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Some Statistical Considerations on Window Width and Matching Stability of Images.

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SYNOPSIS

In stereo matching of images, sample cross-covariances are used commonly as a criterion for deciding whether matched points are truly conjugate. Hereupon window width is a serious parameters to dominate matching stability. This paper argues about relation of matching stability with window width in terms of statistical behavior of sample covariances. For simple circumstances of analysis auto-covariances of a single image are considered instead of cross-covariances of stereo ones. First the mean and variance of sample auto-covariances are derived with parameters, window width and positional lag. Secondly they are evaluated from the correlation function estimated on an aerial image under the assumption of ergodicity to observe how they vary according as two parameters vary. From this result a variation factor is proved usefull to estimate appropriate window width.

1. INTRODUCTION

The major subject in stereo plotting study is to find the efficient way of identify conjugate points on stereo photographs. Among the methods proposed in present, the fundamental one is so-called area matching: i.e., a correlation window and the corresponding search

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window are settled on the left and right images respectively, and then the former is shifted inside the latter to find the maximum correlation peak.[(1),(2)] For area matching method correlation window width is one of the most predominant parameters. Improperly small width tends to cause mismatchings because of deficient textures, whereas improperly large window tends to cause coarse matchings because of correlation deterioration due to terrain reliefs. Then how large window is appropriate for stereo matching? The appropriate window width depends both on the spectral distribution property of terrain reliefs and of image densities. If the terrain is flat, it is evident that larger window make matching more precice and stable. Therefore the distribution of image densities regulates permissible lower limit of window width, and the distribution of terrain reliefs regulates upper limit.[(3)] This paper offers some indications toward the former problem.

We consider the images to be samples of 2-dimensional stationary process. By Imposing a few more essential assumptions on this, the statistics about sample cross-covariances are derived, which are used commonly as a matching criterion. And the appropriate window width will be suggested from a viewpoint of a variation factor. However it should be noted that the stochastic properties of images viewing natural scenes are generally too complex to apply the results obtained through the discussion quantitatively to them.

2. MEAN AND VARIANCE OF SAMPLE CROSS-COVARIANCES

If the terrain is flat and the ground objects do not posess the prominent property of directional reflectivity, then we can consider auto-correlations of a single image instead of cross-correlations of stereo ones. Hereafter we regard an image as a sample of 2-dimensional stationary stochastic field.

The image field is assumed noiseless, and averaged to zero. As seen in Fig.1 we consider two windows which is of the same size L x L and have relative positional lag τ_1,τ_2 in t_1,t_2 coordinate system respectively. Let a density at (t_1,t_2) be denoted by $x(t_1, t_2)$, and a sample cross-covariance (SSC) is defined as thus;

$$\left.\begin{aligned} C &= v_{12} - m_1 m_2 \\ v_{12} &= \frac{1}{4L^2} \iint x(t_1+\tau_1, t_2+\tau_2)\, x(t_1, t_2)\, dt_1 dt_2 \end{aligned}\right\}$$

$$\left.\begin{aligned} m_1 &= \frac{1}{4L^2}\iint x(t_1, t_2)\,dt_1 dt_2 \\ m_2 &= \frac{1}{4L^2}\iint x(t_1+\tau_1, t_2+\tau_2)\,dt_1 dt_2. \end{aligned}\right\} \quad (1)$$

Integration domain is the inside of each window. The mean and variance of SCC can be obtainted by calculation of the mean and variance of v_{12}, $m_1 m_2$ as follows.

Mean of v_{12} : $E(v_{12})$

$$E(v_{12}) = \frac{1}{4L^2}\iint E(x(t_1+\tau_1, t_2+\tau_2)\, x(t_1, t_2))\,dt_1 dt_2. \quad (2)$$

Eq.2 is reduced to

$$E(v_{12}) = \frac{1}{4L^2}\iint R(\tau_1, \tau_2)\,dt_1 dt_2$$

$$= R(\tau_1, \tau_2) \quad (2')$$

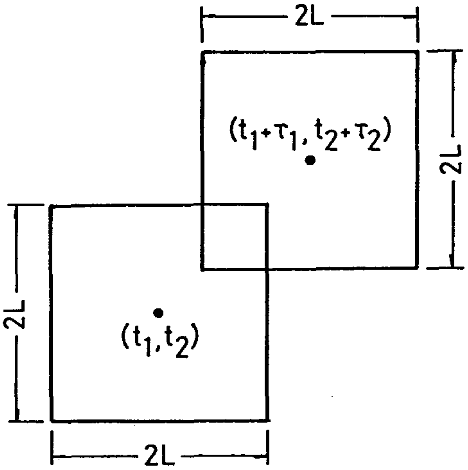


Fig.1 Correlation windows with lag τ_1, τ_2

from definition of auto-correlation function $R(\tau_1, \tau_2)$. For simplicity auto-correlation function $R(\tau_1, \tau_2)$ is assumed separable in t_1 and t_2 directions, i.e.,

$$R(\tau_1, \tau_2) = R_1(\tau_1) R_2(\tau_2). \quad (3)$$

Variation of v_{12} : $V(v_{12})$

$$V(v_{12}) = E(v_{12}^2) - (E(v_{12}))^2. \quad (4)$$

For calculating $E(v_{12}^2)$, the 4-th moment of $x(t)$ should be known. In general this is desparately difficult. But if densities are distributed to normal, the 4-th moment can be decomposed to the product sum of 2-nd moments, that is, if a multi-variate $\boldsymbol{x}=(x_1,x_2,x_3,x_4)$ is distributed to normal with zero mean,

$$E(x_1x_2x_3x_4) = \mu_{12}\ \mu_{34} + \mu_{13}\ \mu_{24} + \mu_{14}\ \mu_{23}, \tag{5}$$

where $\mu_{ij}=E(x_i\ x_j)$.[4] Hence $E(v_{12}^2)$ becomes

$$E(v_{12}^2)=\frac{1}{(4L^2)^2}\int\int\int\int E(x(t_1+\tau_1,t_2+\tau_2)x(t_1,t_2)x(t_3+\tau_1,t_4+\tau_2)x(t_3,t_4)dt_1dt_2$$

$$dt_3dt_4=\frac{1}{(4L^2)^2}\int\int(R(\tau_1,\tau_2)R(\tau_1,\tau_2)+R(t_1-t_3,t_2-t_4)R(t_1-t_3-t_4)+R(t_1+\tau_1-t_3,$$

$$t_2+\tau_2-t_4)R(t_1-t_3-\tau_1,t_2-t_4-\tau_2))dt_1dt_2dt_3dt_4=R_1^2(\tau_1)R_2^2(\tau_2)+\frac{1}{(4L^2)^2}\int\int R_1^2$$

$$(t_1-t_3)dt_1dt_3\int\int R_2^2(t_2-t_4)dt_2dt_3+\int\int R_1(t_1-t_3+\tau_1)R_1(t_1-t_3+\tau_2)dt_1dt_3$$

$$\int\int R_2(t_2-t_4+\tau_2)R_2(t_2-t_4-\tau_2)dt_2dt_4. \tag{6}$$

$V(v_{12})$ in Eq.4 can be expressed as follows according to Eq.6 and Eq.2' after a simple variable transformation;

$$V(v_{12}) = I_{11}(0)\ I_{22}(0) + I_{11}(\tau_1)\ I_{22}(\tau_2) \tag{7}$$

subject to

$$\left.\begin{aligned} I_{11}(\tau) &= \int_{-1}^{+1}(1-|u|)R_1(2Lu+\tau)R_1(2Lu-\tau)du \\ I_{22}(\tau) &= \int_{-1}^{+1}(1-|u|)R_2(2Lu+\tau)R_2(2Lu-\tau)du \end{aligned}\right\} \tag{8}$$

Mean of m_1m_2 : $E(m_1m_2)$

$$E(m_1m_2) = \frac{1}{(4L^2)^2}\int\int\int\int E(x(t_1+\tau_1,t_2+\tau_2)x(t_3,t_4)dt_1dt_2dt_3dt_4$$

$$= \frac{1}{4L^2}\int\int R_1(t_1-t_3+\tau_1)dt_1dt_3\cdot\frac{1}{4L^2}\int\int R_2(t_2-t_4+\tau_2)dt_2dt_4$$

$$= I_1(\tau_1)I_2(\tau_2) \tag{9}$$

subject to

$$\left.\begin{aligned} I_1(\tau) &= \int_{-1}^{+1}(1-|u|)R_1(2Lu+\tau)du \\ I_2(\tau) &= \int_{-1}^{+1}(1-|u|)R_2(2Lu+\tau)du. \end{aligned}\right\} \quad (10)$$

<u>Variance of m_1m_2 : $V(m_1m_2)$</u>

We abbriviate the similar calculation process and show the results.

$$V(m_1m_2) = E(m_1^2m_2^2) - (E(m_1m_2))^2 = I_1^2(0)I_2^2(0) + I_1^2(\tau_1)I_2^2(\tau_2). \quad (11)$$

<u>Covariance of v_{12} and m_1m_2 : $Cov(v_{12},m_1m_2)$</u>

Like the above the calculation results are,

$$Cov(v_{12},m_1m_2)=E(v_{12}m_1m_2)-E(v_{12})E(m_1m_2)=J_1(\tau_1)J_2(\tau_2)+J_1(0)J_2(0) \quad (12)$$

subject to

$$J_i(\tau) = \frac{1}{(2L)^3}\iiint R_i(t_1-t_5+\tau)R_i(t_1-t_3-\tau)dt_1dt_3dt_5$$

$$= \int_{\text{I}}(1-|v|)R_i(2Lv+\tau)R_i(2Lw-\tau)dvdw+\int_{\text{II}}(1-|w|)R_i(2Lv+\tau)R_i(2Lw-\tau)$$

$$dvdw+\int_{\text{III}}(1-|v-w|)R_i(2Lv+\tau)R_i(2Lw-\tau)dvdw \qquad (i=1,2), \quad (13)$$

where integeation domains I, II, and III are illustrated in Fig 2.

Eventually the mean and variance of covariance c can be expressed respectively using the above equations as thus;

$$E(C) = E(v_{12})-E(m_1m_2)$$
$$= R_1(\tau_1)R_2(\tau_2)-I_1(\tau_1)I_2(\tau_2) \quad (14)$$

$$V(C) = V(v_{12})+V(m_1m_2)-2Cov(v_{12},m_1m_2)$$
$$=I_{11}(0)I_{22}(0)+I_{11}(\tau_1)I_{22}(\tau_2)$$
$$+I_1^2(0)I_2^2(0)+I_1^2(\tau_1)I_2^2(\tau_2)$$
$$-2J_1(\tau_1)J_2(\tau_2)-2J_1(0)J_2(0). \quad (15)$$

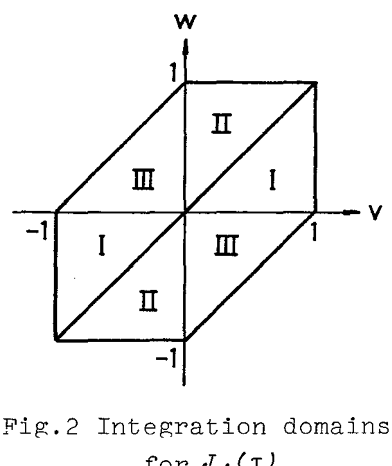


Fig.2 Integration domains for $J_i(\tau)$

3. MATCHING STABILITY

On stereo images after orientation, matching can be executed one-dimensionally in the x direction based on Epipoler geometry. Therefore we can let $\tau_2=0$ in Eq.14,15. Further if an image signal might be assumed Markovic, it is well-known the correlation function $R_1(\tau)$ becomes exponential type[5];

$$R_1(\tau) = e^{-\alpha|\tau|} \quad ; \quad \alpha>0. \tag{16}$$

Since matching stabilty might be discussed vividly together with a numerical example, auto-correlation functions were estimated for typical flat and hilly regions in a middle scale (1:25,000) aerial photograph. The photograph is shown in Fig.3, the regions used in the experiment being enclosed with squares. The image are digitized with 50μm pixel width which corresponds to 1.2m on the ground. If the ergodic assumption associated with 2-nd moment might be permitted, the correlation functions can be estimated from SCC's as thus,[6]

$$\tilde{R}_1(\tau) = \frac{1}{N_1 N_2} \sum_{J=0}^{N_2-1} \sum_{I=0}^{N_1-\tau-1} x(I+\tau,J)x(I,J)$$

$$R_1(\tau) = \frac{\tilde{R}_1(\tau)}{\tilde{R}_1(0)}. \tag{17}$$

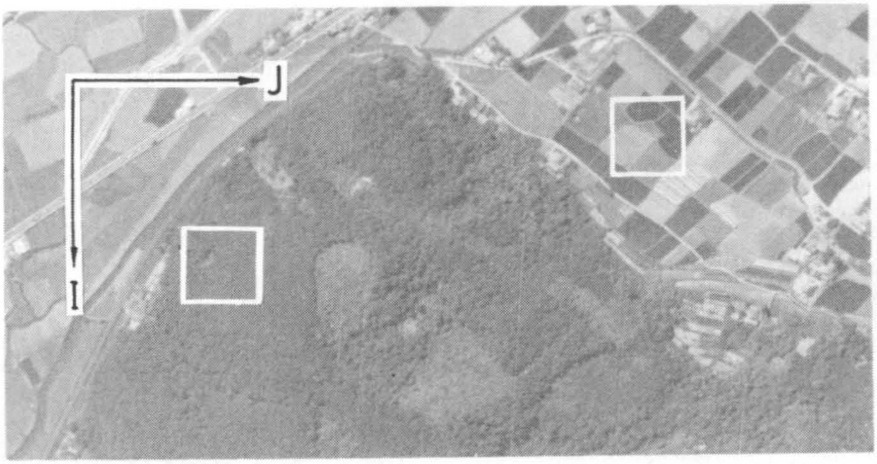


Fig.3 A photograph used for evaluation of correlation functions
The flat and hilly regions are enclosed with squares containing 100 x 100 pixels respectively.

The estimated correlation functions for two regions are shown in Fig.4 (a)(b) respectively. In the flat region, the neighbouring pixels are highly correlated, whereas the correlation function of the hilly region looks near a dalta function.

By use of least squares method, the values of the coefficient α were determind, as $\alpha=0.13$ and $\alpha=0.80$ for the flat and hilly regions respectively. Since the correlation function in the hilly region fit poorly to exponential curve, the first 5 lags were used for fitting.

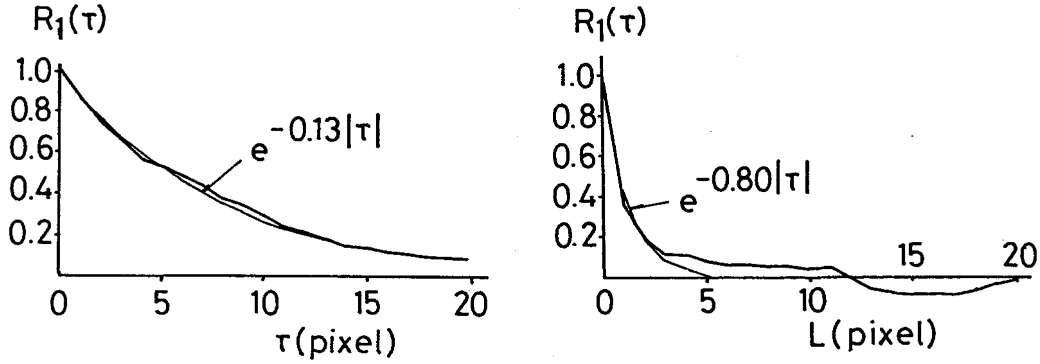
For these two cases, the mean $E(C)$ and standard deviation $\sqrt{V(C)}$ of auto-covariance were calculated for some values of parameters, window width L and positional lag τ, by mumerical method using a computer. The results are shown in Fig.5(a)(b) and Fig.6(a)(b), from which the noticeable points follow;

·Fig.5 shows that the means $E(C)$ converge to constant vallues as L increases, especially the curves in the hilly region, where the image signal is close to white noise, converge rapidly for $L \geqq 10$. On the other hand associated with τ, as the matter of course, $E(C)$ nears to 0 as τ increase.

·Fig.5 shows the standard deviations $\sqrt{V(C)}$ increase from 0 as L increases, reach the peak at $L=2$ and $L=8$ for two regions respectively, and decrease to 0 again at infinite L.

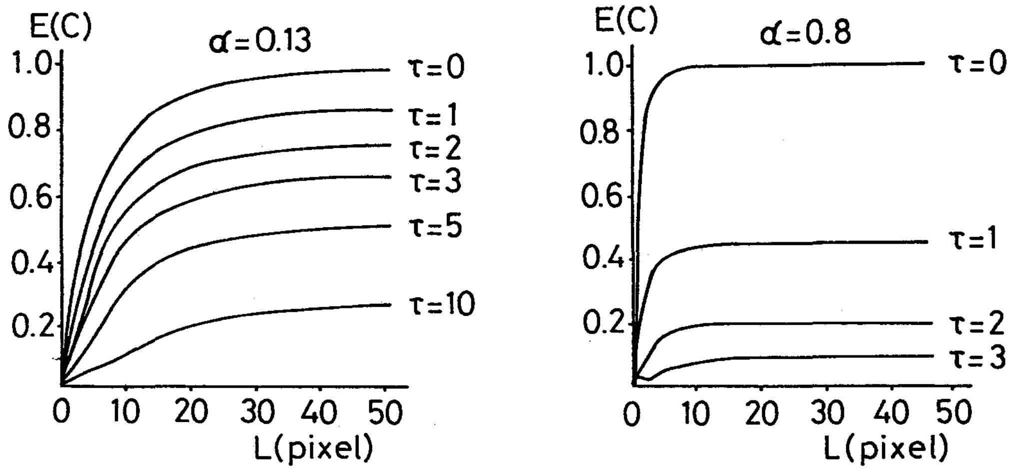
One criterion for stable matching is the values of the mean $E(C)$. in the vicinity of $\tau=0$, the larger $E(C)$ implies the more stable matching.

The other is the values of the standard deviation $\sqrt{V(C)}$. The larger $\sqrt{V(C)}$ implies the less stable matching. Therefore we can suggest to refer to a variation factor $\sqrt{V(C)}/E(C)$ as an unified criterion for stable matching. Fig.7(a)(b) shows the relation of variation factors for respective regions to window width L.
But attention should be paid the ordinate in Fig.7 denotes inverse of variation factor for illustrative convenience. The figure implies variation factors increase monotononsly as L increases, and decrease as τ increases. We can suggest that for sustaining variation factors under 1/3 for τ less than 3 pixel, for example, more than 20 pixel of L is needed.



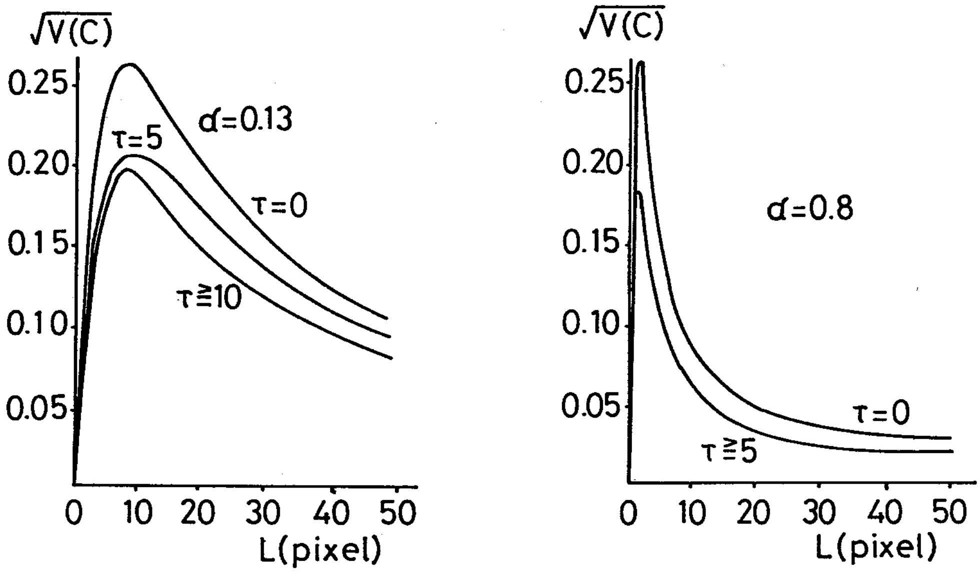
(a) flat region (b) hilly region

Fig.4 Estimated correlation functions $R_1(\tau)$



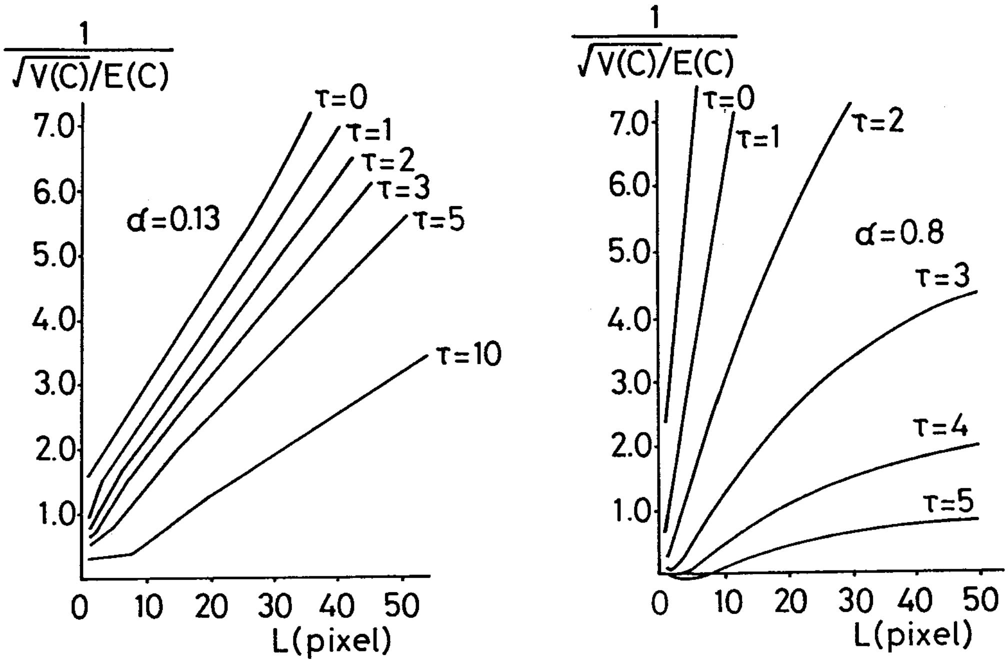
(a) flat region (b) hilly region

Fig.5 Means of covariances $E(C)$



(a) flat region (b) hilly region

Fig.6 Standard deviations of covariances $\sqrt{V(C)}$



(a) flat region (b) hilly region

Fig.7 Variation factors of covariances $\sqrt{V(C)}/E(C)$

4. CONCLUSION

This paper is intended to reveal a quantitative relation of appropriate window width to density distribution property of an image in area matching of stereo images. For this purpose statistical behavior of sample auto-covariances of an single image instead of stereo images was investigated. First the formulae to compute a mean and variance of sample cross-covariances using auto-correlation function were derived. Secondly the mean and variance were evaluated from an aerial photograph. It was shown both the mean and variance vary jointly depending on window width. As a result it was proved more usefull to use the variation factor combining these statistics as designator of appropriate width of windows.

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