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The Effect of Inner Mobility of Shops on Tax Revenue

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1. Introduction

It is easily understood that when the number of customers to a sector of service industry decreases, the profits of the sector will in general also decrease, but this at a greater rate than that of the customers. This is because of fixed cost. Hence we will observe a severer effect on tax revenue.

Suppose, however, that in this service sector there are two types of shops, each offering distinct services at differential prices with fixed cost possibly being also unequal. And imagine the decrease of customers is experienced by one of these two types, not both. Then having less profits, those shops belonging to the damaged type may try, if feasible, to shift to the other type, thus alleviating losses on the whole. This can also mitigate the effect on tax revenue.

The authors have in mind the case of pubs and restaurants where two types are the 'first-rate (more expensive with more services)' and the 'popular' class. And in the days of recession, people tend to choose those

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from the latter, and less customers to gorgeous pubs.

In this note we present a simple model of a sector comprising two types of shops, and conduct an elementary comparative statics analysis. Most results are natural consequences, which can be inferred without using equations. And yet we supply proofs based upon equations and inequalities, which would hopefully be useful to students dealing with more complicated models.

2. Model

In our model sector, there exist two groups of shops called type A and B. Let us first write down their total group profits :

$$\pi_A = (p_A - v)Q_A - f_A N_A \ , \tag{1}$$

$$\pi_B = (p_B - v)(Q - Q_A) - f_B(N - N_A) \ . \tag{2}$$

Here notation is as follows :

π_i : total group profits of type $i\,(i = A\,, B)$,

p_i : price offered by type $i\,(i = A\,, B)$,

f_i : fixed cost per shop belonging to type $i\,(i = A\,, B)$,

v : cost per customer common to both types,

Q_i : number of customers to type $i\,(i = A\,, B)$,

Q : total number of customers to the sector $Q = Q_A + Q_B$,

N_i : number of shops belonging to $i\,(i = A\,, B)$,

N : total number of shops in the sector, $N = N_A + N_B$.

We assume that $p_A > p_B > v$ and $f_A > f_B$, and first suppose that p_A, p_B, f_A, f_B, Q_A, Q_B, v, and N are all positive constants, and determine N_A using the following equilibrium relation.

$$\frac{\pi_A}{N_A} = \frac{\pi_B}{N - N_A} \tag{3}$$

Eq. (3) simply requires that the average profits per shop are equal in both types. Substituting eqs. (1) and (2) into (3), we have

$$F(N_A) \equiv (f_A - f_B)N_A^2 - (f_A N - f_B N + (p_B - v)Q + (p_A - p_B)Q_A)N_A$$
$$+ (p_A - v)Q_A N = 0$$

This quadratic equation has a positive solution in the open interval $(0, N)$. This can be verified because $F(N) = -(p_B - v)Q_B N$, that is, negative.

3. Stability of Equilibrium

Let us formulate the adjustment equation in terms of a difference equation as

$$N_A(t+1) = N_A(t) + s \cdot \left(\frac{\pi_A(N_A(t))}{N_A(t)} - \frac{\pi_B(N_A(t))}{N - N_A(t)}\right),$$

where s is a given positive constant expressing the speed of adjustment. A sufficient condition for local stability is

$$-1 < \{1 - s \cdot \left(\frac{(p_A - v)Q_A}{N_A^2} + \frac{(p_B - v)(Q - Q_A)}{(N - N_A)^2}\right)\} < 1,$$

where N_A is the equilibrium value. This condition is clearly satisfied when the speed of adjustment s is small enough, and in fact gives a global stability because of the uniqueness of equilibrium, provided Q_i / N_i is bounded on and near the boundary (Fujimoto (1986)).

The variable N_A should take on only integer values, but the stability property will not be affected in a destructive way even if a certain mechanism is introduced to guarantee N_A be always an integer. In a higher dimension this may present an interesting problem.

4. Comparative Statics

Now suppose one of the parameters, Q_A, is increased. By a simple reasoning, we know N_A will also go up. This can be confirmed because

$$\frac{dN_A}{dQ_A} = -\frac{\partial F/\partial Q_A}{\partial F/\partial N_A}$$

$$= -\frac{-(p_A - p_B)N_A + (p_A - v)N}{2(f_A - f_B)N_A - (f_A N - f_B N + (p_B - v)Q + (p_A - p_B)Q_A)}$$

$$= \frac{(p_A - v)N_B + (p_B - v)N_A}{\{(p_A - v)Q_A - f_A N_A\} + \{(p_B - v)Q_B - f_B N_B\} + (f_A N_B + f_B N_A)} > 0. \quad (4)$$

In deriving this inequality, we assume that $\pi_A > 0$ and $\pi_B > 0$ at the equilibrium. On the $Q_A - N_A$ plane, N_A is an increasing function of Q_A, and is denoted as $N_A(Q_A)$.

Next we increase Q. Keeping Q_A fixed, i. e., increasing Q_B only, we obtain

$$\frac{dN_A}{dQ} = -\frac{\partial F/\partial Q}{\partial F/\partial N_A} = -\frac{-(p_B - v)N_A}{negative\ value} < 0.$$

Looked from another angle, this tells us that the function $N_A(Q_A)$ shifts downward when Q increases.

Then, N is increased. We have

$$1 > \frac{dN_A}{dN} = -\frac{\{(p_A - v)Q_A - f_A N_A\} + f_B N_A}{negative\ value} > 0.$$

The left-hand inequality follows because $-\partial F/\partial N_A$ consists of three positive terms (see the above eq. (4)) which subsume the numerator as a proper subset. Therefore, when N increases, both N_A and N_B will expand. This tells us that the function $N_A(Q_A)$ shifts upward when N increases.

The effect on the average profits per shop can be examined as follows.

$$\frac{d(\pi_A/N_A)}{dQ} = \frac{(p_A - v)Q_A(-dN_A/dQ)}{N_A^2} > 0 \text{ (with } Q_A \text{ fixed)};$$

$$\frac{d(\pi_A/N_A)}{dN} = \frac{(p_A - v)Q_A(-dN_A/dN)}{N_A^2} < 0 \text{ (with } Q_A \text{ fixed)}.$$

When the number of customers for type A is kept constant, the increase of total number of customers raises the average profits per shop of type A because the number of shops of type A shrinks, while the increase of total number of shops will lower the average profits per shop because the shops belonging to each type will also increase.

5. Effect on Profits

While N_A is unchanged, the elasticity of π_A with respect to Q_A is

$$\frac{Q_A}{\pi_A}\cdot\frac{d\pi_A}{dQ_A} = \frac{(p_A - v)Q_A}{(p_A - v)Q_A - f_A N_A} > 1 \ ,$$

and so the effect is enlarged on account of the existence of fixed cost. In our model, however, shops migrate to another group looking for higher profits, and thus making N_A greater. Now the elasticity becomes

$$\frac{Q_A}{\pi_A}\cdot\frac{d\pi_A}{dQ_A} = \frac{(p_A - v)Q_A - f_A Q_A(dN_A/dQ_A)}{(p_A - v)Q_A - f_A N_A} < (\geq)1 \ \text{ if } \frac{Q_A}{N_A}\cdot\frac{dN_A}{dQ_A} > (\leq)1,$$

that is, if the elasticity of N_A with respect to Q_A is greater than unity, the elasticity of π_A with respect to Q_A is smaller than unity. (We may here assume that the numerator remains positive.) Whatever is the size of the elasticity, it can be shown that

$$\left.\frac{Q_A}{\pi_A}\cdot\frac{d\pi_A}{dQ_A}\right|_{N_A = const} > \frac{Q_A}{\pi_A}\cdot\frac{d\pi_A}{dQ_A}.$$

This is a partial story as we neglect the profits by those shops of type B :

when Q_A and N_A get larger, Q_B and N_B become less. Hence let us check the sectoral total profits. With N_A unchanged,

$$\frac{Q_A}{\pi_A + \pi_B} \cdot \frac{d(\pi_A + \pi_B)}{dQ_A} = \frac{(p_A - v)Q_A - (p_B - v)Q_A}{\pi_A + \pi_B} = \frac{(p_A - p_B)Q_A}{\pi_A + \pi_B} > 0.$$

When migration between groups is allowed, we get

$$\frac{Q_A}{\pi_A + \pi_B} \cdot \frac{d(\pi_A + \pi_B)}{dQ_A} = \frac{(p_A - p_B)Q_A - (f_A - f_B)Q_A(dN_A/dQ_A)}{\pi_A + \pi_B}.$$

Again it follows

$$\frac{Q_A}{\pi_A + \pi_B} \cdot \left.\frac{d(\pi_A + \pi_B)}{dQ_A}\right|_{N_A = const} > \frac{Q_A}{\pi_A + \pi_B} \cdot \frac{d(\pi_A + \pi_B)}{dQ_A}.$$

This inequality asserts that when the customers shift from A group to B group, the size of decrease in the sectoral profits is smaller when migration is possible than when it is not. Hence the tax revenue has less effect with shiftability between two types.

So far, we have considered the case in which Q_A changes while Q is kept constant. Let us now examine the case where Q changes with Q_A fixed. In a similar manner we can obtain, with no migration,

$$\frac{Q}{\pi_A + \pi_B} \cdot \frac{d(\pi_A + \pi_B)}{dQ} = \frac{(p_B - v)Q}{\pi_A + \pi_B} > 0,$$

while we have, with migration allowed,

$$\frac{Q}{\pi_A + \pi_B} \cdot \frac{d(\pi_A + \pi_B)}{dQ} = \frac{(p_B - v)Q - (f_A - f_B)Q\,(dN_A/dQ)}{\pi_A + \pi_B} > 0.$$

In this case,

$$\frac{Q}{\pi_A + \pi_B} \cdot \left.\frac{d(\pi_A + \pi_B)}{dQ}\right|_{N_A = const} < \frac{Q}{\pi_A + \pi_B} \cdot \frac{d(\pi_A + \pi_B)}{dQ}.$$

If people move out of the shops of popular class and do not return to the sector, the shiftability of shops between two classes exacerbates the situation for the inland tax revenue.

When we are to consider hard times of slump, customers are likely to leave the shops belonging to the 'gorgeous' class rather than the popular class, and so we had better take up the case where Q_A decreases while Q_B remains unchanged. This is symmetrical to the case considered just in the above. All we have to do is to assume $f_A < f_B$ instead of $f_A > f_B$. Therefore the result is

$$\frac{Q}{\pi_A + \pi_B} \cdot \frac{d(\pi_A + \pi_B)}{dQ}\bigg|_{N_A = const} > \frac{Q}{\pi_A + \pi_B} \cdot \frac{d(\pi_A + \pi_B)}{dQ}$$

Thanks to shiftability, the effect on tax revenue is milder when some people stop using the first-rate shops in recession.

6. Remarks

We have considered a simple model of a service sector in which there are two groups of firms with different revenue and cost structures. It is assumed that a firm or a shop can shift from one group to the other without any cost. This implies a shop does not need much space to render its services, and the contents of services do not require much fixed equipment. This is the case with pubs. And the two groups are the 'first–rate' class and the 'popular' class.

Our main result is that by shiftability between two groups, the sector may incur less loss against the decrease in customers, especially that in the first-rate class. This means the effect on tax revenue is also milder with shiftability.

Our model in this note is quite restrictive because the size of customers is exogenously given and does not respond to price differentials. Moreover in the analysis of comparative statics, the number of shops is fixed with no entry and no bankruptcy. These constraints should be removed in a more general framework.

Reference

Fujimoto, T. (1986). "Global Asymptotic Stability of Nonlinear Difference Equations", *Economics Letters* 22, pp. 247–250.

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The typical sector which the authors have in mind is that of pubs and restaurants where two types are the 'first-rate (more expensive with more services)' and the 'popular' class.

Based on this model, we conduct an elementary comparative statics analysis. Most results are natural consequences, which can be inferred without using equations. And yet we supply proofs based upon equations and inequalities.

Our main result is that with migration between two groups, the sector may incur less loss against the decrease in customers, especially that in the first-rate class. This means the effect on tax revenue is also milder with shiftability.

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