r=53.85, z=0, standstill).



(a) measurement system



(b) stator and rotor

Fig. 11. Measurement system for braking torque.

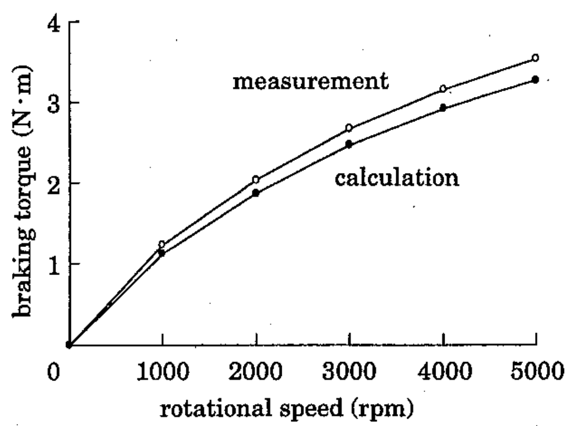


Fig. 12. Comparison of braking torque.

VI. CONCLUSIONS

The dc steady state eddy current analysis of a permanent magnet type of retarder is carried out using a moving coordinate system. The results obtained can be summarized as follows:

(1) The method of dc steady state analysis using a moving coordinate system is described. Although the coefficient matrix is unsymmetric, the dc steady state flux and eddy current distributions are directly obtained without time iteration.

(2) The stable flux distributions without spurious oscillations can be obtained by using a moving coordinate system at high Peclet number.

(3) The calculated braking torque is fairly good agreement with measured one.

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VIII. BIOGRAPHIES

Kazuhiro Muramatsu was born in Yamanashi Prefecture, Japan, on August 15, 1965. He received the B.E., M.E. and D.E. degrees in electrical engineering from Okayama University in 1988 , 1990 and 1993 respectively.

From 1990 to 1992, he was with ALPS Electric Co., Ltd.. From 1992 to 1994, he was with TOHOKU ALPS Co., Ltd.. Since 1994, he has been an Assistant Professor at the Department of Electrical Engineering, Okayama University. His major field of interest is 3-D eddy current finite element analysis.

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Dr. Takahashi was awarded the 1983 IEEJ Book of the Year Award. He is a vice-chairman of the TEAM Workshop Planning Board.

Tomohiro Hashio was born in Hyogo Prefecture, Japan, on October 20, 1973. He received the B.E. in electrical engineering from Yamaguchi University in 1996.

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He entered Isuzu Motors, Ltd. in 1969, then moved to Isuzu Advanced Engineering Center, Ltd., and now belong to Vehicle Department as chief engineer.

Makoto Ogawa was born in Tokyo, Japan on July 26, 1967. He received B.E. and M.E. degrees in chemical engineering from Nihon University in 1990 and 1992 respectively.

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Shin Kobayashi was born in Nagano Prefecture, Japan, on October 31, 1970. He received B.E. degree in mechanical engineering from Science University of Tokyo in 1994.

He entered Isuzu Advanced Engineering Center, Ltd. in 1994 and mainly engaged in computer simulation.

Tohru Kuwahara was born in Yamaguchi Prefecture, Japan, on August 6, 1947. He received B.E. degree in mechanical engineering from Shinshu University in 1970.

He entered Isuzu Motors, Ltd. in 1970 and mainly worked in Heavy Duty Vehicle Engineering and Drivetrain Engineering Departments.

Mr. Kuwahara was awarded JSAE and JSME medal in 1993 for developing permanent magnet type of retarder.