Three-dimensional analysis of eddy current distributions by the boundary element method using vector variables

H. Tsuboi
Okayama University

T. Misaki
Okayama University

THREE-DIMENSIONAL ANALYSIS OF EDDY CURRENT DISTRIBUTIONS BY THE BOUNDARY ELEMENT METHOD USING VECTOR VARIABLES

H. Tsuboi and T. Misaki

Department of Electrical Engineering, Okayama University
Tsushima, Okayama 700, Japan

Abstract - This paper describes a boundary element method using vector variables for three-dimensional analysis of eddy current distributions. In the boundary element method, electric field vectors and magnetic flux density vectors are used as unknown vector variables on the boundaries of two materials. The formulation is performed by using the vector Green's theorem. After determining electric field vectors and magnetic flux density vectors on the boundaries, eddy current distributions in the conductor are computed. The computation results of a conducting sphere model agreed exactly with analytical solutions. As an example of three dimensional eddy current problems, the analysis of a conducting cube model was done.

INTRODUCTION

The computation of three-dimensional eddy current distributions is becoming important in the design of electrical machinery and apparatus. For solving three-dimensional electromagnetic field problems based on Maxwell's equations, it is necessary to introduce unknown vector variables such as vector potential, electric field vector and magnetic flux density vector.

In the boundary element method, we introduce electric field vectors and magnetic flux density vectors as unknown vector variables on the boundaries of two materials.

The boundaries of two materials are divided into a number of triangular elements. And electric field vectors and magnetic flux density vectors on the triangular element are approximated through the use of a linear function of coordinates. The formulation of the boundary element method is performed by using the vectors Green's theorem. After introducing boundary conditions of electric field and magnetic flux density, final simultaneous equations are obtained. The eddy current distribution in the conductor is computed by using electric field vectors and magnetic flux density vectors on the boundaries. Here, a conducting sphere model and a conducting cube model were chosen as examples of the above problems.

FORMULATION

Maxwell's equations for sinusoidal time dependence, which are fundamental equations in the eddy current problems, are given by

$$\nabla\times\vec{E} = -j\omega\vec{B} \tag{1}$$

$$\nabla\times\vec{B} = j\omega\mu(\varepsilon-j\sigma/\omega)\vec{E} + \mu\vec{J}_s \tag{2}$$

$$\nabla\cdot\vec{E} = 0 \tag{3}$$

$$\nabla\cdot\vec{B} = 0 \tag{4}$$

where $\vec{E}$ is the electric field,
$\vec{B}$ is the magnetic flux density,
$\vec{J}_s$ is the source current density,
μ is the permeability,
ε is the permittivity,
σ is the conductivity,
ω is the angular frequency,
j is the complex operator.

On the other hand, the eddy current density, $\vec{J}_e$, in the conductor is computed by multiplying $\vec{E}$ by σ:

$$\vec{J}_e = \sigma\vec{E} \tag{5}$$

The formulation of the boundary element method is based on the vector Green's theorem[1]. Electric field, $\vec{E}_i$, and magnetic flux density, $\vec{B}_i$, at any computation point, i, in the region, V, which is enclosed by the surface, S, are obtained by the use of the vector Green's theorem as follows[2]:

$$\frac{\Omega_i}{4\pi}\vec{E}_i = \int_S\{j\omega(\vec{n}\times\vec{B})\phi-(\vec{n}\times\vec{E})\times\nabla\phi-(\vec{n}\cdot\vec{E})\nabla\phi\}dS - \int_V j\omega\mu\vec{J}_s\phi\, dv \tag{6}$$

$$\frac{\Omega_i}{4\pi}\vec{B}_i = -\int_S\{j\omega\mu(\varepsilon-j\sigma/\omega)(\vec{n}\times\vec{E})\phi+(\vec{n}\times\vec{B})\times\nabla\phi+(\vec{n}\cdot\vec{B})\nabla\phi\}dS + \int_V\mu\vec{J}_s\times\nabla\phi\, dv + \vec{B}_0 \tag{7}$$

where Ω_i is the solid angle subtended by S at i,
$\vec{n}$ is the unit normal vector at source point,
ϕ is the fundamental solution,
$\vec{B}_0$ is the impressed magnetic flux density.

And ϕ is given by

$$\phi = \frac{\exp\{-j\omega\sqrt{\mu(\varepsilon-j\sigma/\omega)}\,r\}}{4\pi r} \tag{8}$$

where r is the distance between the source point and the computation point. The integrations which appear in Eqs. (6) and (7) are evaluated by numerical integrations.

In the boundary element method, only the boundary surface, S, is divided into a number of triangular elements[2]. And electric field, $\vec{E}^e$, and magnetic flux density, $\vec{B}^e$, on the triangular element are approximated through the use of the linear function of coordinates as follows:

$$\vec{E}^e = L_1\vec{E}_1 + L_2\vec{E}_2 + L_3\vec{E}_3 \tag{9}$$

$$\vec{B}^e = L_1\vec{B}_1 + L_2\vec{B}_2 + L_3\vec{B}_3 \tag{10}$$

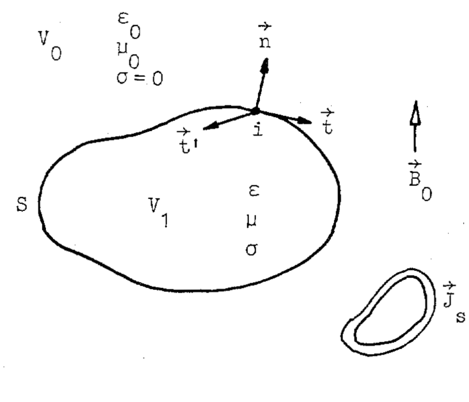

Fig. 1 Typical eddy current problem

0018-9464/87/0900-3044$01.00©1987 IEEE

where L_1, L_2 and L_3 are triangular coordinates, $\vec{E}_1$, $\vec{E}_2$ and $\vec{E}_3$ are electric field vectors at vertices, 1, 2 and 3, respectively, and $\vec{B}_1$, $\vec{B}_2$ and $\vec{B}_3$ are magnetic flux density vectors at vertices. Accordingly, the electric field vector and magnetic flux density vector at the vertex or node are assumed as unknown vectors.

The simultaneous equations are formed by using Eqs. (6) and (7), and boundary conditions are introduced. The typical eddy current problem is shown in Fig. 1. In this case, the boundary conditions of electric field and magnetic flux density are given by

$$(\varepsilon - j\sigma/\omega)\vec{E}_{i1}\cdot\vec{n} = \varepsilon_o\vec{E}_{i0}\cdot\vec{n},$$

$$\vec{E}_{i1}\cdot\vec{t} = \vec{E}_{i0}\cdot\vec{t}. \qquad (11)$$

$$\vec{B}_{i1}\cdot\vec{n} = \vec{B}_{i0}\cdot\vec{n},$$

$$\vec{B}_{i1}\cdot\vec{t}/\mu = \vec{B}_{i0}\cdot\vec{t}/\mu_0. \qquad (12)$$

After solving final simultaneous equations, eddy current density in the conductor is computed by using Eqs. (5) and (6).

COMPUTATION RESULTS

In order to verify the accuracy of the boundary element method, a conducting sphere model was chosen as an example of the eddy current problem. The conducting sphere model in a uniform alternating magnetic field is shown in Fig.2. The number of triangular elements on an eighth part of the sphere surface is 36 and the number of nodes, at which unknown electric field vectors and magnetic flux density vectors are defined, is 28.

The computation results of eddy current density and magnetic flux density are shown in Fig. 3. In this case, the frequencies of impressed magnetic field are 50 (Hz) and 500 (Hz). The computation results agree exactly with analytical solutions[3].

An eighth part of a conducting cube model, which is a three-dimensional eddy current problem, is shown in Fig. 4. The number of triangular elements and that of nodes on an eighth part of the cube surface are 150 and 91, respectively. The distribution of eddy current density and that of magnetic flux density are shown in Fig. 5 and Fig. 6, respectively. These computation results have reasonable physical meanings.

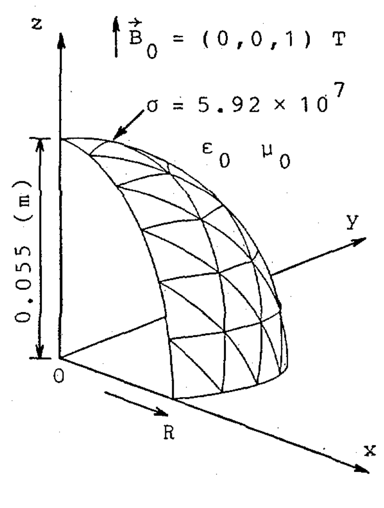

Fig. 2 A conducting sphere model

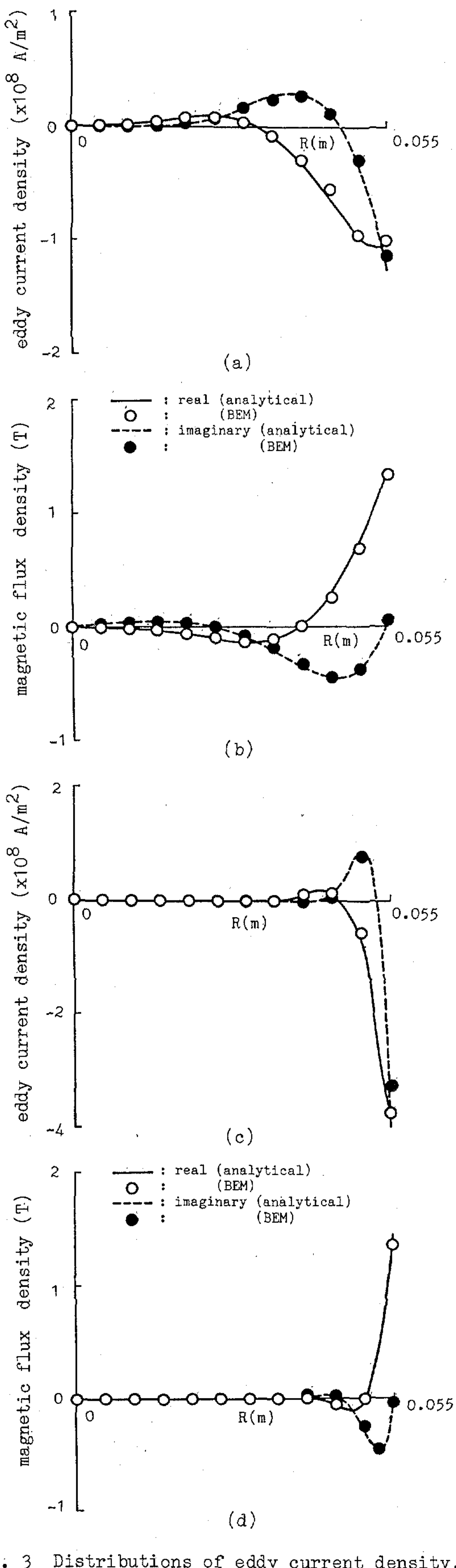

Fig. 3 Distributions of eddy current density, J_e, and magnetic flux density, B, on x-y plane, (a) J_e (f=50 Hz), (b) B (f=50 Hz), (c) J_e (f=500 Hz), (d) B (f=500 Hz)

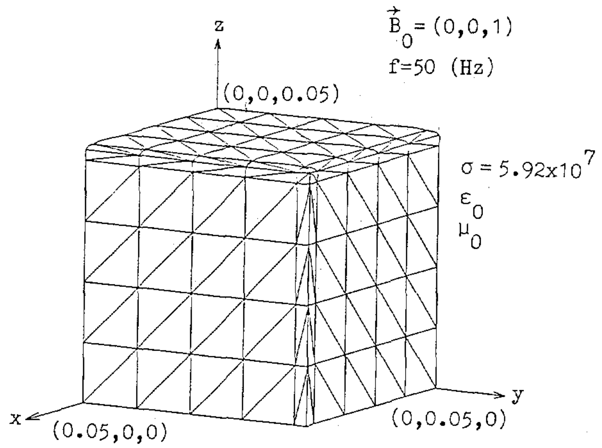

Fig. 4 A conducting cube model

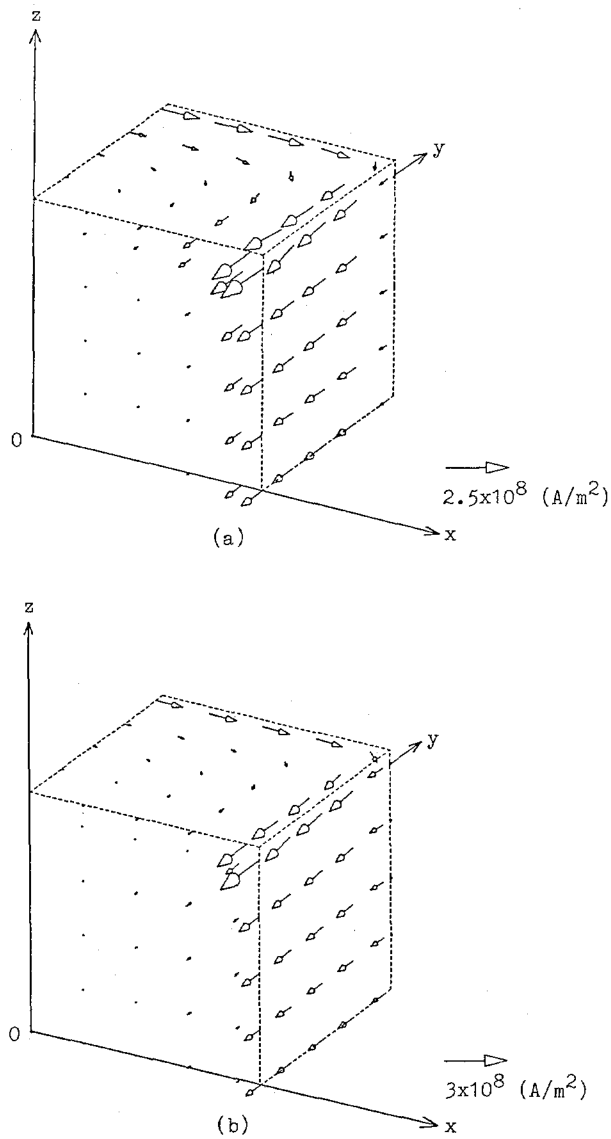

Fig. 5 Distributions of eddy current density vector, (a) real components, (b) imaginary components.

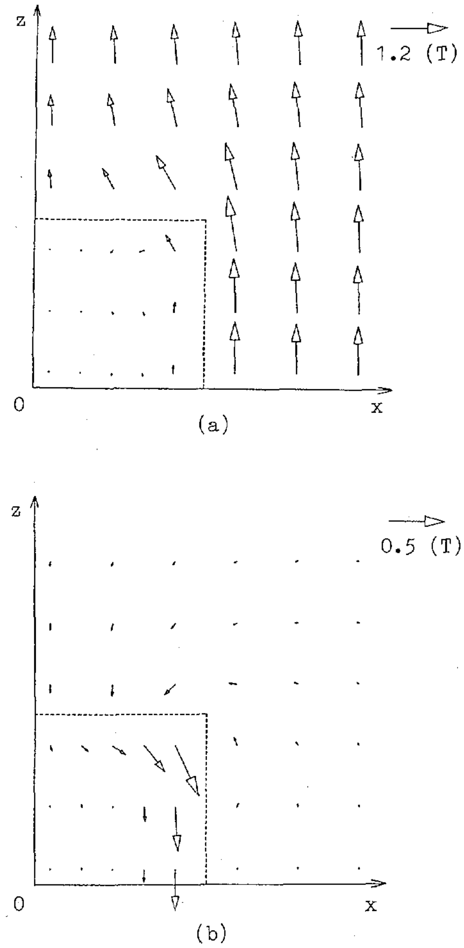

Fig. 6 Distributions of magnetic flux density vector, (a) real components, (b) imaginary components.

CONCLUSION

The paper described the boundary element method using vector variables for computing three-dimensional eddy current distributions. The computation results of the conducting sphere model agreed exactly with analytical solutions. Finally, the computation results of the conducting cube model, which is a three-dimensional model, were shown.

The boundary element method can be applied easily to three-dimensional problems because the computation is performed by using only the data of triangular elements on the boundaries of two materials. The advantage of this method is that electric field and magnetic flux density are solved directly and boundary conditions are physically clear. We expect that this method may be applied to various fields of industry.

REFERENCES

[1] J. A. Stratton: Electromagnetic Theory, McGraw-Hill, New York (1941)

[2] T. Misaki and H. Tsuboi: "Computation of 3-Dimensional Eddy Current Problems by using Boundary Element Method," IEEE Transactions on Magnetics, Vol. MAG-21, pp. 2227-2230 (1985)

[3] T. Morisue and M. Fukumi: "3D eddy current calculation using the magnetic vector potential," Workshop on electromagnetic field computation, Schenectady Section of IEEE, October 1986