Estimation of the screening length and the electric charge on particles in single-layered dusty plasma crystals

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A theoretical approach which has been successful in reproducing results of molecular-dynamics simulations on dusty plasmas is applied to estimate the screening length and the electric charge of two-dimensional dust crystals of melamine particles in the discharge chamber experiment. It has been found that the screening length is of the same order of magnitude as the inter-particle distance and the electric charge decreases on increasing number density of dust particles.

Dusty plasma has attracted our attention because of its aspect as strongly coupled plasma and of clearly observed structure and dynamic properties. We have been investigating the structure formation and melting of dusty plasma in two and three dimensional finite and infinite system by molecular dynamic simulations and theoretical analyses. In experimental dusty plasmas, horizontally layered structures have been observed.[1] We have reproduced layered structures by molecular dynamic simulations modeling the system as charged particles interacting each other via screened Coulomb interaction and vertically confined by the gravity and the electric field due to electrode. The number of layers changes discretely with two system parameters, the strength of confining and the screening length. We have developed the theory with intralayer cohesive energy and succeeded in reproducing results of our simulations.[2, 3] We also have performed the numerical simulation and theoretical analyses on the mixture of dust particles which have different charge-to-mass ratio and found the separation of particles by the gravity.[4]

The dusty plasma system takes the single-layered two-dimensional structure at the limit of strong confinement. Such a single-layered finite dusty plasma can be modeled as charged particles in two-dimensional parabolic potential which interact with each other via the Yukawa potential $q^2 \exp(-r/\lambda)/r$. At low temperatures, this system is characterized by only two parameters, $\alpha = q^2/(k\lambda^3)$ and N, where q is the electric charge on a dust particle, λ the screening length, k the parabolic confining parameter, and N the number of particles. Simulations have been performed for the system containing up to 10^4 particles to obtain the equilibrium states. For large system, we have applied the Fast Multipole Method to compute mutual interactions.[5, 6] In this article, we discuss the estimation of the screening length λ and the electric charge q of experimentally observed single-layered dusty plasmas reported recently by Hebner et al.[7]

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based on our theoretical approach.

Our theoretical analysis reproduces the results of simulation almost completely for large system when parameter α exceeds 1. The essential part of our theory is to take the correlation energy between dust particles into account. The detailed formulation of this theory is described in Ref.[6]. It gives the distribution of particles as the dimensionless density function $\rho(R/\lambda)\lambda^2$ by

$$\rho\left(\frac{R}{\lambda}\right)\lambda^2 = \frac{1}{4\pi\alpha}\left[\left(\frac{R_m}{\lambda}\right)^2 - \left(\frac{R}{\lambda}\right)^2\right] + \frac{3a}{4\pi}\left[\frac{1}{4\pi\alpha}\left[\left(\frac{R_m}{\lambda}\right)^2 - \left(\frac{R}{\lambda}\right)^2\right] + \left(\frac{a}{8\pi}\right)^2\right]^{1/2} + \frac{5a^2}{32\pi^2},$$

where R is a distanse from the center of the system and R_m is the maximum radius of the system determined by

$$8\alpha N = \left(\frac{R_m}{\lambda}\right)^4 + \frac{2a}{\sqrt{\pi}}\alpha^{1/2}\left\{\left[\left(\frac{R_m}{\lambda}\right)^2 + \frac{a^2}{16\pi}\alpha\right]^{3/2} - \left(\frac{a^2}{16\pi}\alpha\right)^{3/2}\right\} + \frac{5a^2}{4\pi}\alpha\left(\frac{R_m}{\lambda}\right)^2.$$

Here a is a parameter determined numerically by cohesive energy of the two-dimensional Yukawa lattice and is approximately equal to $\sqrt{\pi}$. Some typical examples of simulation and theory are shown in Fig.1.

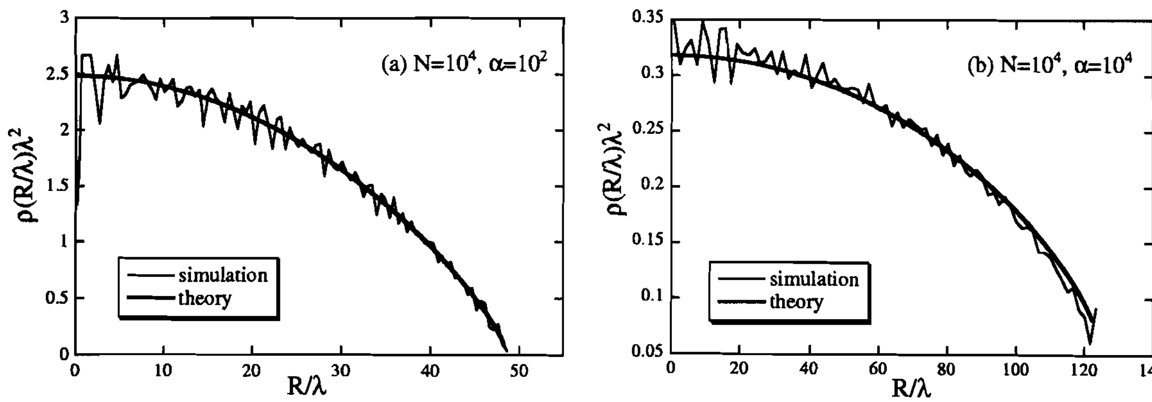

FIG. 1: Examples of radial distributions of number density obtained by simulation and theory including 10^4 particles. Distance from the center of the distribution is normalized by screening length λ, and (a) $\alpha = 10^2$ and (b) $\alpha = 10^4$.

Hebner et al. have observed single-layered dusty plasma crystals in parallel-plate discharge chamber. Some of their experimental data are listed in Table I. The values of nearest neighbor separation at the center of distribution s_0 and maximum radius $r_{\max}$ are taken from Figs. 5 and 6 in Ref.[7] at each number of particles N. The density at the center $\rho_0 = \rho(R = 0)$ is calculated from s_0 using $\rho_0 = 2/(\sqrt{3}s_0^2)$ as the particles are ordered into the regular triangular lattice at the central region.

Assuming that nearest neighbor particles interact via the screened Coulomb interaction, they have obtained the screening length λ and the charge q. The results are shown in Table II.

We now apply our theory to their experi-

TABLE I: Experimental data[7]

N	s_0 (mm)	$r_{\rm max}$ (mm)	ρ_0 (1/mm^2)
1161	0.41	10.31	6.80
434	0.52	6.88	4.35
276	0.58	5.88	3.41
205	0.61	5.35	3.08
106	0.71	4.18	2.28

TABLE II: Estimation in Ref.[7]

relation	λ (mm)	q (e)
(1)	0.260	23500
(2)	0.249±0.010	23400±500
(3)	0.277±0.017	22700±1000

ments and estimate the screening length and the electric charge on dust particles. For given values of two parameters of the system at low temperatures, α and N, we obtain from our theory the distribution of particles in terms of the screening length λ which is also to be determined. We here note that the product of the square of maximum radius $r_{\rm max}^2$ and the density at the center ρ_0 is a function of N and α. Since N is known, we estimate the value of α searching the best fit of $\rho_0 r_{\rm max}^2$ between their experiment and the theory. After determining α, we can derive λ and q from the experimental values of ρ_0 or $r_{\rm max}$ and k: The confining parameter $k = m_d g/R_c$ is equal to 8.6×10^{-12}kg/sec^2, where m_d=4.4×10^{-13}kg is mass of a melamine particle of which diameter is 8.34μm, g is gravity acceleration, and R_c=0.5m is curvature of the electrode. The results are shown in Table III. In the case of $N = 1161$, the estimated value of α is 0.029 and is beyond the applicability of our theory. The obtained values of screening length and the electric charge are shown in Fig.2 as the functions of particle number N.

TABLE III: Our estimation of screening length and electric charge.

N	$\rho_0 r_{\rm max}^2$ (experiment)	estimated α	$\rho_0 r_{\rm max}^2$ (theory)	$\rho_0\lambda^2$ (theory)	$r_{\rm max}/\lambda$ (theory)	λ(mm)	$q(e)$
1161	723.7	0.029	723.7	46.00	3.97	*	*
434	206.3	463	206.3	0.310	25.79	0.267	1.81×10^4
276	118.0	1200	118.0	0.184	25.30	0.232	2.37×10^4
205	88.3	808	88.3	0.191	21.53	0.249	2.16×10^4
106	39.7	2200	39.7	0.118	18.39	0.227	3.10×10^4

* Our theory is not applicable to the case where $\alpha < 1$.

It is found that the electric charge decreases and the screening length slightly increases with the increase in the number and the number density of particles. Our theoretical approach takes the inter-particle interaction not only between neighboring particles but also between distant ones. Since the screening lengths obtained above are the same order of magnitude with the inter-particle separation s_0 in the experiments, the interaction beyond the nearest neighbor cannot be neglected. We also note that, based on our theo-

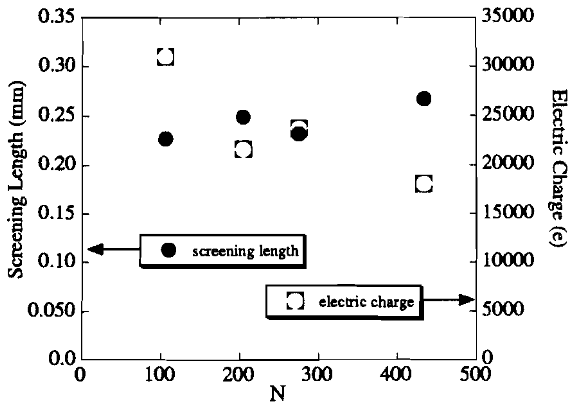

FIG. 2: Estimated screening length and electric charge as the function of N.

retical analysis, the dependency of the charge and screening length on the number density is clarified. Our theory is thus shown to be useful to estimate the screening length and the electric charge in experimentally observed dusty plasmas.

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