Sensitivity Analysis and its Application to the Control of Inner Furnace Temperature Distribution

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Nowadays, various kind of reactor furnaces are widely used for the production in industry. The raw materials charged into the furnace generate reaction heat produced by blowing gas. Generally speaking, the reaction heat generated in the furnace is remarkably high. Therefore the occurrence of an inappropriate temperature distribution in the furnace may make damage or serious accident of the furnace. This is the motivation of furnace control. The author is considering the application of studied results to the furnace control of Blast Furnace in steel industry. To the propose, the approximated and simplified Macro Model of the Blast Furnace is constructed which has the function of representation of qualitative characteristics of the furnace in dynamical sense. The furnace temperature, distribution greatly effects both on the producing and the product quality of the furnace. Needless to say, stable furnace operation is indispensable for the economical prosperity of the industry. In this paper, macro simulation of the furnace is developed to support the analysis and design of the furnace control. Using the simulator, the stability and the control characteristics for inner furnace temperature distribution are analysed quantitatively.

1 Introduction

The modeled furnace is equipped with multiple tuyeres at the bottom. The volume and distribution of gas blow at these tuyeres can be regulated individually. The control factors for furnace are 1) Volume of gas blow into the furnace, 2) Distribution of gas blow for each tuyere of the furnace, 3) Calorific value of blowing gas, 4) Material charge condition from top of the furnace. All of them are boundary conditions for the furnace. In other words, though it is necessary to control the inner temperature distribution, it is can be adjusted only by changing these boundaries. Construction of this paper is as follows.

* Creation of Macro-Model for Furnace - section 2
* Sensitivity Analysis of Control Factors - section 4
* Control of Inner Furnace Temperature using Boundary Data -section 5

First, the simulator which uses control factors as boundary conditions for gas flow and temperature distribution of furnace is created. The purpose of intro-
duction of a Macro-Model is to control the temperature distribution on whole region in the furnace by changing control factors appropriately. Additionally, the process of macro expression for gas flow, heat transfer and reactivity is described and numerical results by Macro-Model are shown. Next section 3 describes sensitivity analysis about 3 control factors which is described upon. Referring 4.1～4.4, the effects of each control factors for temperature distribution are examined by the case study under the various control factors. On furnace, because each control factors are given independently and also variously, factors temperature distribution would be more complex. The effects of control factors for these complex temperature distribution is examined by the sensitivity analysis. Finally, control of inner furnace temperature distribution by changing control factors is proposed by considering sensitivity analysis of them. Where, the method of estimation of inner state using boundary data is proposed. 5.3 describes the method of determination of control factors using estimated inner data by the model. The proposed method determines numerically them in the quantity using the error between current and desired temperature distribution and the results of their sensitivity analysis.

2 Gas flow and heat transfer simulator for furnace

2.1 2-dimensional Macro-Model for furnaces

Gas and temperature distribution in the furnace represented blast furnace are different distributions by gas blow in the bottom and material distribution. And so this paper proposes Macro-Model [1] [2] which calculates 2-dimensional macro gas flow and temperature distribution using control factors, gas blow and material charge, for boundary conditions of calculation. Width direction of Macro-Model is x axis and height direction is y axis. In a real furnace, since height(H) and width(D) is about H/D = 2, x direction has 23 nodes and y direction has 46 nodes at Macro-Model. Referring to Fig.1, there are 5 tuyeres in the bottom of Macro-Model and a discharge equipment of gas flow in the top of it. Additionally gas flow equation adopts Ergun equation [3] [4], temperature distribution has gas and solid layer, therefore Macro-Model is 2 layer model.

2.2 Gas flow equation considering pressure drop through columns packed with granular material

The fluid through the column packed with solid particle like iron ore causes pressure drop by its kinetic energy loss. Ergun equation shows the experimental relations between the pressure drop and a diameter of packed particle, fractional void volume. In this paper, the stress which effects flow is considered in order to treat 2-dimensional gas flow as macro distribution. Gas flow equation used in this paper is as follows.

\[
\frac{\partial V}{\partial t} = -2\nabla p - (f_1 + f_2|V|)V + \frac{1}{Re} \Delta V \quad (1)
\]

\( V \): Gas flow rate, \( p \): Pressure,
\( Re \): Reynolds number

Where \( f_1, f_2 \) are the pressure drop by the motion of gas flow through the column packed. They are shown in the following equation.

\[
f_1 = 150 \frac{(1 - \epsilon)^2 \mu}{c^3} \frac{D_p}{\rho}, \quad f_2 = 1.75 \frac{1 - \epsilon}{c^3} \frac{\rho}{D_p} \quad (2)
\]

\( D_p \): Diameter, \( \rho \): Density of gas,
\( E \): Activity energy, \( R \): Gas constant

Referring equation (1), gas flow rate is proportional to pressure gradient substructed by pressure drop in the right-hand side second term. Gas viscosity is considered in the third term of right-hand side.
2.3 Reaction model of material considering reactivity

The high temperature gas which is blowed into furnace bottom with high energy accelerates reactions of charged material. Arrhenius plot [5] is adopted in the reactivity of solid particle. In addition, the reactivity [6] [7] [8] [9] is improved, when particle size of the solid particle is smaller, and the flow rate of reactant gas which flows along the solid particle is bigger. The reactivity of solid particle is expressed as reactivity coefficient $k$ in the following equation.

$$ k = \frac{\alpha}{D_p} \exp \left( \frac{-E}{R \cdot T_g} \right) \cdot \beta V \quad (\alpha, \beta : \text{const}) \quad (3) $$

Additionally, the particle of material melts in the high temperature gas and the melting is factor which prevents the reactivity gas from flowing. When this behavior is considered in macro manner, it assumes that as if diameter of the material was larger by melting and expressed in equation (4) and shown in Fig.2.

$$ D_p = \delta \exp \left( \frac{(T_s - 1500)^2}{100000} \right) + D_p^0 \quad (4) $$

Where, $D_p^0$ is the equivalent diameter of material which is charged into the furnace.

Fig.2 The increase of equivalent diameter of material by melting

The diameter of material increases by $\delta$ according to temperature distribution. Therefore, the calorific value per unit product of the material particle becomes the function of material particle size, gas temperature and gas flow rate. The schematic diagram of the calorific value considering the reactivity for the change of $T_g, D_p$ is shown in Fig.3. In Fig.3, transverse axis expresses size of material, longitudinal axis expresses gas temperature and vertical axis expresses the calorific value.

Fig.3 Calorific value per unit product of the material particle

2.4 Heat transfer model for gas and solid

The energy from the reaction of material is transferred to the surrounding gas, and extended along the gas flow. These heat transfer between solid and gas is expressed using heat transfer coefficient $h$ in equation (5).

$$ h(T_g - T_s) \quad (5) $$

In addition, heat transfer coefficient $h$ is expressed in the following equation as a function of material particle size and gas temperature which surrounds the particle. [10]

$$ \frac{h \cdot D_p}{\lambda_g} = 2 + 1.1 \left( \frac{D_p \cdot V}{\nu} \right)^{0.6} \left( \frac{\rho \cdot \mu}{\lambda_g} \right)^{1/3} \quad (6) $$

The heat transfer equation for solid layer in the furnace is expressed in the equation (7) considering reaction model of material and heat transfer between gas and solid layer.

$$ \frac{\partial T_s}{\partial t} = \lambda_s \Delta T_s + k \cdot Q + h(T_g - T_s) \quad (7) $$

$T_s, T_g$ : Solid, Gas temperature

In this paper, it is assumed that gas is incompressible, and the temperature variation does not effect the motion of flow. Therefore heat energy is one-sidedly transported by the flow, and it's called the forced convection. For such gas layer, the heat transfer equation is expressed in the equation (8).
\[
\frac{\partial T_g}{\partial t} = -(\nabla \cdot \nabla)T_g + \lambda_g \Delta T_g - h(T_g - T_s) \tag{8}
\]

Where, Macro-Model is verified by the reference literature [11].

3 Stability of numerical calculation

This paper uses difference equation for mathematical model. Here, the stability of the numerical calculation is examined.

3.1 Stability of heat transfer equations

To begin with, the stability of heat transfer equation is described. Heat transfer equations of Gas and Solid are expressed in the following equation.

\[
\begin{bmatrix}
A_g & 0 \\
-h\Delta t & A_s
\end{bmatrix}
\begin{bmatrix}
T_g^{n+1} \\
T_s^{n+1}
\end{bmatrix}
= \begin{bmatrix}
1 & h\Delta t \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
T_g^n \\
T_s^n
\end{bmatrix}
+ \begin{bmatrix}
0 \\
kQ\Delta t
\end{bmatrix} \tag{9}
\]

Where, \(A, B\) and \(C\) are defined as follows.

\[
A = \begin{bmatrix}
A_g & 0 \\
-h\Delta t & A_s
\end{bmatrix}, \quad B = \begin{bmatrix}
1 & h\Delta t \\
0 & 1
\end{bmatrix}, \quad C = \begin{bmatrix}
0 \\
kQ\Delta t
\end{bmatrix}
\]

\[
T^{n+1} = A^{-1}BT^n + A^{-1}C \tag{10}
\]

About the stability of numerical calculation, the eigenvalue \(\lambda_i\) of matrix \([A^{-1}B]\) is examined. The maximum and minimum values of absolute eigenvalues for each \(\Delta t\) and \(\Delta l (= \Delta x = \Delta y)\) are plotted in Fig.4. In Fig.4, horizontal axis expresses \(\Delta l\) and vertical axis expresses \(\Delta t\). Additionally, solid circles show converged results of numerical calculation, and solid crosses show diverged. The sufficies written in bracket ( ) are \((\max |\lambda_i|, \min |\lambda_i|)\). This paper uses implicit method for the heat transfer equation. Generally speaking, the implicit method is stable without constraints. However referring Fig.4, stability limit exist.

The transitions of inner temperature \(T_g(2, 12)\) during 10 steps are shown in Fig.5. The solid circles show converged calculation and the solid crosses show diverged one.

3.2 Stability of gas flow equation

Next, the stability of gas flow equation is described. For gas flow equation, the explicit method is used. Therefore it is necessary that time width \(\Delta t\) is small so that the calculation could have stable solution. This paper uses \(\Delta t = 0.002\) by experimental results. So the stability of gas flow equation for \(\Delta l\) is described. The average of inner gas flow rate are shown in Fig.6.
Referring Fig.6, when \( \Delta t \) is small, gas flow solution is converged to constant value. Especially numerical solutions for \( \Delta t = 0.05 \) and 0.1 are same results. Therefore \( \Delta t = 0.1 \) is adopted for gas flow equation.

### 3.3 Flow chart of numerical calculation

As stated above, the mathematical model uses \( \Delta t = 0.1 \). And time width must be \( \Delta t = 0.002 \) for stable gas flow calculation. On the other hand, it is possible to calculate the heat transfer equations at \( \Delta t = 0.01 \). Since the calculation using \( \Delta t = 0.002 \) takes a lot of iterations for the calculation of solid temperature, converged gas flow solution was used. The flow chart of these calculation is shown in Fig.7.

![Flow chart of calculation using converged numerical gas flow solution](image)

The error of numerical solution between method of converged gas flow at \( \Delta t = 0.01 \) and method of \( \Delta t = 0.002 \) is 1% or less, and Therefore the method using converged gas flow solution is considered to be reasonable.

### 4 Sensitivity analysis for control of furnace

The boundary conditions of proposed Macro-Model is control factors of furnace. This paper treats volume of gas blow, weight of gas blow, temperature of gas blow and material charge as control factors. Then, sensitivity analysis of these control factors for inner furnace temperature distribution is carried out by case studies. The standard control conditions are shown following.

- gas volume: \( V_{in} = 6 \) unit
- gas weight: evenness from 1~5
- gas temp.: \( T_{in} = 1200 \) °C
- material charge: even material

#### 4.1 Volume of gas blow

First it is necessary to determine the volume of gas blow into the furnace. For the control of temperature distribution, the capacity of gas blow is set such that \( 3 \leq V_{vol} \leq 10 \) unit and the sensitivity analysis is carried out for volume of gas blow. Fig.8 shows equi-temperature line of 1400 °C gas temperature distribution which is the steady state for each volume of gas blow. Referring Fig.8, the more the volume of gas blow is large, the more the heat transfer for furnace height is large. In other words, volume of gas blow for furnace affects to heat transfer for furnace height.
4.2 Weight of gas blow for tuyeres

Next analysis is gas temperature distribution by the change in weight of gas blow for tuyere at the bottom of the furnace. When the weight of whole gas blow is set with 1, the weight for each tuyere is weight of gas blow $u = [u_1 \cdots u_6]$. Therefore, the volume of gas blow from each tuyeres $V_i$ is in the equation (11).

$$V_i = u_i \times V_i \quad \text{for } i = 1 \cdots 5 \quad (11)$$

subject to $\|u\| = 1$

Where, $V_i$ is whole volume for furnace.

Fig.9 shows equi-temperature line of 1400°C gas temperature distribution only for blowing from tuyere $\overline{1}$, and also temperature distribution for tuyere $\overline{2} \cdots \overline{6}$.

![Equi-temperature line on 1400°C for each gas blow temperature](image)

Fig.9 Equi-temperature on 1400°C for each gas blow temperature

Referring Fig.9, heat transfer in the furnace width distribution is different by the effects for deviation of gas blow. In other words, weight of gas blow for each tuyere effects the heat transfer in furnace width.

4.3 Temperature of gas blow

The temperature of gas blow to the furnace is defined in the equation (12).

$$1150 \leq T_{in} \leq 1250 \, ^\circ C \quad (12)$$

Fig.10 shows equi-temperature line on 1400°C of temperature distribution for gas blow temperature $T_{in} = 1150 \, ^\circ C$, and also temperature distributions for $T_{in} = 1175, 1200, 1225, 1250 \, ^\circ C$.

![Equi-temperature on 1400°C for each gas blow temperature](image)

Fig.10 Equi-temperature on 1400°C for each gas blow temperature

4.4 Material charge for furnace width

For control of temperature distribution in furnace, it is assumed that 2 kinds of material which reactivity differs are changed into furnace. Where, the difference of reactivity is a difference of material's size, in short, the more material's size is small, the more the reactivity is high. (See Fig.3)

![Pattern of material charge for furnace width](image)

Fig.11 Pattern of material charge for furnace width

Fig.12 Equi-temperature on 1400°C for each pattern of material charge
Sensitivity for material charge which differs in the furnace width distribution is examined. First, this paper describes the pattern of material charge for furnace. Fig.11 shows 2-dimensional plane of the furnace. Referring Fig.11, it is possible that the high reactivity material charge on each node 6~10, 10~14, 14~18 for furnace width. Where, each charge pattern is named W1, W2 and W3. Gas temperature distributions which the high reactivity material charges each charge pattern W1, W2, W3 are shown in Fig.12 by distribution in width direction at half level of furnace height.

Referring Fig.12, to the region where the high reactivity material is supplied, center of the temperature distribution changes. In other words, charging high reactivity material for furnace width effects to heat transfer for furnace width direction.

5 Control of temperature distribution by adjusting control factors

Fig.13 shows inner furnace and the relation between state ① and state ②, transverse axis expresses direction of furnace width, longitudinal axis expresses direction of furnace height and vertical axis expresses inner temperature. Additionally, marker ○ or × shows temperature on node(y,x). State ① is initial temperature, and state ② is desired temperature. Referring Fig.13, this paper proposes the method of determining control factors numerically in order to change inner furnace temperature distribution from state ① to state ②. The method of determination of control factors based on the sensitivity analysis for inner furnace temperature distribution will be described.

5.1 Control system for furnace

For control of inner furnace temperature, it is necessary to grasp the inner temperature distribution. However in operating furnace, the inner data of furnace is not measurable.
Now in this study, inner temperature distribution is estimated using wall temperature and gas flow, what is called boundary data, in order to determine control factors. Fig.14 shows the schematic diagram of control system for furnace including Estimating Model by boundary data. In the flow chart shown in Fig.15, furnace model is Macro-Model and named it Furnace Model.

5.2 Estimation of furnace inner state using boundary data

The informations given by the Furnace Model is

1. Gas and solid temperature on the wall of furnace
2. Volume and weight of gas blow for each tuyere
3. Temperature of gas blow
4. Gas flow rate at the top of the furnace

The history on which material has been charged is known. Estimating Model is also Macro-Model. For estimating inner state based on these informations, Estimating Model is used as well as Furnace Model. Referring Fig.16, the same control factors are given to both Furnace Model and Estimating Model as boundary conditions. Under the given boundary conditions, Estimating Model is solved and outputs a numerical solution of gas temperature.

5.3 Method of determinating control conditions

This study uses the results of sensitivity analysis and analysis of dynamic characteristics in order to determine control factors. Therefore this paper proposes the evaluation functions including these information, and numerical determination of control factors. Where, for the evaluation of inner temperature distribution, the 50 representative data of estimated state by Estimating Model is $\hat{x} = [\hat{z}_1, \hat{z}_2, \cdots, \hat{z}_{50}]^T$, the one of desired temperature distribution is $x^d = [x^d_1, x^d_2, \cdots, x^d_{50}]^T$. 
5.3.1 Determination of gas blow volume

In section 4.1, it is revealed that heat transfer in furnace can be controlled by changing volume of gas blow. This relationship between volume of gas blow and inner gas temperature puts in "Relation Matrix of gas blow volume and temperature distribution" \( C_v \). First, the transition of the inner gas temperature for the change of gas blow volume is examined at Macro-Model. Referring Fig.17, volume of gas blow changed to \( V_{in} \) on steady state of gas temperature for volume of gas blow \( V_{in} = 6 \text{unit} \).

In Fig.18, the relationship between the variation of gas blow volume \( \Delta V_{in} = V_{in}' - V_{in} \) and the variation of gas temperature \( \Delta T' = T_{res} - T_{ini} \) by changing volume of gas blow is noticed. The variations of gas temperature \( \Delta T' \) are plotted for the variation of gas blow volume \( \Delta V_{in} \), and shown in Fig.19.

![Fig.17 Steady-state of gas temperature on \( V_{in} = 6 \text{unit} \), \( T_{in} = 1200 \degree C \)](image)

In addition, using the least squares method, relational expression shown in equation (13) is deduced.

\[
\Delta T' = 60.0 \cdot \Delta V_{in} = c_v \cdot \Delta V_{in} \quad (13)
\]

Equation (13) shows the variation of gas temperature induced from change of gas blow volume by \( \Delta V_{in} \).

Secondly the error between current gas temperature \( T_g(23, 12) \) and desired temperature \( T_g^0(23, 12) \) is noticed. The error between current and desired temperature is defined in

\[
\Delta T_g(23, 12) = T_g(23, 12) - T_g^0(23, 12) \quad (14)
\]

and the evaluation function in which volume of gas blow is determined as in equation (15).

\[
\{ T_g + c_v \cdot \Delta V_{in} - T_g^0 \}^2 \rightarrow \text{min} \quad (15)
\]

Equation (15) determines the variation of gas blow volume \( \Delta V_{in} \) in equation (13) so that the error between current inner temperature and desired temperature is made up for.

In fact, the variation of inner gas temperature by changing gas blow volume is examined on 50 representative nodes. Therefore the relationship between the
Variation of gas blow volume $\Delta V_{in}$ and the variation of gas temperature $\Delta T'$ is given for the 50 nodes as well as equation (13), and shown in following.

$$\Delta T'_i = c_i \cdot \Delta V_{in} \quad \text{(for } i = 1, \cdots, 50) \quad (16)$$

The evaluation function on $i$ node is shown in equation (17) using data $\hat{z}_i$ of estimating data by estimating model and data $x_i^0$ of desired temperature distribution.

$$\{\hat{z}_i + c_i \cdot \Delta V_{in} - x_i^0\}^2 \rightarrow \min \quad \text{(for } i = 1, \cdots, 50) \quad (17)$$

In addition, it is equivalent to equation (18) that equation (17) is applied to whole region of furnace.

$$\|\hat{x} + C_{V} \cdot \Delta V_{in} - x^0\| \rightarrow \min \quad (18)$$

Then by the given $\Delta V_{in}$ in equation (18), the volume of gas blow for furnace at the next step $V_{in}^{n+1}$ is shown in following equation.

$$V_{in}^{n+1} = V_{in}^{n} + \Delta V_{in} \quad (19)$$

### 5.3.2 Determination of weight of gas blow

In section 4.2, it is revealed that heat transfer for furnace width direction can be controlled by changing weight of gas blow for tuyeres. This relationship between weight of gas blow and inner gas temperature distribution puts in "Matrix of weight of gas blow and temperature distribution" $C_G$. To begin with, the weight of gas blow for each tuyere is expressed in $u_g = [u_1^0 \ u_2^0 \ \cdots \ 0 \ 0 \ 0 \ 0]^T$. For example, weight of gas blow for using only tuyere 1 is expressed in $u_1^0 = [1 \ 0 \ 0 \ 0 \ 0]^T$. Where, weight of gas blow is the weighted coefficient for each tuyere satisfying in following equation.

$$||u_g||_1 = 1, \ 0 < u_g^i < 1 \quad \text{(for } i = \{1 \sim 5\}) \quad (20)$$

The representative 50 data of temperature for weight of gas blow $[1 \ 0 \ 0 \ 0 \ 0]^T$, in other words from tuyere 1, puts in first row of $C_G$. These actions carries out for tuyere 2~5 similarly, and then "Matrix of weight of gas blow and temperature distribution" $C_G$ which has 50 column and 5 rows is obtained.

$$C_G = [x_1^0 \ x_2^0 \ \cdots \ x_5^0] \quad (21)$$

When an arbitrary weight of gas blow is multipled matrix $C_G$, the multiplied result approximately expresses gas temperature corresponding to the weight of gas blow. The evaluation function for determination of weight of gas blow is defined in equation (22) using "Relation matrix of weight of gas blow and temperature distribution" $C_G$.

$$\|\hat{x} + C_G \cdot \Delta u_g - x^0\| \rightarrow \min \quad (22)$$

The constraints are shown in following equations.

$$||u_g^n + \Delta u_g||_1 = 1, \quad 0 < u_g^i + \Delta u_g^i < 1 \quad \text{(for } i = 1, \cdots, 5) \quad (23)$$

Where, $u_g^n$ is a current($n$ step) weight of gas blow, and $\Delta u_g$ is variation of weight of gas blow. Equation (22) combines temperature distribution data for each weight of gas blow so that the error between current inner temperature and desired temperature is made up for. Therefore by the given $\Delta u_g$, the weight of gas blow for furnace $u_g^{n+1}$ on next step($n + 1$) step is shown in the following equation.

$$u_g^{n+1} = u_g^n + \Delta u_g \quad (24)$$

### 5.3.3 Determination of gas blow temperature

In section 4.3, it is confirmed that heat transfer in furnace can be controlled by changing temperature of gas blow. This relationship between temperature of gas blow and inner gas temperature is expressed as "Relation Matrix of gas blow temperature and temperature distribution" $C_Q$. In Macro-Model, the transition of inner gas temperature for the change of temperature of gas blow is examined. Temperature of gas blow changed from $T_{in}(=1200) \degree C$ to $T'_{in}$ on steady state.

Fig.20 shows the transition of inner gas temperature $T_g(23, 12)$, and there are the difference of steady-state gas temperature by each temperature of gas blow.
Equation (26) determines the variation of gas blow temperature $\Delta T_{in}$ so that the error between current inner temperature and desired temperature is made up for. Then by the given $\Delta T_{in}$, the temperature of gas blow for furnace on next step $T_{in}^{n+1}$ is shown in the following equation.

$$T_{in}^{n+1} = T_{in}^n + \Delta T_{in}$$  \hspace{1cm} (27)

5.3.4 Determination of material charge

In section 4.4, it is possible to control heat transfer for furnace width by changing high reactivity material. This relationship between charge of high reactivity material and inner gas temperature puts in "Relation Matrix of material charge and temperature distribution" $C_M$. First, material charge is expressed in $\mathbf{u}_m = [u_{m1}^W u_{m2}^W u_{m3}^W]^T$. For example, in Fig.11, material charge $W1$ is expressed by $u_{m1}^W = [1 0]$. Where, weight of material charge is the weight coefficient for each charging pattern satisfying in following equation.

$$\|\mathbf{u}_m\|_1 = 1, \quad 0 < u_{m1}^j < 1$$ \hspace{1cm} (28)

As well as previous description, the representative 50 data of gas temperature for charging pattern $[1 0 0]^T$ puts in first row of $C_M$. These actions carries out for each charging pattern $W2, W3$ similarly, and then "Matrix of material charge and temperature distribution" $C_M$ which has 50 columns and 3 rows is obtained.

$$C_M = [x_{W1} \ x_{W2} \ x_{W3}]$$ \hspace{1cm} (29)

When an arbitrary charging pattern is multiplied matrix $C_M$ shown in equation (29), the result approxi-
mately expresses gas temperature for the charging pattern. The evaluation function for determining material charge is defined in equation (30) using \( C_M \).

\[
\| \tilde{x} + C_M \cdot u_m - x^0 \| \rightarrow \min \quad (30)
\]

The constraints are shown in following equations.

\[
\| u_m \|_1 = 1, \quad 0 < u_m' + \Delta u_m' < 1 \quad (for \ j = W1 \sim W3) \quad (31)
\]

Equation (30) combines temperature distribution data for each material charge so that the error between current inner temperature and desired temperature is made up for. Then \( u_m \) is given.

6 Experimental results

In this section, it is described that the control of inner furnace temperature for 2 control factors, volume of gas blow and temperature of gas blow. To begin with, the control of gas blow volume is described, and next is gas blow temperature. Where, the desired inner temperature distribution is shown in Fig.17, the control factors are shown in Table.1

<table>
<thead>
<tr>
<th>Gas blow vol.</th>
<th>Gas blow temp.</th>
<th>Gas blow weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_m = 6 ) unit</td>
<td>( T_m = 1200 ) °C</td>
<td>1~5</td>
</tr>
</tbody>
</table>

6.1 Control of temperature by volume of gas blow

State ① in Fig.13 which is the initial temperature distribution is shown in the Fig.22 in order to examine control of temperature by volume of gas blow. The distribution of temperature is also steady state under \( V_m = 4 \) unit.

The gas volume of temperature distribution in Fig.22 is \( V_m = 4 \) unit and is smaller than that \( V_m = 6 \) unit on desired value. Therefore, the temperature of initial state is lower than the desired value in whole region of furnace.

Under control time \( T_c = 20 \) hour, the transitions of determined volume of gas blow are shown in the Fig.24.

The transitions of temperature on node (23,12) which is center node of furnace for change of gas blow...
volume are shown in the Fig.25.

![Fig.25 Inner temperature for each time](image)

Since initial inner temperature is lower than desired one, gas blow volume increases so that inner temperature rises. When the inner temperature is almost close to desired one, the increment of gas blow volume stops at $V_{in} = 6$ unit. In other words, control factor is adjusted by proposed determination using evaluation function.

6.2 Control of temperature distribution by temperature of gas blow

State 1 in Fig.13 which is the initial temperature distribution is shown in the Fig.26 in order to examine control of temperature by temperature of gas blow.

![Fig.26 Steady-state of $V_{in} = 6, T_{in} = 1150$](image)

![Fig.27 Effects of gas blow temperature control](image)

The distribution of temperature is also steady state under $T_{in} = 1150$ °C. The temperature distribution shown in Fig.26 is $T_{in} = 1150$ °C and is smaller than the one of $T_{in} = 1200$ °C on desired value. Therefore, the temperature of initial state is lower than the desired value in whole region of furnace. The transitions of determined temperature of gas blow are shown in the Fig.28. And the transitions of temperature on node (23,12) which is center node of furnace for change of gas blow temperature are shown in the Fig.29.

![Fig.28 Gas blow temperature for each time](image)

![Fig.29 Inner temperature for each time](image)

In the same gas blow volume control, gas blow temperature is adjusted so that inner temperature should be close to desired temperature. Therefore, inner temperature distribution is controlled by also gas blow temperature.

6.3 The control performances

The control performances of gas blow volume and temperature are evaluated. The error between inner furnace temperature ($x$) on each time and desired temperature ($x_d$) is defined as an evaluation index, and the behavior for time course by the change of the process...
requirement has examined.

\[ f = (x - x_0)^T Q (x - x_0) \]  

(32)

\[ \frac{df}{dt} = x^T Q (x - x_0) + (x - x_0)^T Q \dot{x} \]  

(33)

The control performance of gas blow volume is shown in real line, and the one of gas blow temperature is in dotted line in Fig.30.

Referring Fig.30, 31, inner furnace temperature is close to desired temperature by changing control factors. Additionally referring Fig.30 and Table.2, effect of gas blow volume for inner temperature is larger than the one of gas blow temperature. Therefore it's better that gas blow volume is adjusted in order to change inner temperature largely.

7 Conclusion

In this paper, macro simulation of the furnace has been studied in order to control inner furnace temperature distribution. The Macro-Model expresses main part of inner phenomenon, for example gas flow distribution and heat transfer of furnace in macro manner. To the purpose, Macro-Model uses control factors of the furnace as boundary conditions. Using this Macro-Model, the stability of numerical calculation and the control characteristics for the inner furnace temperature are examined. The numerical calculation of Macro-Model resulted in stable, steady state and transient inner temperature are obtained under the given control factors. To begin with, using characteristics of steady state temperature distribution, sensitivity analysis are carried out. By sensitivity analysis, it is possible that effects of control factors on inner furnace state has been examined. Since the investigation is one of the index of the control of the furnace, it is very important. Next, using characteristics of transient temperature distribution, inner furnace temperature is controlled by changing control factors. By proposed method, control factors are determined numerically by evaluation function. In other words, the appropriate control amount is obtained quantitatively by them. However, in the current stage, the control of inner furnace temperature remains only by one control factor. In the future, the control by multiple control factors will be examined. Additionally, in mathematical model for the furnace, the reasons why the inner temperature is stable or diverges are examined at the point of the inner furnace conditions, for example the pressure drop and the material distribution. Reflecting these factors to the model, the stability limit of inner furnace temperature control was examined.
Bibliography


