ON THE \((1 - C_2)\) CONDITION

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Abstract. In this paper, we give some results on \((1 - C_2)\)-modules and \(1\)-continuous modules.

1. Introduction

All rings are associated with identity, and all modules are unital right modules. By \(M_R, (\_R M)\) we indicate that \(M\) is a right (left) module over a ring \(R\). The Jacobson radical, the uniform dimension and the endomorphism ring of \(M\) are denoted by \(J(M), u - \text{dim}(M)\) and \(\text{End}(M)\), respectively. For a module \(M\) (over a ring \(R\)), we consider the following conditions:

\(1\) Every uniform submodule of \(M\) is essential in a direct summand of \(M\).
\(2\) Every submodule of \(M\) is essential in a direct summand of \(M\).
\(3\) Every submodule isomorphic to a direct summand of \(M\) is itself a direct summand of \(M\).
\(4\) For any direct summands \(A, B\) of \(M\) with \(A \cap B = 0, A \oplus B\) is also a direct summand of \(M\).

A module \(M\) is defined to be a \((1 - C_1)\)-module if it satisfies the condition \(1 - C_1\). If \(M\) satisfies \(C_1\), then \(M\) is said to be a \(CS\)-module (or an extending module). \(M\) is defined to be a continuous module if it satisfies the conditions \((C_1)\) and \((C_2)\). If \(M\) satisfies \((C_1)\) and \((C_3)\), then \(M\) is said to be a quasi-continuous module. We call a module \(M\) a \((C_2)\)-module if it satisfies the condition \((C_2)\). We have the following implications:

Injective \(\Rightarrow\) quasi-injective \(\Rightarrow\) continuous \(\Rightarrow\) quasi-continuous \(\Rightarrow\) CS \(\Rightarrow\) \((1 - C_1)\),

and \((C_2)\) \(\Rightarrow\) \((C_3)\).

For a set \(A\) and a module \(M\), \(M^{(A)}\) denotes the direct sum of \(|A|\) copies of \(M\). A module \(M\) is called a \((\text{countably})\) \(-\text{quasi-injective}\) if \(M^{(A)}\) (resp. \(M^{(\mathbb{N})}\) is a quasi-injective -module for every set \(A\) (note that \(\mathbb{N}\) denotes the set of all natural numbers). Similarly, a module \(M\) is called a \((\text{countably})\) \(-\text{(1-C1)}\) if \(M^{(A)}\) (resp. \(M^{(\mathbb{N})}\) is a \((1 - C_1)\)-module for every set \(A\).

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In Section 2, we give several properties on the \((1-C_2)\)-modules, (strongly) 1-continuous modules, and discuss the question of when a 1-continuous module is continuous \((1-C_2)\)-module is \((C_2)\)-module)?

2. \((1-C_2)\) CONDITION

In this section, we consider the following condition for a module \(M\).

\((1-C_2)\) Every uniform submodule isomorphic to a direct summand of \(M\) is itself a direct summand of \(M\).

A module \(M\) is defined to be a \((1-C_2)\)-module if it satisfies the condition \((1-C_2)\). If \(M\) satisfies \((1-C_1)\) and \((1-C_2)\) conditions, then \(M\) is said to be a \((1-C_2)\)-continuous module. \(M\) is defined to be a strongly \((1-C_2)\)-continuous module if it satisfies the conditions \((C_1)\) and \((1-C_2)\). A ring \(R\) is called a right (left) \((1-C_2)\)-continuous ring if \(RR\) (resp. \(RR\)) is a \((1-C_2)\)-continuous module.

We have the following implications:

Continuous \(\Rightarrow\) strongly \(1\)-continuous \(\Rightarrow\) \(1\)-continuous, and \((C_2)\) \(\Rightarrow\) \((1-C_2)\).

Remark 2.1. By [4, Corollary 7.8], let \(M\) be a right \(R\)-module with finite uniform dimension, \(M\) is a \((1-C_1)\)-module if and only if \(M\) is CS. Therefore, \(M\) has finite uniform dimension then \(M\) is a \((1-C_2)\)-continuous module if and only if \(M\) is strongly \(1\)-continuous. In general, if \(M\) satisfies the condition \((1-C_2)\), \(M\) may not satisfy the condition \((C_2)\). By the definitions \((1-C_2)\)-module, \(1\)-continuous module and strongly \(1\)-continuous module, we have:

Lemma 2.2. Let \(M\) be a right \(R\)-module and \(N\) is a direct summand of \(M\). If \(M\) is a \((1-C_2)\)-module \((1\)-continuous, strongly \(1\)-continuous\) then \(N\) is also \((1-C_2)\)-module \((1\)-continuous, strongly \(1\)-continuous\).

Theorem 2.3. Let \(U = \bigoplus_{i=1}^{n} U_i\) where each \(U_i\) is a uniform module, then the following conditions are equivalent:

(i) \(U\) is a \((C_2)\)-module;
(ii) \(U\) is a \((1-C_2)\)-module and \(U\) satisfies the condition \((C_3)\).

Proof. (i) \(\implies\) (ii). It is obvious.

(ii) \(\implies\) (i). We show that \(U\) is a \((C_2)\)-module, i.e., for two submodules \(X, Y\) of \(U\), with \(X \cong Y\) and \(Y\) is a direct summand of \(U\), \(X\) is also a direct summand of \(U\). Note that \(Y\) is a closed submodule of \(M\), there is a subset \(F\) of \(\{1, ..., n\}\) such that \(Y \oplus (\bigoplus_{i \in F} U_i)\) is an essential submodule of \(U\). But \(Y, \bigoplus_{i \in F} U_i\) are direct summands of \(U\) and \(U\) satisfies the condition \((C_3)\), we imply \(Y \oplus (\bigoplus_{i \in F} U_i) = U\). If \(F = \{1, ..., n\}\) then \(X = Y = 0\), as desired.

If \(F \neq \{1, ..., n\}\) and set \(J = \{1, ..., n\} \setminus F\), then \(U = Y \oplus (\bigoplus_{i \in F} U_i) = \bigoplus_{i \in J} U_i \oplus (\bigoplus_{i \in F} U_i)\). Hence, \(X \cong Y \cong U/ \bigoplus_{i \in F} U_i \cong \bigoplus_{i \in F} U_i = Z\). Suppose that \(J = \{1, ..., k\}\) with \(1 \leq k \leq n\), i.e., \(Z = U_1 \oplus \cdots \oplus U_k\). Let \(\varphi : Z \longrightarrow X\), and set \(X_i = \varphi(U_i)\) then \(X_i \cong U_i\) for any \(i = 1, ..., k\). We imply \(X = \varphi(Z) =\)
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\[ \varphi(U_1 \oplus \ldots \oplus U_k) = \varphi(U_1) \oplus \ldots \oplus \varphi(U_k) = X_1 \oplus \ldots \oplus X_k. \]

By \( X_i \) is a uniform submodule of \( U \), \( X_i \cong U_i \). Let \( L' = L \cap U_i \) and \( L'' \neq 0 \), hence \( U_j \) is not uniform module, a contradiction. Therefore \( U_i \) does not embed in a proper submodule of \( U_j \), proving (i). \( \square \)

**Theorem 2.4.** Let \( U = \bigoplus_{i=1}^n U_i \) where each \( U_i \) is a uniform module, then the following conditions are equivalent:

(i) \( U \) is a continuous module;

(ii) \( U \) is a \( 1 \)-continuous module.

Proof. (i) \( \implies \) (ii). It is obvious.

(ii) \( \implies \) (i). We show that \( S = \text{End}(U_i) \) is a local ring for any \( i = 1, \ldots, n \).

We first prove a claim that \( U_i \) does not embed in a proper submodule of \( U_i \).

Let \( f : U_i \longrightarrow U_i \) be a monomorphism with \( f(U_i) \) is a proper submodule of \( U_i \). Set \( f(U_i) = V \), then \( V \neq 0 \), proper submodule of \( U_i \) and \( V \cong U_i \). By hypothesis, \( U_i \) is a \((1−C_2)\)-module, and hence \( V \) is a direct summand of \( U_i \), i.e., \( U_i \) is not uniform module, a contradiction. Therefore, \( U_i \) does not embed in a proper submodule of \( U_i \).

Let \( g \in S \) and suppose that \( g \) is not an isomorphism. It suffices to show that \( 1−g \) is an isomorphism. Note that, \( g \) is not a monomorphism. Then, since \( Keg(g) \) is a nonzero submodule, it is essential in the uniform module \( U_i \). We always have \( Keg(g) \cap Keg(1−g) = 0 \), it follows that \( Ker(1−g) = 0 \), i.e. \( 1−g \) is a monomorphism. But \( U_i \) does not embed in a proper submodule of \( U_i \), \( 1−g \) must be onto, and so \( 1−g \) is an isomorphism, as required.

Let \( U_{ij} = U_i \oplus U_j \) with \( i, j \in \{1, \ldots, n\} \) and \( i \neq j \). We show that \( U_{ij} \) satisfies the condition \((C_3)\), i.e., for two direct summands \( S_1, S_2 \) of \( U_{ij} \) with \( S_1 \cap S_2 = 0 \), \( S_1 \oplus S_2 \) is also a direct summand of \( U_{ij} \). Note that, since \( u−\dim(U_{ij}) = 2 \), the following cases are trivial:

1) Either one of the \( S'_i \) has uniform dimension 2, consequently the other \( S_i \) is zero, or

2) One of the \( S'_i \) is zero.

Hence we consider the case that both \( S_1, S_2 \) are uniform. We prove that \( U_i \) does not embed in a proper submodule of \( U_j \). Let \( h : U_i \longrightarrow U_j \) be a monomorphism with \( h(U_i) \) is a proper submodule of \( U_j \). Set \( h(U_i) = L \), then \( L \neq 0 \), proper submodule of \( U_j \) and \( L \cong U_i \). By hypothesis, \( U \) is a \((1−C_2)\)-module and \( U_{ij} \) is a direct summand of \( U \), \( U_{ij} \) is also \((1−C_2)\)-module. Note that \( L \) is a uniform submodule of \( U_{ij} \) and \( L \cong U_i \) with \( U_i \) is a direct summand of \( U_{ij} \), \( L \) is also direct summand of \( U_{ij} \). Set \( U_{ij} = L \oplus L' \), then by modularity we get \( U_j = L \oplus L'' \) with \( L'' = U_j \cap L' \). Note that \( L'' \) is also proper submodule of \( U_j \) and \( L'' \neq 0 \), hence \( U_j \) is not uniform module, a contradiction. Therefore \( U_i \) does not embed in a proper submodule of \( U_j \).
Similarly, $U_j$ does not embed in a proper submodule of $U_i$. Note that, $U_i$ (and $U_j$) does not embed in a proper submodule of $U_i$ (resp. $U_j$).

Note that, $End(U_i)$ and $End(U_j)$ are local rings, by Azumaya’s Lemma ([1, 12.6, 12.7]), we have $U_{ij} = S_2 \oplus K = S_2 \oplus U_i$ or $S_2 \oplus K = S_2 \oplus U_j$. Since $i$ and $j$ can interchange with each other, we need only consider one of the two possibilities. Let us consider the case $U_{ij} = S_2 \oplus K = S_2 \oplus U_i = U_i \oplus U_j$. Then it follows $S_2 \cong U_j$. Write $U_{ij} = S_1 \oplus H = S_1 \oplus U_i$ or $S_1 \oplus H = S_1 \oplus U_j$.

If $U_{ij} = S_1 \oplus H = S_1 \oplus U_i$, then by modularity we get $S_1 \oplus S_2 = S_1 \oplus W$ where $W = (S_1 \oplus S_2) \cap U_i$. From here we get $W \cong S_2$, this means $U_i$ contains a copy of $S_2 \cong U_j$. By $U_j$ does not embed in a proper submodule of $U_i$, we must have $W = U_i$, and hence $S_1 \oplus S_2 = U_i \oplus U_j = U_{ij}$.

If $U_{ij} = S_1 \oplus H = S_1 \oplus U_j$, then by modularity we get $S_1 \oplus S_2 = S_1 \oplus W'$ where $W' = (S_1 \oplus S_2) \cap U_j$. From here we get $W' \cong S_2$, this means $U_j$ contains a copy of $S_2 \cong U_j$. By $U_j$ does not embed in a proper submodule of $U_j$, we must have $W' = U_j$, and hence $S_1 \oplus S_2 = U_{ij}$.

Thus $U_{ij}$ satisfies (C3). Note that, $U_{ij}$ is a direct summand of $U$ and $U$ is a CS-module (by $U$ has finite dimension and $U$ is a $1 - C_1$-module, thus $U$ is CS-module), $U_{ij}$ is also CS-module, and hence $U_{ij}$ is a quasi-continuous module for any $i, j \in \{1, \ldots, n\}$ and $i \neq j$.

Now, by [6, Corollary 11], thus $U$ is a quasi-continuous module. By Theorem 2.3, $U$ is a continuous module, proving (i). \qed

**Corollary 2.5.** Let $U = \oplus_{i=1}^{n} U_i$ where each $U_i$ is a uniform module, then the following conditions are equivalent:

(i) $U$ is a $\sum$-quasi-injective module;

(ii) $U$ is a $1$-continuous module, countably $\sum -(1 - C_1)$-module.

Proof. (i) $\implies$ (ii). It is obvious.

(ii) $\implies$ (i). By Theorem 2.4, $U$ is a continuous module. By [7, Proposition 2.5], $U$ is a $\sum$-quasi-injective module, proving (i). \qed

A right $R$-module $M$ is called *distributive* if for any submodule $A, B, C$ of $M$ then $A \cap (B + C) = A \cap B + A \cap C$. We say that, $M$ is a UC-module if each of its submodules has a unique closure in $M$.

**Theorem 2.6.** Let $U = \oplus_{i=1}^{n} U_i$ where each $U_i$ is a uniform module. Assume that $U$ is a distributive module, then the following conditions are equivalent:

(i) $U$ is a $(C_2)$-module;

(ii) $U$ is a $(1 - C_2)$-module.

Proof. (i) $\implies$ (ii). It is obvious.

(ii) $\implies$ (i). Similar proof of Theorem 2.4, $U_i$ does not embed in a proper submodule of $U_j$ for any $i, j \in \{1, \ldots, n\}$ and $S = End(U_i)$ is a uniform module for any $i \in \{1, \ldots, n\}$. We first prove a claim that, if $S_1$ and $S_2$ are direct summands of $U$ with $u - dim(S_1) = 1$, $u - dim(S_2) = n - 1$...
and \( S_1 \cap S_2 = 0 \), then \( S_1 \oplus S_2 = U \). By Azumaya’s Lemma, we have \( U = S_2 \oplus K = S_2 \oplus U_i \). Suppose that \( i = 1 \), i.e., \( U = S_2 \oplus U_1 = (\oplus_{i=2}^n U_i) \oplus U_1 \).

Write \( U = S_1 \oplus H = S_1 \oplus (\oplus_{i \in I} U_i) \) with \( I \) being a subset of \( \{1, \ldots, n\} \) and \( \text{card}(I) = n - 1 \). There are cases:

**Case 1.** If \( 1 \notin I \), \( U = S_1 \oplus (U_2 \oplus \ldots \oplus U_n) = U_1 \oplus (U_2 \oplus \ldots \oplus U_n) \). Then it follows from \( S_1 \cong U_1 \). By modularity we get \( S_1 \oplus S_2 = S_2 \oplus V \) where \( V = (S_1 \oplus S_2) \cap U_1 \). From here we get \( V \cong S_1 \), this means \( U_1 \) contains a copy of \( S_1 \cong U_1 \). By \( U_1 \) does not embed in a proper submodule of \( U_1 \), we must have \( V = U_1 \), and hence \( S_1 \oplus S_2 = S_2 \oplus U_1 = U \).

**Case 2.** If \( 1 \in I \), there exist \( k \neq 1 \) such that \( k = \{1, \ldots, n\} \setminus I \), \( U = S_1 \oplus (\oplus_{i \in I} U_i) = U_k \oplus (\oplus_{i \in I} U_i) \). Then it follows \( S_1 \cong U_k \). By modularity we get \( S_1 \oplus S_2 = S_2 \oplus V' \) where \( V' = (S_1 \oplus S_2) \cap U_1 \). From here we get \( V' \cong S_1 \), this means \( U_1 \) contains a copy of \( S_1 \cong U_k \). By \( U_k \) does not embed in a proper submodule of \( U_1 \), we must have \( V' = U_1 \), and hence \( S_1 \oplus S_2 = U \) as required.

We aim show next that \( U \) satisfies the condition \((C_3)\), i.e., for two direct summands of \( X_1, X_2 \) of \( U \) with \( X_1 \cap X_2 = 0 \), \( X_1 \oplus X_2 \) is also direct summand of \( U \). By Azumaya’s Lemma, we have \( U = X_1 \oplus K = X_1 \oplus (\oplus_{i \in I} U_i) = (\oplus_{i \in F} U_i) \oplus (\oplus_{i \in J} U_i) \) (where \( F = \{1, \ldots, n\} \setminus J \) and \( U = X_2 \oplus L = X_2 \oplus (\oplus_{j \in E} U_j) \oplus (\oplus_{j \notin E} U_j) \) (where \( E = \{1, \ldots, n\} \setminus D \)).

We imply \( X_1 \cong \oplus_{i \notin F} U_i \) and \( X_2 \cong \oplus_{j \in E} U_j \). Suppose that \( E = \{1, \ldots, t\} \) and let \( \varphi : \oplus_{j=1}^t U_j \to X_2 \) be an isomorphism and set \( Y_j = \varphi(U_j) \), we have \( Y_j \cong U_j \) and \( X_2 = \oplus_{j=1}^t Y_j \). By hypothesis \( X_2 \) is a direct summand of \( U \), thus \( Y_j \) is also direct summand of \( U \) for any \( j \in \{1, \ldots, t\} \). We show that \( X_1 \oplus X_2 = X_1 \oplus (Y_1 \oplus \ldots \oplus Y_t) \) is a direct summand of \( U \).

We prove that \( X_1 \oplus Y_1 \) is a direct summand of \( U \). By Azumaya’s Lemma, we have \( U = Y_1 \oplus W = Y_1 \oplus (\oplus_{p \in P} U_p) = U_\alpha \oplus (\oplus_{p \in P} U_p) \), with \( P \) is a subset of \( \{1, \ldots, n\} \) such that \( \text{card}(P) = n - 1 \) and \( \alpha = \{1, \ldots, n\} \setminus P \). Note that, \( \text{card}(P \cap J) \geq \text{card}(J) - 1 = m \). Suppose that \( \{1, \ldots, m\} \subseteq (P \cap J) \), i.e., \( U = (X_1 \oplus (U_1 \oplus \ldots \oplus U_m)) \oplus U_\beta = Z \oplus U_\beta \) with \( \beta = J \setminus \{1, \ldots, m\} \) and \( Z = X_1 \oplus (U_1 \oplus \ldots \oplus U_m) \).

By \( U \) is a distributive module, we have \( Z \cap Y_1 = (X_1 \oplus (U_1 \oplus \ldots \oplus U_m)) \cap Y_1 = (X_1 \cap Y_1) \oplus ((U_1 \oplus \ldots \oplus U_m) \cap Y_1) = 0 \).

Note that, \( Z, Y_1 \) are direct summands of \( U \) with \( u - \text{dim}(Z) = n - 1 \) and \( u - \text{dim}(Y_1) = 1 \), \( U = Z \oplus Y_1 = (X_1 \oplus (U_1 \oplus \ldots \oplus U_m)) \oplus Y_1 = (X_1 \oplus Y_1) \oplus (U_1 \oplus \ldots \oplus U_m) \). Therefore, \( X_1 \oplus Y_1 \) is a direct summand of \( U \). By induction, we have \( X_1 \oplus X_2 = X_1 \oplus (Y_1 \oplus \ldots \oplus Y_t) = (X_1 \oplus Y_1 \oplus \ldots \oplus Y_{t-1}) \oplus Y_t \) is a direct summand of \( U \). Thus \( U \) satisfies the condition \((C_3)\).

Finally, we show that \( U \) satisfies the condition \((C_2)\). By hypothesis (ii) and \( U \) satisfies \((C_3)\), thus \( U \) is a \((1 - C_2)\)-module (see Theorem 2.3), proving (i). \( \square \)
Theorem 2.7. Let \( U_1, \ldots, U_n \) be uniform local modules such that \( U_i \) does not embed in \( J(U_j) \) for any \( i, j = 1, \ldots, n \). If \( U = \oplus_{i=1}^n U_i \) is a UC distributive module then it is a continuous module.

Proof. We first prove a claim that \( U \) is a CS module. Let \( A \) be a uniform closed submodule of \( U \). Let the \( X_i = A \cap U_i \) for any \( i \in \{1, \ldots, n\} \). Suppose that \( X_i = 0 \) for every \( i \in \{1, \ldots, n\} \) and \( A \cap U_i \neq 0 \). By property \( A \) and \( U_i \) are closed uniform submodules of \( U \), thus \( X_i \) is an essential submodule of \( A \) and \( X_i \) is also essential submodule of \( U_i \). Hence \( A \) and \( U_i \) are closure of \( X_i \) in \( U \), \( U \) is an UC module we get \( A = U_i \). This implies that \( A \) is a direct summand of \( U \), i.e., \( U \) is a \( (1 - C_1) \)-module. By \( U \) has finite dimension, \( U \) is CS module (see [4, Corollary 7.8]), as required.

We aim to show next that \( S = \text{End}(U_i) \) is a local ring for any \( l \in \{1, \ldots, n\} \).

Let \( f \in S \) and suppose that \( f \) is not an isomorphism. It suffices to show that \( 1 - f \) is an isomorphism.

Suppose that \( f \) is a monomorphism. Then \( f \) is not onto, and \( f : U_l \rightarrow J(U_l) \) is an embedding, a contradiction. Thus \( f \) is not a monomorphism.

Then, since \( \text{Ker}(f) \) is a nonzero submodule, it is essential in the uniform local module \( U_l \). Thus, since we always have \( \text{Ker}(f) \cap \text{Ker}(1 - f) = 0 \), it follows that \( \text{Ker}(1 - f) = 0 \), i.e., \( 1 - f \) is a monomorphism. But, since \( U_l \) does not embed in \( J(U_l) \), \( 1 - f \) must be onto, and so \( 1 - f \) is an isomorphism. Thus, \( S \) is a local ring.

Now, we show that \( U \) is a \( (1 - C_2) \)-module, i.e., for two uniform submodules \( V, W \) of \( U \), with \( V \cong W \) and \( W \) is a direct summand of \( U \), \( V \) is also a direct summand of \( U \). By Azumaya’s Lemma, we have \( U = W \oplus W' = W \oplus (\oplus_{j \in J} U_j) = U_k \oplus (\oplus_{j \in J} U_j) \) where \( J \) is a subset of \( \{1, \ldots, n\} \) with \( \text{card}(J) = n - 1 \) and \( k = \{1, \ldots, n\} \setminus J \). Hence \( V \cong W \cong U_k \). Let \( V^* \) be a closure of \( V \) in \( U \). By \( U \) is a CS module, thus \( V^* \) is a direct summand of \( U \). Similarly, there exists \( s \in \{1, \ldots, n\} \) such that \( V^* = U_s \), this means \( U_s \) contains a copy of \( W \cong U_k \). If \( V \) is a proper submodule of \( U_s \), then \( U_k \) embed in \( J(U_s) \), a contradiction. We must have \( V = U_s \), and hence \( V \) is a direct summand of \( U \). Thus, \( U \) is a \( (1 - C_2) \)-module, i.e., \( U \) is a \( 1 \)-continuous module (by \( U \) is a CS module).

Finally, by Theorem 2.4 thus \( U \) is a continuous module. \( \square \)

Corollary 2.8. Let \( U_1, \ldots, U_n \) be uniform local modules such that \( U_i \) does not embed in \( J(U_j) \) for any \( i, j = 1, \ldots, n \). If \( U = \oplus_{i=1}^n U_i \) is a UC distributive module then the following conditions are equivalent:

(i) \( U \) is a \( \sum \)-quasi-injective module;

(ii) \( U \) is a countably \( \sum \)-(1 - \( C_1 \))-module.

Proof. (i) \( \implies \) (ii). It is obvious.
(ii) $\implies$ (i). By Theorem 2.7, $U$ is a continuous module. By [7, Proposition 2.5], $U$ is a $\sum$-quasi-injective module, proving (i). $\square$

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