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Calculation of Circuit Constants for Impulse Voltage Generator by Means of Computer

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In this paper, we describe the way to compute circuit constants of the impulse voltage generator by means of the digital computer, when an impulse voltage waveform is given. The definition of waveform is to be revised, and this definition is adopted to our computation. From the results, we can see the influence of revising definition upon circuit constants. We also devised graphs, from which we can easily determine the L-C-R circuit constants. (see Fig. 2 (a))

§ 1. Introduction

It is very complex to calculate circuit constants of an impulse voltage generator, when the voltage waveform is given. In this paper, we describe the method to calculate circuit constants for two different circuit connections by means of the digital computer.

The definition of an impulse voltage waveform is given in Japanese Electrotechnical Committee (JEC) 106. In this definition, duration of wave front (T_f) is defined as the value which the interval between 90 % and 10 % of crest value is divided by 0.8. In the near future, this definition will be revised. In the definition to be revised, duration of wave front is defined as the value which the interval between 90 % and 30 % of crest value is divided by 0.6, as shown in Fig. 10. Hereafter the former is named 10 %—90 % method and the latter is named 30%-90% method. Nowadays in order to obtain circut constants L, R_0, R_s for the given waveform, coefficients $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{k}$, etc. (see Eqs. ③, ④, ⑦, etc.) are used.

The calculated results, following to 30%—90% method, show that coefficients $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{k}$, etc, must be varied to some extent. Consequently circuit constants (L, R_0, R_s) vary in some degree. In order to examine the calculated results, we analyse some typical waveforms by the analogue computer.

§ 2. The computing method, using digital computer

(1) The procedure to compute

When α_1, α_2 are given, circuit constants are calculated with the next procedure using Eqs. ②~⑦ (c. f. appendix).

$$\alpha_1,\ \alpha_2 \rightarrow \boldsymbol{k} \rightarrow A \rightarrow t_1/t_m,\ t_2/t_m,\ t_3/t_m \rightarrow \boldsymbol{a},\ \boldsymbol{b} \rightarrow T_f,\ T_t$$

On the other hand, T_f, T_t are given, the circuit constants are calculated with the next procedure.

$$T_f,\ T_t \rightarrow T_t/T_f \rightarrow \boldsymbol{k} \rightarrow A \rightarrow t_1/t_m,\ t_2/t_m,\ t_3/t_m \rightarrow \boldsymbol{a},\ \boldsymbol{b} \rightarrow T_f,\ T_t$$

In the first case, starting from α_1, α_2, coefficient $\boldsymbol{k}$ is determined from Eq. ⑦. In the second case, starting from T_f, T_t however, it is difficult to obtain the value of $\boldsymbol{k}$, in spite of being the function of T_t/T_f. Because $\boldsymbol{k}$ is the solution of simultaneous Eqs. ②~⑦ with exponential function. Therefore, $\boldsymbol{k}$ is assumed as follow,

$$(1/\boldsymbol{k})\log \boldsymbol{k} = (T_f/T_t)\ \ln 2. \qquad (1)$$

Then T_f, T_t are calculated for this $\boldsymbol{k}$. If this calculated T_f, T_t do not correspond exactly to given T_f, T_t, we must calculate again by substituting more accurate $\boldsymbol{k}$ into Eq. ⑦. Therefore, the latter procedure requires many repetitions. In practice, it is often required to determine the circuit constants from T_f, T_t. So we describe about this case. The flow chart is shown in Fig. 1.

t_1/t_m, t_2/t_m and t_3/t_m are given by Eq. ②. But they are exponential functions, so we begin next approximate equations, and reach final t_1/t_m, t_2/t_m, t_3/t_m by repeating the same procedure.

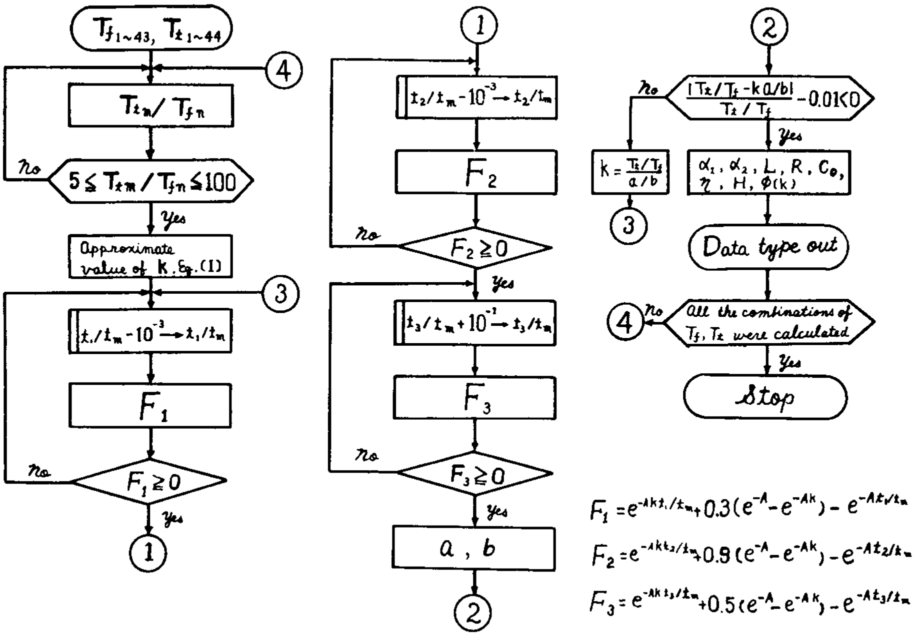

Fig. 1. Flow chart

$$\left.\begin{aligned} t_1/t_m &= 0.04 \log T_t/T_f + 0.14 \\ t_2/t_m &= 0.15 \log T_t/T_f + 0.7 \\ t_3/t_m &= 0.5\ T_t/T_f \end{aligned}\right\} \quad (2)$$

Where

$$0 < t_1/t_m < t_2/t_m < 1 < t_3/t_m$$

Hereafter, t_1/t_m, t_2/t_m, t_3/t_m are replaced briefly with $\mathbf{t}_1$, $\mathbf{t}_2$, $\mathbf{t}_3$ respectively.

In above equations, $\mathbf{t}_1$, $\mathbf{t}_2$ are larger than the expected values and $\mathbf{t}_3$ is smaller than that. We desire $\mathbf{t}_n$ (n=1, 2, 3), for which F_1, F_2, F_3 (see Fig. 1) converge to zero. If we decrease $\mathbf{t}_1$ and $\mathbf{t}_2$ by 10^{-3} and increase $\mathbf{t}_3$ by 10^{-1}, F_1, F_2, F_3 vary gradually from negative to positive. So we can obtain most suitable $\mathbf{t}_n$ (n=1, 2, 3), when F_1, F_2, F_3 change their signs respectively. When $\boldsymbol{k}$ satisfies,

$$\frac{|\,T_t/T_f - \boldsymbol{k}\boldsymbol{a}/\boldsymbol{b}\,|}{T_t/T_f} < 10^{-2}$$

and $\mathbf{t}_1$, $\mathbf{t}_2$, $\mathbf{t}_3$ are accurate, the error of $\boldsymbol{k}$ is within about 1%. But these $\mathbf{t}_1$, $\mathbf{t}_2$, $\mathbf{t}_3$ contain errors themselves, therefore, the error of $\boldsymbol{k}$ becomes within 1.5%.

(2) The ranges of computation

T_f: 0.5~2,000 μs (43 values which are actually used)

T_t; 3.0~10,000 μs (44 values which are actually used)

$5 \leq T_t/T_f \leq 100$

Fig. 2 (a), 2 (b) show the circuits for the impulse voltage generator under cosideration.

(a) circuit: $\boldsymbol{k}$, $\mathbf{t}_1$, $\mathbf{t}_2$, $\mathbf{t}_3$, $\boldsymbol{a}$, $\boldsymbol{b}$, α_1, α_2, L, R_0+R_s, $\phi(\boldsymbol{k})$, H and η are computed when T_f, T_t and C are given.

(b) circuit: $\boldsymbol{k}$, $\mathbf{t}_1$, $\mathbf{t}_2$, $\mathbf{t}_3$, $\boldsymbol{a}$, $\boldsymbol{b}$, α_1, α, R_0, R_s, $\phi(\boldsymbol{k})$, H and η are computed when T_f, T_t, C and C_0 are given.

§ 3. The results of computation

(1) The variations of circuit constants due to revising the definition of the waveform

Obtained coefficients $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{k}$ and $\phi(\boldsymbol{k})$ are shown in Figs. 3~5. Over the region where the value T_t/T_f is larger than 100, computed points are dotted. Using these graphs, it is easy to determine the circuit constants of an impulse generator when the waveform is given under 30%—90% method.

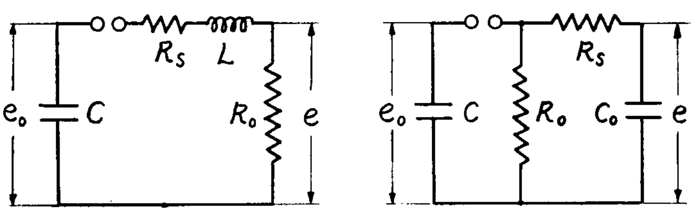

Fig. 2. Circuits of impulse voltage generator
(a) L-C-R circuit (b) C-R-C circuit

Table 1 shows a comparison

Table 1 The results of computation for typical waveform

Definition	T_t/T_f	k	a	b	$\phi(k)$	t_1	t_2	t_3	$T_f(\mu s)$	$T_t(\mu s)$	$\alpha_1(s^{-1})$	$\alpha_2(s^{-1})$	* $L(mH)$	* $R_0+R_s=R\ (k\Omega)$	** $R_0(k\Omega)$	** $R_s(k\Omega)$
1	5	5.6355	1.27123	1.43271	0.81085	0.03418	0.57943	3.374	50	250	5,084.95	28,654.2	109.81	3.7048	2.9709	12.320
2		7.6019	1.17813	1.79104	0.83221	0.10887	0.56897	3.713			4,712.52	35,820.9	94.78	3.8418	3.2159	9.824
1	6	8.9587	1.09624	1.63667	0.84393	0.03214	0.56264	3.945	500	3,000	365.42	3,273.3	13,376	48.673	41.526	107.37
2		11.5046	1.04489	2.00339	0.86140	0.10295	0.55226	4.372			348.30	4,006.8	11,464	49.930	43.632	87.59
1	10	22.6513	0.88728	2.00972	0.90401	0.02781	0.52034	6.123	350	3,500	253.51	5,742.1	10,991	65.900	60.089	60.972
2		27.7830	0.86884	2.41375	0.91506	0.08989	0.50985	6.880			248.24	6,896.4	9,346	66.774	61.391	50.745
1	26.67	84.0193	0.76016	2.39501	0.95930	0.02223	0.44949	14.211	1.5	40	19,003.9	1,596,670	0.52730	0.85195	0.80290	0.21891
2		100.667	0.75453	2.84819	0.96426	0.07261	0 43948	16.195			18,863.2	1,898,790	0.44670	0.85663	0.80896	0.18406
1	40	134.813	0.73833	2.48836	0.97116	0.02053	0.42346	20.117	1	40	18,458.3	2,488,360	0.34834	0.87324	0.82681	0.14043
2		160.859	0.73454	2.95363	0.97474	0.06726	0.41391	23.006			18,363.3	2,953,630	0.29499	0.87671	0.83114	0.11830
1	50	173.294	0.72972	2.52913	0.97612	0.01970	0.40993	24.360	1	50	14,594.4	2,529,130	0.43347	1.1026	1.0458	0.13816
2		206.422	0.72663	2.99964	0.97910	0.06463	0.40067	27.901			14,532.6	2,999,640	0.36703	1.1063	1.0520	0.11648
1	100	367.801	0.71241	2.62025	0.98669	0.01749	0.37136	44.206	1	100	7,124.06	2,620,250	0.85713	2.2520	2.1427	0.13333
2		436.517	0.71073	3.10214	0.98839	0.05760	0.36309	50.824			7,107.26	3,102,140	0.72569	2.2563	2.1478	0.11262
1	1,000	3,921.14	0.69556	2.72754	0.99814	0.01271	0.27636	329.52	2	2,000	347.78	1,363,770	33.734	46.018	43.898	0.25615
2		4,634.10	0.69532	3.22213	0.99839	0.04216	0.27114	381.56			347.66	1,611,060	28.566	46.032	43.913	0.21683
1	10,000	39,567.2	0.69344	2.74376	0.99975	0.00996	0.21731	2,591.8	1	10,000	69.344	2,743,760	84.093	230.73	220.16	0.12731
2		46,728.5	0.69341	3.24021	0.99979	0.03317	0.21398	3,013.4			69.341	3,240,210	71.212	230.74	220.17	0.10781

Notes: Definition 1 — Waveform is defined by 10%—90% method.
Definition 2 — Waveform is defined by 30%—90% method.
* Constants for L-C-R circuit ($C=0.0625\,\mu F$)
** Constants for C-R-C circuit ($C=0.0625\,\mu F$, $C_0=0.003\mu F$)

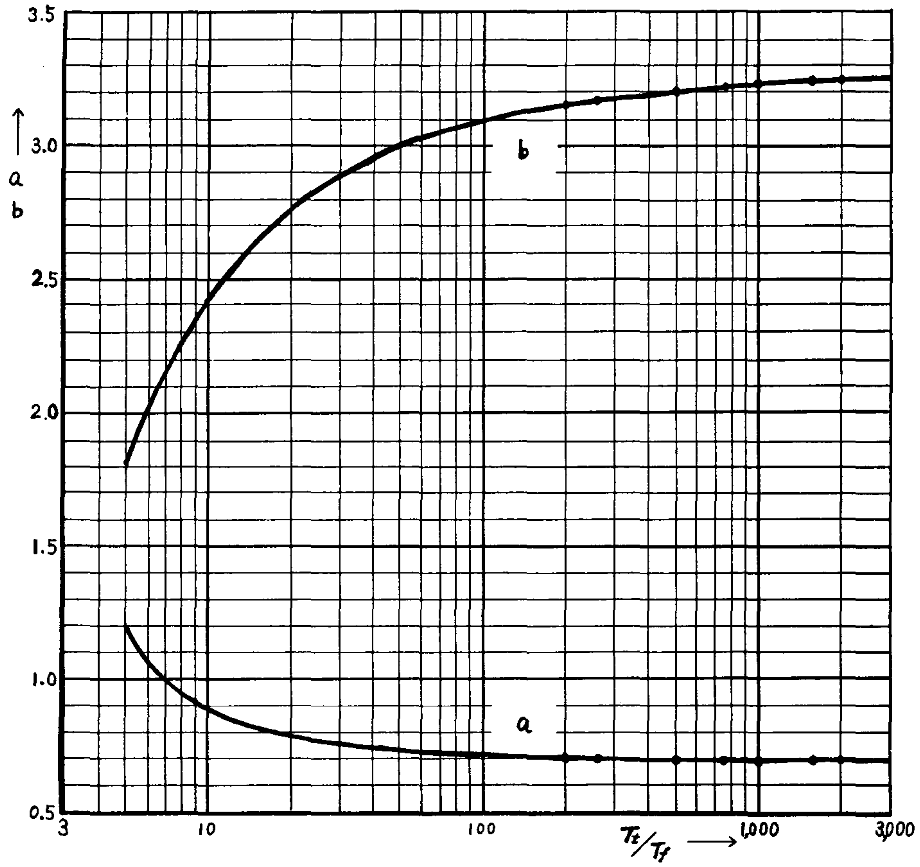

Fig. 3

between circuit constants for the typical waveforms by the 10%—90% method and those by the 30%—90% method.

Particularly in the Fig. 2 (a) circuit, if the waveform is given, the circuit constants can be obtained from Fig. 6 and Fig. 7 immediately.

(2) About computed waveform

The waveform described in general literatures, has the nominal zero point O_1 on the right side of the real zero point O, however in our computation, the waveform always has it on the left side of O, as shown in Fig. 10. Compared with the duration between 0% and 90% of crest value (t_2), the duration between 90% and 100% of crest value (t_m-t_2) is considerably large, for example, with the standard wave, $(t_m-t_2)/t_2$ comes up to about 1.5.

§ 4. Analysis using an analogue computer

To verify the circuit constants computed by the digital computer, an analogue computer was applied. Fig. 8 and Fig. 9 show the block-diagrams of L-C-R and C-R-C circuits respectively. The results for typical waveforms are shown in Table 2, where "Given" shows given waveforms, and "Computed" shows computed waveforms by means of analogue computer, using circuit constants, which are obtained from digital computer. It may be considered that the difference between them is caused by the errors of the analogue computer and of the X-Y recorder.

Table 2

T_f [μsec]		T_t [μsec]	
"Given"	"Computed"	"Given"	"Computed"
1	1.1	40	40
0.5	0.55	50	50
350	355	3,500	3,400
500	490	3,000	3,000

Illustrating the differences between two definitions, we used the analogue computer. Fig. 11 (a curve) shows 1×40 μs waveform, defined by 10%—90% method, and Fig. 11 (b-curve)

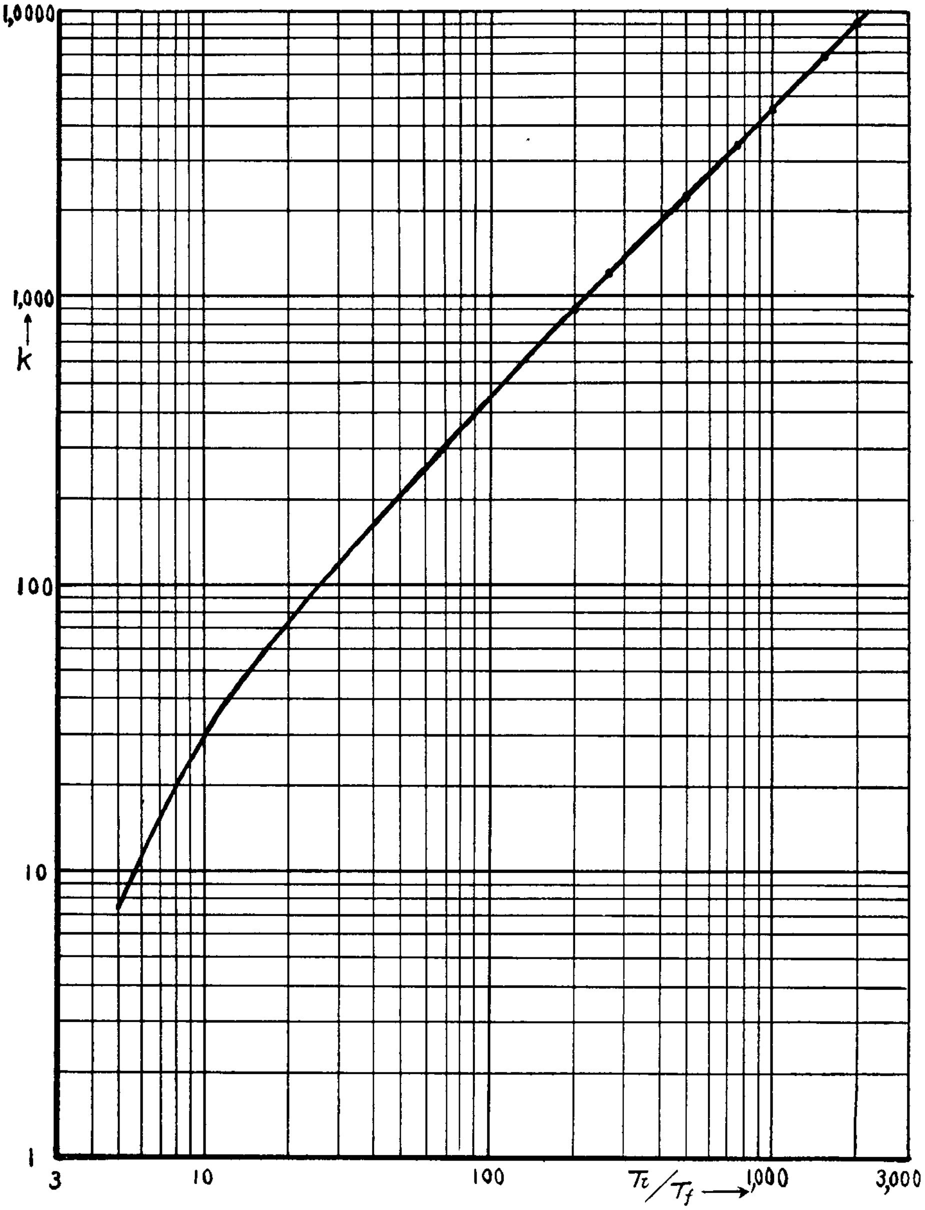

Fig. 4.

shows the same one, defined by 30%—90% method. From those curves, we can understand the difference between two definitions, namely $1\times40\ \mu s$ waveform, defined by 10%—90% method correspond to $1.2\times40.2\ \mu s$ waveform, defined by 30%—90% method.

§ 5. Conclusions

(1) ***a***, ***b***, ***k***, etc. and circuit constants of the impulse voltage generator following to 30%—90% method were calculated with 1.5% errors.

(2) According to these results, coefficients ***b*** and ***k*** are considerably different from values calculated by the traditional 10%—90% method.

(3) We obtained the graphs to determine the constants of Fig. 2 (a) circuit for given T_f and T_t.

(4) The computed results were examined by the analogue computer.

(5) By computing $\mathbf{t}_1$, $\mathbf{t}_2$ and $\mathbf{t}_3$, we could know more accurate waveform for the impulse waveform.

(6) From this, the nominal zero point O_1 is located in the left side of the real zero point O,

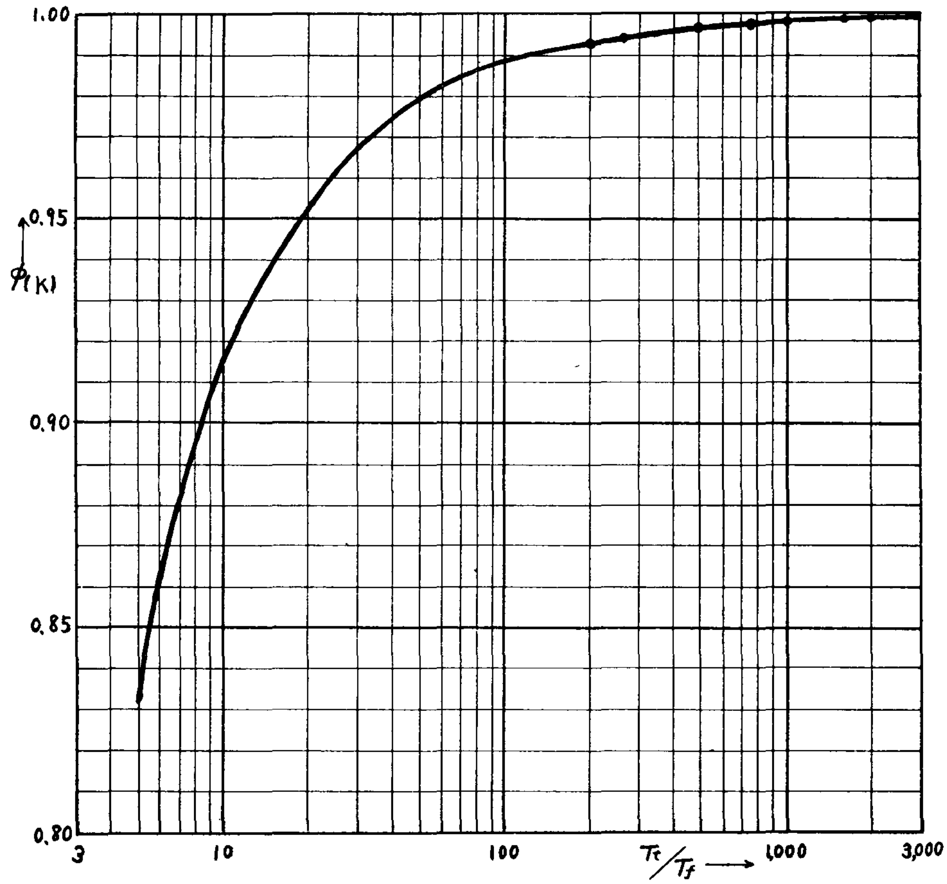

Fig. 5.

and the duration between 90% and 100% crest value is unexpectedly long.

In this paper, we have computed, assuming stray capacitance and inductance are already known. From this time, we are going to investigate them.

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References

1) Rivised JEC 106
2) S. Hōki: Journal of the Institute of Electrical Engineers of Japan, Aug. 1941.
3) H. Yoshida: Tokyo branch of the Institute of Electrical Engineers, 1962.

Appendix

Notations; see Fig. 2 and Fig. 10

Formulation of equations used in writing; When e is out put voltage

$$e=E\{\exp(-\alpha_1 t)-\exp(-\alpha_2 t)\} \quad ①$$

$$\left.\begin{aligned}&\exp(-At_1/t_m)-\exp(-Akt_1/t_m)\\&\quad=0.3\{\exp(-A)-\exp(-Ak)\}\\&\exp(-At_2/t_m)-\exp(-Akt_2/t_m)\\&\quad=0.9\{\exp(-A)-\exp(-Ak)\}\\&\exp(-At_3/t_m)-\exp(-Akt_3/t_m)\\&\quad=0.5\{\exp(-A)-\exp(-Ak)\}\end{aligned}\right\} \quad ②$$

$$a=\alpha_1 T_t=A(t_3/t_m-3t_1/2t_m+t_2/2t_m) \quad ③$$

$$b=\alpha_2 T_f=\frac{10}{6}Ak\,(t_2/t_m-t_1/t_m) \quad ④$$

$$\phi(k)=(k-1)k^{k/(1-k)} \quad ⑤$$

where

$$A=\alpha_1 t_m=(\ln k)/(k-1) \quad ⑥$$

$$k=\alpha_2/\alpha_1=\frac{T_t/T_f}{a/b} \quad ⑦$$

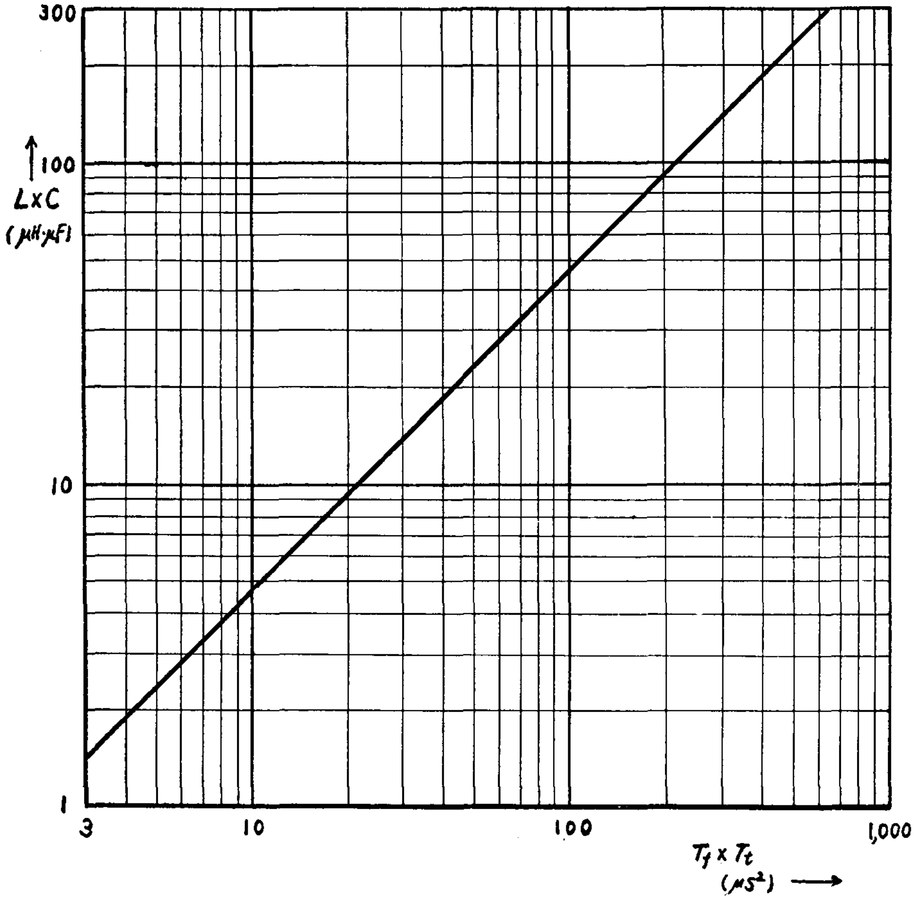


Fig. 6

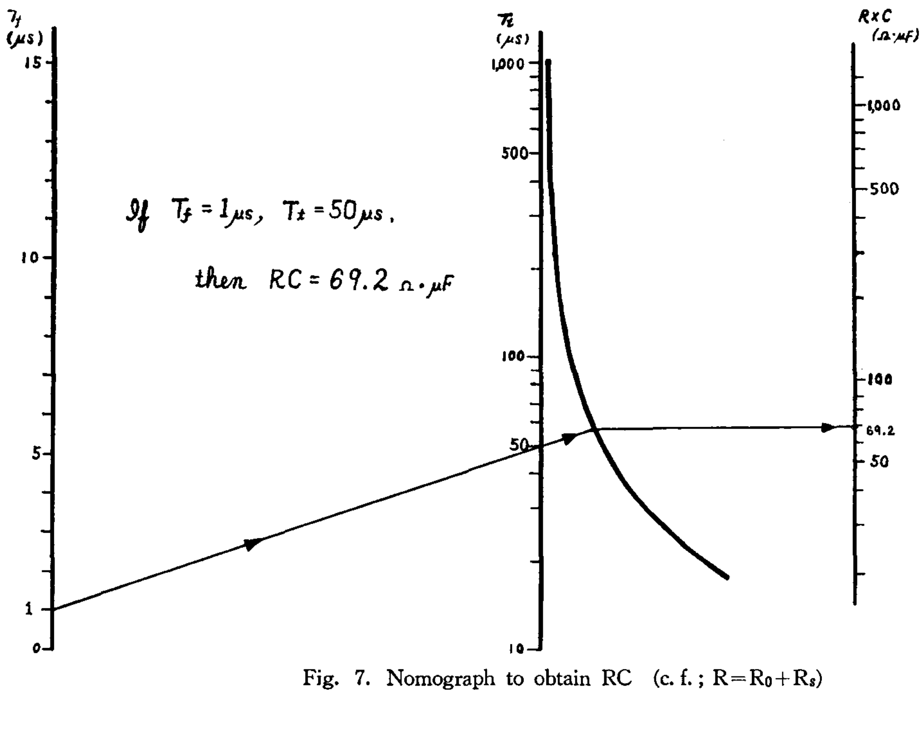
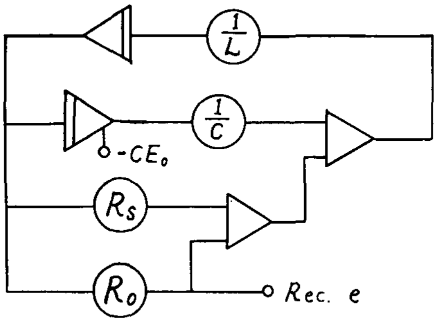
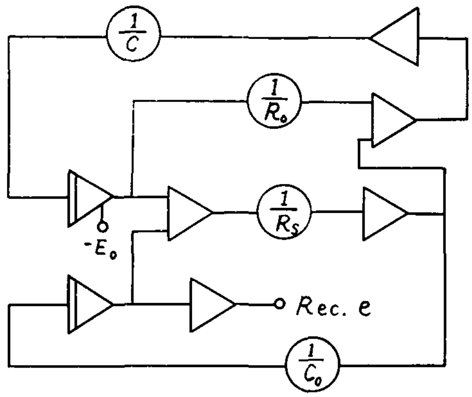


Fig. 7. Nomograph to obtain RC (c. f.; $R=R_0+R_s$)



$$\begin{cases} LC\dfrac{d^2e_0}{dt^2}=(R_0+R_s)\left(-C\dfrac{de_0}{dt}\right)-e_0 \\ e=R_0\left(-C\dfrac{de_0}{dt}\right) \end{cases}$$

Fig. 8. Block-diagram of L-C-R circuit



$$\begin{cases} -C\dfrac{de_0}{dt}=\dfrac{e_0}{R_0}+C_0\dfrac{de}{dt} \\ \dfrac{de}{dt}=\dfrac{1}{C_0R_s}(e_0-e) \end{cases}$$

Fig. 9. Block-diagram of C-R-C circuit

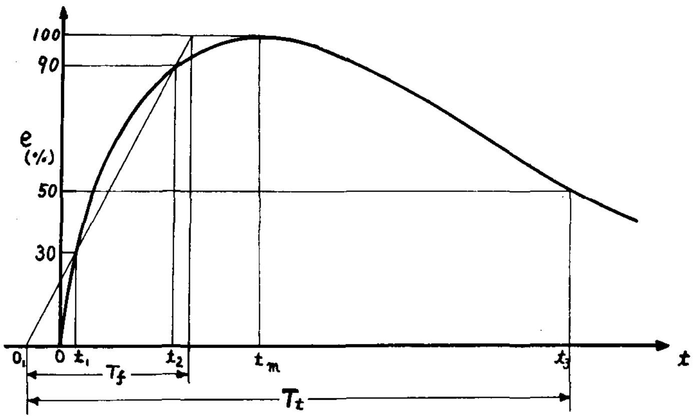


Fig. 10. Definition of waveform

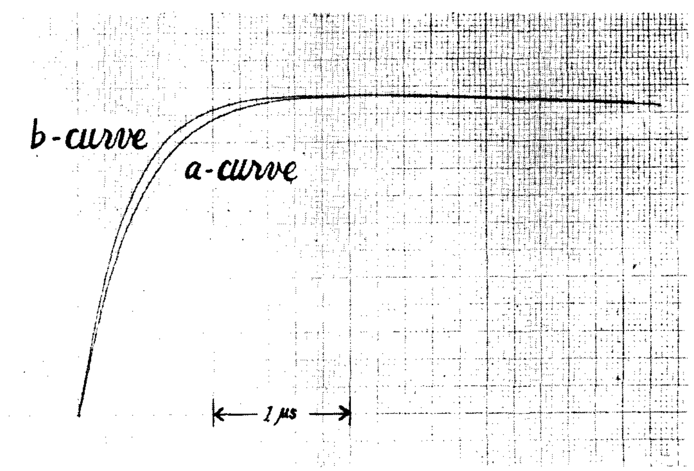


Fig. 11. Waveform differences between 10%−90% method and 30%—90% method obtained by analogue computer

(1) L-C-R circuit (see Fig. 2(a))

$$L=T_fT_t/(abC) \quad ⑧$$

$$R_0+R_s=T_t(1+1/k)/(aC) \quad ⑨$$

$$\gamma=\phi(k)H \quad ⑩$$

where

$$H=R_0/(R_0+R_s) \quad ⑪$$

(2) C-R-C circuit (see Fig. 2(b))

$$R_0=qHT_t/C \quad ⑫$$

$$R_s=\gamma T_f/(C_0H) \quad ⑬$$

$$\gamma=\psi(k)H \quad ⑭$$

where

$$H=(1+K)/2(1+1/\delta) \quad ⑮$$

$$K=\sqrt{1-(1-1/\delta)m} \quad ⑯$$

$$m=4k/(1+k)^2 \quad ⑰$$

$$\delta=C/C_0 \quad ⑱$$

$$q=(1+k)/(ab) \quad ⑲$$

$$\gamma=k/\{b(1+k)\} \quad ⑳$$