Title

A new phantom and empirical formula for apparent diffusion coefficient measurement of a 3 tesla MRI scanner

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Abstract
The aim of this study was to create a new phantom for a 3 tesla (T) magnetic resonance (MR) device for apparent diffusion coefficient (ADC) calculation using diffusion-weighted imaging (DWI), and to mimic ADC values of normal and tumor tissues at various temperatures including physiological body temperature of 37 °C. We made the phantom using sucrose of several concentrations from 0 to 1.2 M and performed DWI under monitoring the various phantom temperature. We calculated the accurate ADC values using DWIs of phantoms, and made an empirical formula to calculate the ADC values of phantoms from arbitrary sucrose concentration and arbitrary phantom temperature. The empirical formula could produce the ADC values of the range from 0.33–3.02 × 10⁻³ mm²/sec, which covered the range of ADC values of human body that were measured clinically by 3T MR imaging in the previous studies. Our phantom and empirical formula might be available to mimic the ADC values of the clinical human lesion by 3T MR imaging.

Key words: sucrose, phantom, apparent diffusion coefficient value, diffusion-weighted imaging, magnetic resonance imaging, 3 tesla
Introduction
Magnetic resonance (MR) diffusion-weighted imaging (DWI) has been increasingly performed for clinical purposes such as the detection of tumors and cerebrovascular diseases. The apparent diffusion coefficient (ADC) value, which is calculated based on DWI using several b-values, is useful for discriminating whether the lesion is benign or malignant and determining a therapeutic effect of a tumor. Recently popularized 3 tesla (T) MR devices show a performance advantage when calculating accurate ADC values. Several clinical studies have revealed that ADC values from 3T MR imaging have the diagnostic value as a quantitative parameter (1-8). However, to our knowledge there is no report of an ADC phantom for 3T MR imaging. Regarding ADC phantoms for 1.5T MR imaging, Tamura, et al. (9) reported on a phantom that used gelatin and sucrose. Matsuya, et al. (10) reported on a phantom using polyethylene glycol for 1.5T MR imaging, and created empirical formulas to calculate polyethylene glycol concentration, which provide arbitrary ADC values at any measurement temperature. In principle, the ADC value of a phantom differs due to its temperature. In the present study, we developed an ADC phantom using sucrose for 3T MR imaging, which produces arbitrary ADC values due to a range of phantom temperature (28–39 °C), which includes physiological body temperature. This is the first temperature-controlled ADC phantom for 3T MR imaging. It mimics ADC values of normal and tumor tissues of the human body, and the developed empirical formula enables the calculation of sucrose concentration that provides arbitrary ADC values at any phantom temperature.

Materials and methods
Sucrose phantoms
To create sucrose phantoms, sucrose (S0389-500G, Sigma, St. Louis, MO), NaN₃ (28-1789-5, Sigma-Aldrich Co., Tokyo, Japan) as an antiseptic, and distilled water were heated and stirred until dissolved. The solution was cooled, and the final concentrations of sucrose and NaN₃ were adjusted to be 0.2, 0.4, 0.6, 0.8, 1.0 and 1.2 M, and 0.03 w/w%. These solutions were filled into phantom cases (No1-4628-11, AS ONE Co., Osaka, Japan; Fig. 1(A)) as sucrose phantoms.

Preparation for MR imaging of sucrose phantoms
Sucrose phantoms were placed into a container filled with 0.9 M sucrose solution and 0.03 w/w% NaN₃. The container could hold a maximum of 16 phantoms (Fig. 1(B)).
**Heating system**

The phantom case container was enclosed in a heating box (Fig. 1(C)) made of Styrofoam that was created in-house. The container was heated in the gantry of an MR scanner via a tube that was connected to a circulating temperature-regulated water bath (Thermo-Mate BF-41; Yamato Scientific Co., Ltd., Tokyo, Japan; Fig. 1(D)) to keep the desired phantom temperature constant during MR imaging.

**Real-time phantom temperature monitoring**

Optical fiber thermometers (Fluoroptic™ thermometer m600; Luxtron Co., Mountain View, CA; Fig. 1(E)) were put into phantoms. Phantom temperature was monitored every 30 s during MR imaging to ensure a constant temperature.

**MR imaging**

A clinical 3T MRI unit (Magnetom Skyra; Siemens, Erlangen, Germany) with a head coil was used for MR imaging. DWIs were acquired by a three-scan trace, in the phase-encoding, readout and slice-selective directions, via a single-shot echo-planar imaging sequence. The scan parameters were set as follows: 8000 ms of relation time, 100 ms of echo time, 220 × 220 mm field of view, 160 × 112 matrix, b values of 0, 300, 600, 900, 1200, 1500, 1800, 2100, 2400, 2700, and 3000 s/mm², a thickness of 5 mm, one excitation number, 26.2 ms diffusion gradient pulse duration (δ) and 47.1 ms diffusion time (Δ), which was the interval between onsets of diffusion gradient pulses. Each DW image of a maximum of four phantoms was taken at about each 1 °C interval to cover the physiological body temperature within the range from 28 to 39 °C.

**Accurate measurement of ADC values**

We placed the regions of interest (ROIs; Fig. 1(F)) 7.27 mm square at the position of the thermometer on each phantom DW image. The averaged signal intensity in each ROI was obtained using Image-J software (National Institutes of Health, Bethesda, MD). The logarithms of these signal intensities were plotted as a function of the 11 b-values of 0, 300, 600, 900, 1200, 1500, 1800, 2100, 2400, 2700, and 3000 s/mm². The slope of the regression line, which is defined as the ADC value, and its R² value were obtained by the least-squares method. The ten sets of ADC values and their R² values were obtained for each set of data from 11 DW images using 11 b-values to 2 DW images using 2 b-values in order of decreasing b-values. When R² values exceeded 0.99 according to a decrease in b-values, the ADC values from its set of b-values was defined to be accurate; that is, we used the b-value within the range that the signal intensities
remained above the noise, and the slope of the logarithms of the signal intensities versus b-values became linear. We used these accurate ADC values to create the following empirical formula.

**Empirical formula for calculating phantom ADC values**

ADC values of phantoms were plotted as a function of temperature from 28–39 °C at 1° intervals for each sucrose concentration of 0, 0.2, 0.4, 0.6, 0.8, 1.0, and 1.2 M. The linear equations were made for each sucrose concentration based on a first-order approximation of the relationship between the ADC values and phantom temperature. First-order coefficients and intercepts of the seven linear equations were also plotted as a function of sucrose concentrations. Two formulas were then created, with one based on the fourth-order approximation of the relationship between first-order coefficients and sucrose concentrations, and another between intercepts and sucrose concentrations. Using these two formulas, an empirical formula was created for calculating ADC values of phantoms that were made of arbitrary sucrose concentrations at arbitrary phantom temperatures.

**Accuracy validation of the empirical formula**

To validate the accuracy of the empirical formula, we created new phantoms using sucrose concentrations of 0.2, 0.4, 0.6, 0.8, 1.0, and 1.2 M. We made three phantoms using all sucrose concentrations three times independently, and obtained mean ADC values at each concentration. The ADC values of these verification phantoms were measured at phantom temperatures ranging from 28–39 °C at 1° intervals. We compared the experimental mean ADC values of these verification phantoms and the ADC values calculated with the empirical formula by substituting sucrose concentrations and phantom temperatures at measurement, and then validated the relationship between ADC values calculated with the empirical formula and the range of standard deviations of the experimental ADC values of the verification phantoms.

**Results**

**Calculation accuracy of ADC values**

For each concentration and temperature of the sucrose phantom, ADC values were calculated. The ten sets of ADC values and their $R^2$ values were obtained by the least-squares method for each set of data from 11 DW images using 11 b-values to 2 DW images using 2 b-values in order of decreasing b-values. Figure 2 indicates the
calculation procedure of the ADC value of a 0.2 M phantom at a temperature of 37.09°C as an example. Among ten sets of ADC values and their $R^2$ values, when the maximum b-value decreased to 1500 s/mm$^2$ (Fig. 2(E)), the $R^2$ value obtained for the set of data from 6 DW images using 6 b-values exceeded 0.99 to become 0.9935. According to the slope calculation using this set, the ADC value of the 0.2 M phantom became $3.72 \times 10^{-3}$, which was decided to be accurate. Finally, ADC values were decided for all concentrations and temperatures as shown in Fig. 3(A).

*Change of ADC values of sucrose phantoms by temperature*

ADC values of 0, 0.2, 0.4, 0.6, 0.8, 1.0 and 1.2 M phantoms were plotted in Fig. 3(A) as a function of temperature. The ADC values of phantoms of each sucrose concentration rose as phantom temperature increased. The increasing rate of the ADC value per 1°C increased as the sucrose concentration decreased.

*Development of an empirical formula to calculate ADC values*

We made seven linear equations based on a first-order approximation of the relationship between the ADC values and phantom temperature (t) for each sucrose concentration (s) as shown in Fig. 3(A). The values of these $R^2$ were within the range of 0.9379–0.9801. First-order coefficients (A) and intercepts (B) of the seven linear equations were plotted as a function of sucrose concentrations (s) as shown in Fig. 3(B) and 3(C), respectively. We created each formula based on a fourth-order approximation of the relationship between first-order coefficients, or intercepts and sucrose concentrations. $R^2$ values were 0.9638 and 0.9862, respectively. Using these relational formulas, we created an empirical formula for calculating the ADC values of phantoms made of arbitrary sucrose concentration (s) at arbitrary phantom temperature (t), as follows:

$$\text{ADC value} \times 10^{-3} \text{ mm}^2/\text{sec} = At + B$$

$$A = a_1s^4 - a_2s^3 + a_3s^2 - a_4s + a_5$$
$$a_1 = 8.96519842127907 \times 10^{-7},$$
$$a_2 = 2.94479295800953 \times 10^{-5},$$
$$a_3 = 6.94789261608819 \times 10^{-5},$$
$$a_4 = 6.5038339758676 \times 10^{-5},$$
$$a_5 = 6.22597789270809 \times 10^{-5}$$

$$B = -b_1s^4 + b_2s^3 - b_3s^2 + b_4s + b_5$$
$$b_1 = 5.75284527700504 \times 10^{-4},$$
$$b_2 = 2.48741270074326 \times 10^{-3}.$$
\[ b_3 = 3.1259071150129 \times 10^{-3}, \]
\[ b_4 = 1.19937338765919 \times 10^{-4}, \]
\[ b_5 = 5.94518521028771 \times 10^{-4} \]

**Accuracy validation of empirical formula**

Figure 4(A) indicates the calculated ADC values using the empirical formula shown as the three-dimensional graph with the relationship among ADC values, sucrose concentration, and phantom temperature. The ADC values decrease according to an increase in sucrose concentration and decrease in phantom temperature. Figure 4(B) indicates the relationship between the ADC values, which have been used to make the empirical formula, and the ADC values calculated using the empirical formula. The formula seems to mimic well all ADC values that have been used to create it initially. Figure 4(C) indicates the relationship between the ADC values measured using verification phantoms and the ADC values calculated using the empirical formula. 66.67% of calculated ADC values were less than one standard deviation (SD) away from the mean of measured ADC values of verification phantoms; 97.22% of calculated ADC values were less than two SDs away from the mean, and 100% of calculated ADC values were less than 3 SDs away from the mean.

**Discussion**

To the best of our knowledge, this is the first study of the ADC phantoms for DWIs with 3T MR imaging. We created the ADC phantoms for 3T MRI using sucrose, and made an empirical formula for calculating ADC values between 0.33–3.02 × 10^{-3} at arbitrary sucrose concentrations between 0–1.2 M and arbitrary phantom temperature between 28–39 °C including a physiological temperature of 37 °C to mimic normal and tumor tissue of the human body.

Sucrose, a large molecule with the formula \( C_{12}H_{22}O_{11} \), is a safe and inexpensive material, and it is easy to adjust the concentration of its solution. The diffusion coefficient of the material (D) was related to the temperature (t), the viscosity of the medium (\( \eta \)), and the radius of the diffusion molecule (r) with the Stokes-Einstein equation (11):

\[ D = \frac{k t}{6 \pi \eta r}, \]

where \( k \) is the Boltzmann constant \((1.3805 \times 10^{-23} \text{ J K}^{-1})\). Therefore, we selected the sucrose that has a large molecular size, 0.9 nm in diameter, as the material of phantoms to decrease the ADC values (12).

According to Stokes-Einstein equation, ADC values are affected by the temperature of
the objects in question. As the ADC values used in clinical MR diagnosis are measured for the human body at 37 °C, ADC phantoms that mimic human body tissue should be comparable. Sasaki et al. (13) measured the ADC values of bio-phantoms using human Burkitt’s lymphoma cells at 37 °C, otherwise most in vitro studies performed the ADC measurement at a lower temperature (14-16). Tamura, et al. (9) reported an ADC phantom using 10–50 wt% sucrose for 1.5T MRI, which cover the range of ADC values between 0.2–1.8 × 10⁻³ mm²/s for temperatures between 6–20 °C. In our pre-examination, we measured the ADC values at temperatures from 6–39 °C. The R² values of the first-order approximation of the relationship between the ADC values and phantom temperature were low for phantoms of high sucrose concentration at temperatures < 27 °C. Therefore, we used a range of temperature between 28–39 °C to create an empirical formula.

This empirical formula covered ADC values from 0.672.47 × 10⁻³ mm²/sec at a physiological temperature of 37 °C. Our ADC phantoms nearly covered the ADC values of normal and tumor tissues of the human body that are measured clinically by 3T MRI as summarized in Table 1 (1,3,5-8,17-19). The table indicates sucrose concentration of ADC phantoms at 37°C, which mimic each tissue of the human body by using our empirical formula.

One limitation of this study is that our sucrose phantom produces an ADC value due to change in free diffusion alone. The actual in vivo diffusion in the human body is affected by not only the change of free diffusion but also various factors including perfusion and the change of restricted diffusion due to cellular membrane structures, cell density, etc (20-26). This new ADC phantom and the empirical formula for 3T MRI has potential to be used in many applications.

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References


Figure legends

Figure 1. The phantom and methods for the experiments.

(A) The sucrose phantom in its case.

(B) The phantom case container. Up to 16 sucrose phantoms could be placed into this container filled with 0.9 M sucrose solution including 0.03 w/w% NaN₃.

(C) The heating box made of Styrofoam, which encloses the phantom case container. The container could be heated in the MR gantry via a tube that was connected to a circulating temperature-regulated water bath.

(D) The circulating temperature-regulated water bath.

(E) The optical fiber thermometer for temperature monitoring, which was put in a phantom.

(F) The ROIs 7.27 mm square at the position of the thermometer on each DW image.
Figure 2. Calculation of ADC values of a 0.2 M phantom at 37.09 °C.

The vertical axis indicates the logarithm of the signal intensity of the ROI in the DW image of the phantom. The horizontal axis indicates b-values. ♦: The data that were used for the least-squares method to obtain the regression line and the $R^2$ value. ◇: The data that were not used for the least-squares method to obtain the regression line and the $R^2$ value.
Figure 3. The ADC values of phantoms, and the development of an empirical formula to calculate the ADC values.

(A) The change of ADC values by temperature. The vertical axis indicates the ADC values. The horizontal axis indicates phantom temperature.

♦: 0, ■: 0.2, ▲: 0.4, ×: 0.6, *: 0.8, ●: 1.0 and +: 1.2 M sucrose phantom. Each straight line indicates a first-order approximation of the relationship between ADC values and the phantom temperature for each sucrose concentration. The first-order coefficients and the intercepts of these linear equations are plotted on (B) and (C), respectively. Each $R^2$ value for the first-order approximation was within the range of 0.9379–0.9801.

(B) The relationships between the sucrose concentrations and the first-order coefficients of linear equations from the first-order approximation. Black diamonds indicate first-order coefficients. The curved line indicates the fourth-order approximation, with $R^2 = 0.9638$.

(C) The relationships between sucrose concentrations and the intercepts of the linear equations from the first-order approximation. Black diamonds indicate intercepts. The curved line indicated the fourth-order approximation, with $R^2 = 0.9862$. 
Figure 4. Calculated ADC values and accuracy validation using the empirical formula.
The vertical axis indicates the ADC values. The horizontal axes indicate sucrose concentration and phantom temperature, respectively. Black cubes in each figure indicate the ADC values that were calculated from the empirical formula.
(A) The ADC values which were calculated from the empirical formula.
(B) The relationship between predetermined and calculated ADC values using the empirical formula. The crosses (×) indicate the ADC values used to make the empirical formula.
(C) The relationship between the ADC values measured using verification phantoms and the ADC values calculated using the empirical formula. The crosses (×) and vertical lines indicate the mean and three standard deviations of the ADC values measured using verification phantoms.
<table>
<thead>
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<th>Regions</th>
<th>Mean ADC values (×10⁻³ mm²/sec)</th>
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b-values (sec/mm$^2$): $^1$ 0-800, $^2$ 0-1000, $^3$ 0-1500, $^4$ 0-2000

The superscript at region is the number of reference.