岡山大学経済学会雑誌26(1),1994, 91～96

Marx All Vindicated — Falling Rate of Profit (2)

Takao Fujimoto

1. Let me continue the thesis on falling rate of profit. In this note, we present some numerical examples which show the possibility of lowered profit rate after adopting cost-reducing new processes because of external diseconomies. We do not claim this is an important case, but rather consider it nice to have a couple of concrete examples which satisfy our intuitive deduction on this matter: though cost reducing, the new process may require more of a particular commodity, which then should be produced in a greater amount causing external negative effects on other industries, thus realizing the reduced uniform rate of profit. We will make many explicit/implicit assumptions, which are after all (hopefully) not so lousy.

2. Let us take up an economy where there exist only two commodities, and assume the absence of joint production. Let $B(x)x \equiv Ix$, and

$$A^{\circ}(x)x \equiv \begin{bmatrix} 0.1+0.16x_2/x_1, & 0.4 \\ 0.5, & 0.4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix},$$

where I is the 2 by 2 identity matrix. The superscript $^{\circ}$ means that symbols are those before technical changes, and * those after innovation. Note that external effects are represented in the material input coefficient of the first industry. By writing in this way, it may be easier to grasp externalities for students who are familiar with linear Leontief models. (Our mathematical theorem in [1] can deal with more general cases.) This case shows the existence of external diseconomies because the input coefficient of the first commodity in the first industry, $a_{11}(x)$, is $0.1+0.16x_2/x_1$. As the gross output of commodity 2, x_2, increases, this coefficient becomes larger provided x_1 is kept unchanged. (On the other hand, when x_1 increases while keeping x_2 fixed, the scale merit works.) Now, the equation $x=(1+r)A^{\circ}(x)x$ can be solved: $x^{\circ}=(4/9, 5/9)'$, and $r^{\circ}=0.25$ with $a_{11}(x^{\circ})=0.3$.(Variable vectors are normalized so that the sum of the elements is unity. The uniqueness of solutions is guaranteed by various nonlinear generalizations of Perron-Frobenius theorem.) At this output vector x°, the equilibrium price vector $p^{\circ}=(0.5, 0.5)$ again with the equilibrium profit rate $r^{\circ}=0.25$. Now a cost-reducing technical change in the second industry takes place. That is, $(0.4, 0.4)'$ now becomes $(0.28, 0.5)'$.

$$A^*(x)x \equiv \begin{bmatrix} 0.1+0.16x_2/x_1, & 0.28 \\ 0.5, & 0.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

When this process is fully adopted, the equilibrium output vector $x^*$is $(0.383, 0.617)$, and $r^*=0.235$ with $a_{11}(x^*)=0.358$. With x^*, the equilibrium price vector $p^*=(0.525, 0.475)$, and the profit rate $r^*=0.235$. Thus, the adoption of a cost-reducing process pushes down the equilibrium rate of profit.

Now, $H^*(x)\equiv Ix-A^*(x)x$ is homogeneous of a positive degree(one in this case), and in fact linear, and satisfies our assumption in section 5 of [1]. (The assumption shoud have been strengthened requiring $H_i^*(0)=0$.) To examine our Theorems 3 and 4 in [1], we calculate the Jacobian of $H^*(x)$, $\nabla H^*(x)$. Fortunately in our simple case, $\nabla H^*(x)$ is independent of x, and

$$\nabla H^*(x) \equiv I-(1+r^\circ)\begin{bmatrix}0.1, & 0.44\\ 0.5, & 0.5\end{bmatrix}$$

When $p=(0.41,\ 0.59)$, $p\nabla H^*(x)<0$, thus our sufficient condition is not satisfied, showing the possibility of a falling rate of profit after adopting a cost-reducing process.

Now let us consider the case of external economies.

$$A^\circ(x)x \equiv \begin{bmatrix}0.4-0.08x_2/x_1, & 0.4\\ 0.5, & 0.4\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}, \text{ and}$$

$$A^*(x)x \equiv \begin{bmatrix}0.4-0.08x_2/x_1, & 0.28\\ 0.5, & 0.5\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}.$$

More precisely, $a_{11}(x)$, should be $\max(0,\ 0.1-0.08x_2/x_1)$. Our simplified version, however, does not affect the final outcome, and in fact piecewise differentiability suffices. $x^\circ=(4/9,\ 5/9)'$, $p^\circ=(0.5,\ 0.5)$, $r^\circ=0.25$ with $a_{11}(x^\circ)=0.3$, while $x^*=(0.351,\ 0.649)'$, $p^*=(0.491,\ 0.509)$, $r^*=0.298$ with $a_{11}(x^*)=0.252$. Thus, the profit rate is raised. In this case of external economies, our Theorems 3 and 4 in [1] are of help.

$$\nabla H^*(x) \equiv I-(1+r^\circ)\begin{bmatrix}0.4, & 0.2\\ 0.5, & 0.5\end{bmatrix}$$

With $r^\circ=0.25$, no semi-positive p can satisfy $p\nabla H^*(x)\leq 0$. (In this special case with no joint production and resulting linearity, we can examine whether the Jacobian $\nabla H^*(x)$ satisfies the Hawkins-Simon condition.) Therefore, by Theorem 4 there exists an x^+ such that $H^*(x^+)>0$. Hence, the inequality (3)' in Theorem 3 is satisfied. The equilibrium rate of profit rises after adopting a cost-reducing process in the second industry.

3. Now we can show a proposition for a simple case without joint

production helped by the above numerical examples. Before technical changes we have

$x^o = (1+r^o)A^o(x^o)x^o$, and $p^o = (1+r^o)A^o(x^o)$,

while after the adoption of new processes, which are assumed to be cost-reducing, we should have

$x^* = (1+r^*)A^*(x^*)x^*$, and $p^* = (1+r^*)A^*(x^*)$, together with

$p^o \geq (1+r^o)A^*(x^o)$.

Proposition: In an input-output model with externalities, but without joint production, the uniform rate of profit rises after cost-reducing technical progress, if and only if the quantity augmenting property holds.

The quantity augmenting property here implies the existence of an vector x^+ such that

$x^+ > (1+r^o)A^*(x^*)x^+$.

Let us assume the indecomposability of $A^*(x^*)$. The if part has been proved in [1] almost like a tautology. The only if part is also easy to show thanks to the Perron-Frobenius theorem. All we have to do is to examine the Hawkins-Simon condition for

$I-(1+r^o)A^*(x^*)$,

after solving x^* which realizes the quantity equilibrium (steady balanced growth) with new technology. (In the appendix, we give a PASCAL program, which was used to calculate x^* in the above numerical examples.)

Appendix

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(* 非線形レオンチェフ : TF: 1994-1-19(水 ) *)
(* L=(0.1, 0.1); b=(0,1)' included in A,i.e, a21 & a22. *)
program leonl;
{$C-} (* To make KeyPressed effective. *)
const
 e:real=0.16; (* 外部性係数 All = all + e*X1/X2 *)
 eps: real=1.0E-6;
var
 a11,a12,a21,a22: real;
 x1,x2,x1t,x2t,s: real;
 d: real;
 ch: char;
 i: integer;
begin
  a11:=0.1; a12:=0.4; (* Quantity side. *)
  a21:=0.5; a22:=0.4;
  x1:=0.5; x2:=0.5;
  i:=0;
 repeat
  i:=i+1;
  x1t:=(a11+e*x2/x1)*x1 + a12*x2;
  x2t:=a21*x1 + a22*x2;
  s:=1/(x1t + x2t);
  x1t:=s*x1t;
  x2t:=s*x2t;
  d:=abs(x1t-x1) + abs(x2t-x2);
  writeln('(',i:2,')',': x1 = ',x1t:9:6,'; x2 = ',x2t:9:6,'; s = ',s:9:6);
  x1:=x1t;
  x2:=x2t;
   IF KeyPressed then begin     (* To pause *)
    Read(Kbd,ch);
    if UpCase(ch)='Q' then halt;
    Read(Kbd,ch);
    if UpCase(ch)='Q' then halt;
    end;
 until (d < eps);
 s := a11 + e*x2/x1;
 writeln('    a11* = ',s:9:6);

 a11:=s; s:=a12; a12:=a21; a21:=s; (* 価格 *)
  x1:=0.5; x2:=0.5;
  i:=0;
 repeat
  i:=i+1;
  x1t:=a11*x1 + a12*x2;
  x2t:=a21*x1 + a22*x2;
  s:=1/(x1t + x2t);
  x1t:=s*x1t;
  x2t:=s*x2t;
  d:=abs(x1t-x1) + abs(x2t-x2);
  writeln('(',i:2,')',': p1 = ',x1t:9:6,'; p2 = ',x2t:9:6,'; s = ',s:9:6);
  x1:=x1t;
  x2:=x2t;
   IF KeyPressed then begin     (* To pause *)
    Read(Kbd,ch);
    if UpCase(ch)='Q' then halt;
    Read(Kbd,ch);
    if UpCase(ch)='Q' then halt;
    end;
 until (d < eps);
end.
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Reference

(1) Fujimoto, T. and R. R. Ranade(1991), "Okishio's Theorem Generalized with Joint Production and Externalities", mimeo, University of Okayama. (rev. 1993).