The Provision of Public Goods under the Proportional Taxation System

TADAHIRO UEMATSU*

I Introductiron

The provision of public goods can be peformed either under the benefit principle or the ability-to-pay principle.(1) Under the former, the tax-price of one public good differs from person to another, and is proportionate to the person's marginal substitution of that good to private goods. This is one of the necessary conditions for the "Lindahl Equilibrium" which is on Pareto optimum(Foley〔4〕). Moreover, if we can apply an overall calculation for maximizing the social welfare function, we can also calculate the optimal production and income redistribution in addition to optimal prices, from the viewpoint of maximization of social welfare (Samuelson 〔8〕).

It seems to us, however, that public goods are not actually provided under the benefit principle except in a few cases, in contrast to the great pile of theoretical accumulation. The first reason for this may be due to the so-called "free rider problem" or false assertion of preferences for public goods. Since people will not be excluded from the consumption of

* The author is assistant professor in the school of law and literature of Okayama University. The research is financially supported by the Grant Aid for Scientific Research of the Ministry of Education.

(1) There are many difinitions on public goods. We simply define them as those which are equally enjoyed consuming by all the people in the community.

public good even if they assert less than their actual preferences, the demand for public goods will be less than optimal as a whole. In other words, this is the problem of information costs about preferences. In order to get a picture of the true preferences, government will have to gather a great deal of information, which costs might prevent the provision plan of this public good in some cases.

Secondly, we must not disregard the regressve taxation effect of this principle. For example, people with low income can not supply private alternatives for public goods like public education, public housing or parks, so their marginal substitution of these goods and, therefore, their tax-prices will be fairly high. This difficulty will not be overcome without proper income redistribution, which is not always easily performed.

On the other hand, much more severe assumptions of comparability of interpersonal utility and the constant size of public expenditure often have been presupposed in the ability-to-pay principle theories.(2) But these assumptions do not seem indispensable. Bowen〔1〕presented an excellent model to decide the size of public expenditure by citizens' voting though in a partial equilibrium model. Foley〔3〕, from the quite different point of view, showed the existence of a public competitive equilibrium with proportional taxation. These two have a characteristic to have shown how to decide the size of public expenditure on the ability-to-pay principle.

In this paper I will also analyse the provision of public goods under proportional taxation in a competitive equilibrium. With a simple model, we compare Foley's model with Bowen's one, and find the relation of these two models which seem quite different at a glance. Section II

(2) See Musgrave〔6〕chapter 5 for an excellent discussion about ability-to-pay principle.

presents a basic model from which Foley's public competitive equilibrium and Bowen's voting procedure derive. Section Ⅲ analyses the difference between these two models from our standpoint. And section Ⅳ gives a device for voting in the case with many private goods. Finally, section Ⅴ presents a comparison between these two models in regard to distributional effect and information costs.

Ⅱ Model

For simplicity, we assume there are two private goods $x_j (j=1,2)$, one public good x_g, three citizens $(i=1,2,3)$, one large production institution or firm, and one government. It is also assumed that the first private good x_1 is used for production of the second private good x_2 and public good x_g by this firm. Then consumption vector x^i and initial endowment vector $\bar{x}^i$ of each citizen are written as

$$x^i=(x_1^i,\ x_2^i,\ x_g^i) \qquad (i=1,\ 2,\ 3.)$$

$$\bar{x}^i=(\bar{x}_1^i,\ 0,\ 0) \qquad (0<\bar{x}_1^i\ll\infty)$$

Production vector $y=(y_1, y_2, y_g)$ is restricted in the production set Y by the production function $F(y)\geqq 0$.

$$y\in Y=\{y \mid F(y)\geqq 0\}$$

Supply price vector of private goods and public good is

$$p=(p_1,\ p_2,\ p_g).$$

We also make the following assumptions.

1. Utility funcions of all people $u^i(x^i)$ are continuous, increasing and concave functions of x^i.

2. Production function $F(y)$ is also a continuous, increasing and concave function of y. Production is also assumed to satisfy the con-

straints of the possibility of inaction, the impossibility of land of Cockaigne and the irreversibility of production.

$$0 \in Y, \quad Y \cap R_+ = \{0\}, \quad Y \cap (-Y) = \{0\}.$$

3. The profit of the firm π is completely distributed among three citizens, with distributive share μ_i fixed.

$$\pi = \Sigma \mu_i \pi. \qquad (0 < \mu_i < 1)$$

4. Government makes a final decision about allocation, keeping balanced budget on a proportional income taxation. Expressing the tax rate and the i-th citizen's income as t and M^i, we have

$$p_g x_g = t \ \Sigma M^i.$$

Based on these assumptions, competitive equilibrium with public good is formulated as follows.

Definition Competive Equilibrium

Competitive equilibrium is a triple vector (x, y, p) which satisfies following conditions.

A) Demand does not exceeds supply

$$\sum_i x_j^i \leqq y_j + \sum_i \bar{x}_j^i \qquad (j = 1, 2)$$

$$x_g \leqq y_g.$$

B) Citizen maximizes $u^i(x^i)$ subject to

$$\Sigma p_j x_j^i \leqq (1-t) M^i$$

$$M^i = p_1 \bar{x}_1^i + \mu_i \pi$$

$$p_g x_g = t \Sigma M^i.$$

C) Firm maximizes profit $\pi = \Sigma p_j y_j + p_g y_g$ subject to $F(y) \geqq 0$.

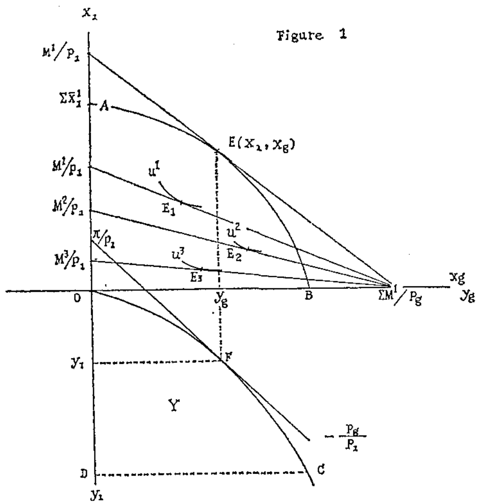
D) Government maximizes social welfare $W = W(u^1, u^2, u^3)$ subject to

$$p_g x_g = t \Sigma M^i$$

$$x_g^1 = x_g^2 = x_g^3 = x_g.$$

Can this competitive equilibrium be proved to exist? Unfortunately, its existence does not seem to be directly assured. Figure 1 discribes the projection into $(x_1; x_g)$ plane with the other private good fixed. From the assumptions 1 and 2, total consumption set and production set are confined to convex, compact sets ABO and OCD respectively. Given price vector (p_1, p_2, p_g), the firm makes an optimal production at point F where hiper-plane of price vector is tangent to production set.

Production at F brings about profit π.



Price vector p also specifies tax rate t and everyone's income M^i from the equations $M^i = \Sigma p_1 \bar{x}_i^1 + \mu_i \pi$, $p_g x_g^i = t \Sigma M^i$, $x_g^i = x_g$, and $x_g \leqq y_g$. Consumers want to maximize their utilities under these prices, tax and income constraints so that they select respectively different consumption E_1, E_2, and E_3. As easily seen in Figure 1, demand and supply of both private good and public good will generally be different. So price vector has to shift. But in which direction? From the property of public good, x_g^i should be equal to x_g. But in Figure 1, $x_g^2 > x_g > x_g^3 > x_g^1$ Then it is not clear whether or not public good should be extensively produced without any exogeneous adjustment means. This means a failure to accomplish a "competitive equilibrium" under the ability-to-pay theory.(3)

Two abjustment procedures can be imagined; one is the governmental selection of the size of public expenditure from the viewpoint of the maximization of social welfare, and the other is the selection by voting among people. The former was studied by Foley 〔3〕, and the latter by Bowen 〔1〕. So we would like to compare them in next section.

Ⅲ Two Adjustment Procedures

In the definition of Foley's public competitiive equilibrium, condition B) and D) of our definition are changed so that citizens select optimal consumption of *private goods* given the supply of public good while government selects optimal consumption of *public good* given information about citizens' preferences.(4) According to Foley 〔3〕 and Homma 〔5〕, this interaction between citizens and government will lead to an equilibrium if the social marginal significance of each citizen is equal to the reciprocal of the marginal utility of his income in equilibrium. This proposition bases on the following proposition about Paréto optimum.

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(3) Under the benefit principle, competitive equilibrium can be attained by the private transactions, if free rider problem is avoided. But under the ability-to-pay principle, it can not generally attained by the private transactions, as we analysed here.

(4) This statement is slightly different from that of Foley's original definition. But it does not distort his intention. Our definition is due to Homma 〔5〕, which succeeded in extention of Foley's original model. We deeply owe to him this paragraph.

Definition *Pareto* *Optimum*

Pareto optimum is defined as the vector (x^1, x^2, x^3, y) which maximizes the social welfare function $W=\Sigma k^i u^i(x^i)$, $(k^i \geqq 0,\ \Sigma k^i=1)$ under the constraints

$$\sum_i x_j^i \leqq y_j + \sum_i \bar{x}_j^i \qquad (i=1,\ 2)$$

$$x_g \leqq y_g \qquad \text{and}$$

$$F(y) \geqq 0.$$

Proposition 1. Under the assumptions 1 and 2, there exists one Pareto optimum for any $\{k^i\}$.

Proof. Under the assumptions 1 and 2, consumption vector (x^1, x^2, x^3) and production vector y are confined to compact, convex sets respectively. Social welfare function W is a continuous function of x^i from the construction of W. Then, by Weierstrass' theorem, there is a vector (x^1, x^2, x^3, y) which maximizes $W=\Sigma k^i u^i(x^i)$ for any $\{k^i\}$.

Q. E. D.

In the one private good-one public good case, Proposition 1 assures the existence of public competitve equilibrium, because in this case citizens have no room to make their own selection after the government decides the size of supply of public good, and therefore Pareto optimum is a necessary and sufficient condition for public competitive equilibrium.

Even if there are more than one private good and one public good, the existence can be proven if the marginal social significance of each citizen is to be equal to the reciprocal of the marginal utility of his income in equilibrium.(5) This might be a little bit severe condition.

On the other hand, Bowen [1] tries to decide the size of public expenditure by citizens' voting, viewing that competitive equilibrium can not be attained directly. Though he analysed it in a partial equilibrium model with fixed income and equal division of cost, we can easily reformulate it in a competitive equilibrium model with a proportinal income tax.

Let us suppose a situation where the congress composed of all the citizens in the community tries to decide the optimal provision of public good. Given information about the firm, the citizen tries to maximize his utility by the selection of the most favourable production and taxation. So he tries to maximize $u^i(x^i)$ subject to

1. budget constraint

$$\Sigma p_j x_j^i \leqq (1-t)M^i$$

$$M^i = p_1 \bar{x}_1^i + \mu_i \pi$$

$$p_g x_g^i = t \Sigma M^i$$

2. feasibility condition

$\pi = \max\ (\Sigma p_j y_j + p_g y_g)$ subject to $F(y) \geqq 0$, and

3. public good condition

$$x_g^i \leqq x_g \leqq y_g .$$

Is it possible for the citizen to find an "optimal" plan of consumption? In order to reply to this question, we have to separate one private good case from more than one private good case.

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(5) See Homma [5] pp54—56.

If there is only one private good besides public good, the supply of public good and proportional taxation substantially confine the citizen's budget constraint set to a subset of total consumption set which is compact. Moreover, from the above conditions 1 and 2, his budget constraint is closed, then it is also compact. By assumption 1, utility function $u^i(x^i)$ is continuous on this set, so a maximal value can be found on a point in this set. This assures "optimal" plan for the i-th citizen.

Let us confirm this assertion by a simple example.

Example 1. Suppose that private good x_1 is the input for production of public good x_g, and that production function is $y_1=-by_g^2$ $(0<b\ll\infty)$. Let us also suppose the i-th citizen's initial endowment is $(\bar{x}_1^i,\ 0)$.

In order to attain optimal production, the firm has to

maximize $\pi=p_1x_1+p_gx_g$

subject to $y_1=-by_g^2$

which brings about an optimal production $(y_1,\ y_g)=(-\frac{p_g^2}{4bp_1^2},\ \frac{p_g}{2bp_1})$ and maximal profit $\pi=p_g^2/4bp_1$.

Total consumption set C is

$$C=\{(x_1,\ x_g)\ |\ x_1+bx_g^2\leqq\Sigma\bar{x}_1^i\}$$

and the i-th citizen's budget constraint set C^i is expressed as follows

$$C^i=\{(x_1^i,\ x_g^i)\ |\ x_1^i\,(\Sigma\bar{x}_1^i+bx_g^{i2})\leqq(\bar{x}^i+b\mu_ix_g^{i2})\ (\Sigma\bar{x}^i-bx_g^{i2})\}$$

$$(0\leqq x_1^i\leqq\bar{x}_1^i,\ 0\leqq x_g^i\leqq\{\Sigma\bar{x}_1^i/b\}^{\frac{1}{2}})$$

So it is clear that C^i is compact.

Figure 2 shows this example. With supply price $(p_1,\ p_g)$, supply of goods which is consistent with maximal profit of firm is point D, and under proportional taxation, the citizen's budget line is GK. But only D^i on this line is feasible for him from the property of public good. Then given price $(p_1,\ p_g)$, he selects D^i. If the price vector changes to

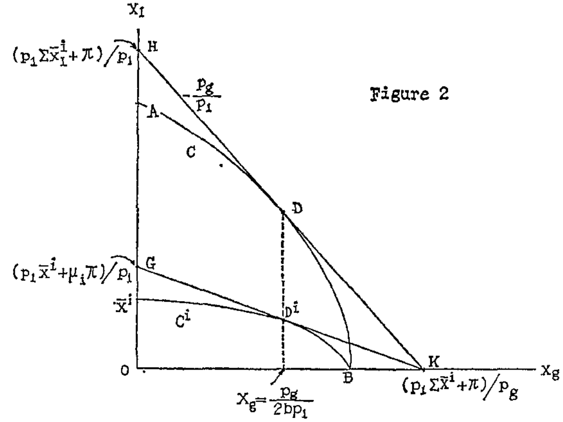


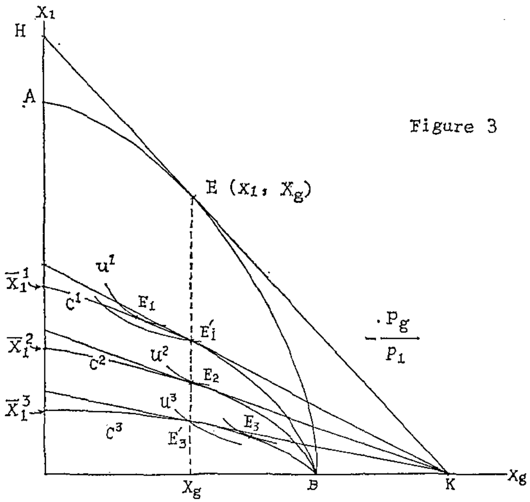
Figure 2

(p_1', p_g'), he will selects another point $D^{i'}$, and these points on budget constraints lines make a budget constraint set C^i. It is easy to see that the utility function has a tangency point with this set C^i. It is an "optimal" plan for him.

Generally speaking, "optimal" public expenditure for the i-th citizen will be different from that for the j-th citizen $(j \neq i)$, however. Then let us adopt a majority voting rule for adjustment. When government offers one plan for the provision of public good, if majority of the people prefers an extension as opposed to reduction, government extends the demand of public good and vice versa. The provision of public good is in an equilibrium if the number of citizens who assert extension is equal to that of citizens who assert reduction. Then, clearly, the existence of an equilibrinm is assured if there is an odd number of people.

Turning back Example 1, look at Figure 3. It shows that voting decides all.

Each citizen asserts his most favourable demand of public good E_1, E_2, or E_3 in which his utility function is tangent to his budget con-



straint set. But voting results in the adoption of the second citizen's proposal x_g^2 (in Figure 3). At the same time, voting decides all other necessary items, the consumption of private good (E_1', E_2, E'_3), the supply of both goods E, the supply price shown by the slope of HK, and the tax rate. It is to be noted that the supply and demand of private good meet in this case.

Then as far as one private good-one public good case is concerned, voting is almighty in that it can decide all the necessary items in the economy. This is a straight extension of Bowen's voting procedure to a competitive equilibrium analysis with proportional taxation.

We summarize this conclusion as a proposition.

Proposition 2. If there is one private good besides one public good, the voting procedure brings about an equilibrium.(6)

(6) This proposition is easily extended to many public goods case so far as there is only one private good, because then budget constraint sets C^i of all citizens are also confined to compact subset of C respectively.

IV Trouble in the Voting Procedure

In this section, it will be revealed that whenever there is more than one private good, voting has to follow a double procedure and then it will have a similar characteristic to the public competitive equilibrium. Let us take up a simple example to see this, at first.

Example 2. Suppose that the first private good is used for the production of the second private good and public good, and that the production function is $y_1 = -(ay_2^2 + by_g^2)$, $(0 < a,\ b \ll \infty)$. Let us also suppose the i-th citizen's initial endowment is $(\bar{x}_1^i,\ 0,\ 0)$.

In order to attain an optimal production, the firm has to

$$\text{maximize } \pi = \Sigma p_j y_j + p_g y_g$$

$$\text{subject to } y_1 = -(ay_2^2 + by_g^2),$$

which brings about an optimal production $(y_1,\ y_2,\ y_g)$

$$= \left(-\frac{bp_2^2 + ap_g^2}{4abp_1^2},\ \frac{p_2}{2ap_1},\ \frac{p_g}{2bp_1}\right) \text{ and maximal profit } \pi = \frac{bp_2^2 +\ p_g^2}{4abp_1}.$$

Total consumption set C is

$$C = \{(x_1,\ x_2,\ x_g) \mid x_1 + ax_2^2 + bx_g^2 \leqq \Sigma \bar{x}_1^i\}$$

But the i-th citizen's budget constraint set C^i is not necessarily a subset of C, because C^i depends on the price vector.

$$C^i = \{(x_1^i,\ x_2^i,\ x_g^i) \mid \frac{p_1 x_1^i + p_2 x_2^i}{p_1(\bar{x}_1^i + \mu_i(\Sigma\bar{x}_1^i - \Sigma x_1^i))} + \frac{p_g^2}{2bp_1^2(2\Sigma\bar{x}_1^i - \Sigma x^i)} \leqq 1\}$$

Therefore, whenever there is more than one private good, citizen may select a point outside of total consumption set C in some price vector and then voting can not determine of demand and supply of private goods. This is the essential trouble with the voting procedure.

As a device, let us consider the following double procedure. First of

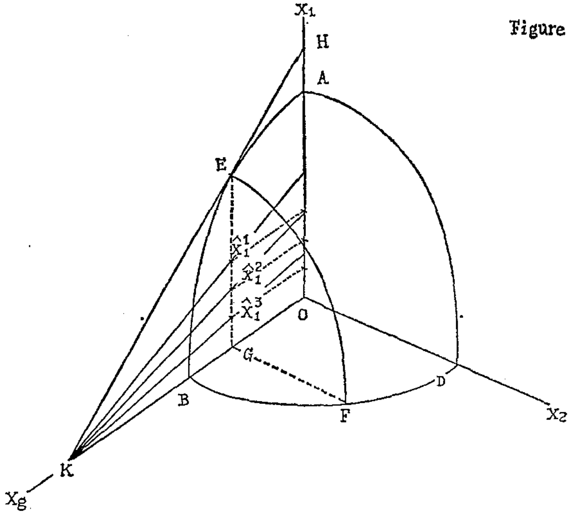


Figure 4

all, people ascertain the total consumption set and their initial endowments value, according to variously supposed price vector (p_1, p_2, p_g). Then they try to decide optmal demand for public good by voting, with disregard to the second private good for the moment. This voting procedure is the same as that in Example 1, and it will decide public expenditure OG (in Figure 4), for instance. Proportional taxation assures financial resources, and it will leave remaining incomes $\hat{x}_1^1$, $\hat{x}_1^2$ and $\hat{x}_1^3$ respectively. Next, when public good as much as OG is produced, the consumption possibility set of private goods is $EF\,G$. Then people try to trade private goods as if they are in a private economy. The trade will lead to a competitive equilibrium, which shows optimal total demand and individual demand for private goods. Finally, the total demand for public good and private good points out optimal production and supply price vector.

To sum up. Whenever there is more than one private good in an economy, voting about optimal demand for public good has to be supplemented by the free trade of private goods. Together voting and trade decide optimal selection. We formulate this as a proposition.

Proposition 3. Whenever there is more than one private good in an economy, voting about public good and free trade of private goods attain together an equilibrium.

V Comparison

In the preceding two sections, we analysed that the provision of public goods can be performed either in a public competitive equilibrium or by voting procedure. In this section we would like to compare them from the viewpoint of distributional effect and information costs.

In regard to distributional effect, public competitive equilibrium intends to respect the initial condition as competitive equilibrium without public goods does so. Since marginal significance k^i is to be equal to the reciprocal of the marginal utility of income, it will be larger for the citizen with higher income than for the one with lower income, if utility functions do not differ one another to a great extent and if they are concave functions. Then in an equilibrium, the citizen with higher initial endowment (i. e. higher income) gets a higher social weight k^i. Social welfare function based on this weight vector $\{k^i\}$ will correspond to an equilibrium respecting initial endowments.

On the other hand, distributive effect of voting procedure is not necessarily clear. In this case optimal selection of the i-th citizen depends on the initial endowment $\bar{x}_1^i$, distributive share μ_i and the configuration of his utility function $u_i(x_i)$.

First of all, if the distributive shares are nearly equal one another, and utility functions are similar among all the citizens, the citizen with initially *medium income* is advantageous, as Bowen showed in his original paper (Bowen [1] section Ⅲ). Secondly if preferences of the people are different even though distributive sahres are equal, the citizen with *medium preference* for public good in an equilibrium will be advantageous. But he may belong either to the highre income class or to the lower one. Finally if μ_i is reversely related to $\bar{x}_1^i$, the economic hierarchy will change as production extends so that we can not prospect in advance which citizen will be most advantageous in an "optimal" provision of public good.

In the second place, in regard to the information coat, voting procedure is much better than public competitive equilibrium. Voting only needs n proposals and the reaction of the citizens to each proposal (n is the number of the citizens).

On the other hand public competitive equilibrium needs a great deal of information on preferences in order to make a social welfare function as we showed in section Ⅲ.

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要　約

一般均衡理論のフレームの中で公共財の供給を扱うときには，なぜか「利益説」的なアプローチが優先する。財政学では支持されてきたはずの「能力説」が，この問題では十分な理論を構築していないためであろうか。しかし，現実には，公共財は「利益説」によってではなく，「能力説」に従って供給されているのであり，この現実を理論的に根拠づけることが必須の条件である。

本稿は，比例税制度を設定して，一般均衡のフレームの中で公共財の供給を分析した，Foley の公共競争均衡と Bowen の投票システムをとりあげ，比較・拡張したものである。筆者はとくに Bowen のモデルに関心をもっているので，これを一般均衡的な設定に拡張してもなお，公共財の最適供給がデザインしうることを明らかにしようと試みた。その際，公共財の数にかかわらず，私的財が一種類だけ存在している場合には，投票システムが資源配分，相対価格，（比例）税率等一切を決定しうる（Proposition 2）のにたいして，私的財が2種類以上存在する場合には，投票制度が公共財の供給量を決め，私的財の市場が私的財にかんする資源配分を決定する，という複合システムが要請されること（Proposition 3）を明らかにした。

この Proposition 3 は，公共財の供給量と税率を投票で決定するという形式で民意を生かし（消費者主権），かつ私的財の取引を市場に委ねることによって，投票制度のもつ強制力を排除するなど，従来の「利益説」による公共財供給理論と十分匹敵できる方法を提示している言えないだろうか。