Cooperative and Noncooperative R&D in Cournot and Bertrand Duopolies with Spillovers, and Their Comparison

Shoji Haruna

1. Introduction

In a lot of literature roles that strategic methods such as research and development (R&D) investment, capacity, and advertising play have been considered, and the interesting result that an overinvestment (or underinvestment) is made in R&D in the nonsimultaneous decisions of output (or price) and such a strategic method had been derived by several papers.

d’Aspremont and Jacquemin (1988) and Kamien et al. (1992) use strategic two-stage game models with R&D investment and compare the R&D investments and outputs under several organizational modes based on such models. Kamien et al. especially extend the d’Aspremont and Jacquemin model to a more general model with product differentiation and $n (> 2)$ firms. They take four modes, say R&D competition, R&D cartelization, RJV competition, and RJV cartelization, with respect to R&D investment decision, where RJV abbreviates research joint venture, and rank among strategic R&D investments and prices (outputs) under them in the Cournot quantity-setting model\(^1\). Moreover, they perform the classification only as to R&D investment in a Bertrand price-setting model, however they do not make a comparison between R&D investments and prices in the Cournot quantity-setting and Bertrand price-setting models at all. Our main purpose is to classify the strategic R&D investments in each of both models and then to compare reciprocally the R&D investments and the prices (outputs) in the models. When firms perform strategic R&D activities, and their spillovers arise, we address the validity of the conventional outcome that prices (outputs) are lower (larger) in Bertrand–price competition than in Cournot–quantity one.

Our analysis is distinguished from those of d’Aspremont and Jacquemin (1988) and Kamien et al. (1992) by the following points. First, their discussions are all or for the most part spent to investigate classifications as to R&D investment and price in Cournot–quantity competition. But we assign our discussion equally to investigation of such classifications in both Cournot–quantity and Bertrand–price competitions. Second, our discussion is extended to the case in which outputs are complements (substitutes) in Cournot–quantity (Bertrand–price) competition.

\(^1\) Kamien et al. (1992) make reference to an RJV in detail.
Secondly, we compare the strategic R&D investments and the prices (outputs) in a quantity-setting model with those in a price-setting model. Then the relationship among firms’ R&D behavior, and demand and competition characteristics is considered. Although the expenditure of R&D in both R&D competition and R&D cartelization is larger in Cournot-quantity competition than in Bertrand-price competition, the conventional result that output (price) is lower (higher) in the former competition than in the latter competition carries over to the case with strategic R&D investment and its spillover effect. It, therefore, follows that welfare is greater in Bertrand-price competition than in Cournot-quantity competition. These results show that commitment by R&D and the spillover effect do not have a strong influence to such an extent that they overthrow the conventional result. Consumer’s surplus is maximized (minimized) in the case of RJV cartelization (R&D competition) in Bertrand-price (Cournot-quantity) competition.

The paper is organized as follows. In Section 2 we rank each of the R&D investments and the prices (outputs) obtained under several behavioral modes as to R&D choice in Cournot-quantity competition, as in both Kamien et al. (1992) and d’Aspremont and Jacquemin (1988). In Section 3 the same classification as in the previous section is made by replacing Cournot-quantity competition with Bertrand-price competition. This is the counterpart of the analysis in Section 2. Section 4 compares the R&D investments and the prices (outputs) in the quantity-setting model with those in the price-setting model. Section 5 concludes.

2. The Cournot-quantity competition

Consider a duopoly of firms 1 and 2 producing differentiated goods $q_i, i = 1, 2$. Their inverse demand functions take the form of

$$p_1 = a - bq_1 - dq_2$$
$$p_2 = a - dq_1 - bq_2,$$

where $p_i$ stands for the price of good $i$, and the parameters are $a > 0$ and $b > 0$. The goods are substitutes, independent, or complements according as $d$ is positive, zero, or negative. We assume $b \geq d$ and $d > 0$ or $d < 0$. Since the case with $d = 0$ corresponds to monopoly, we exclude it.

Firm $i$ produces its output with both constant marginal cost, $c > 0$, and zero fixed cost, and also invests in R&D to reduce its own per-unit costs prior to its output choice. If it intends to lower the per-unit costs by $x_i$, then it must spend $c^I(x_i)$ as R&D expenditure. Now expenditure function $c^I(x_i)$ of R&D investment is assumed to be a convex function of $x_i$, $dc^I(x_i)/dx_i = c^{I'}(x_i) > 0, c^{I'}(x_i) > 0$, and $c^I(0) = 0$: namely, the rate of returns to R&D investment is diminishing.

We suppose that there are spillovers between the firms in the industry concerning the outcome of investment, because it will be impossible for each firm to perfectly appropriate its own technology and skills for cost reduction acquired through its R&D activities even if it takes out patents for them. No small amount of them...
will be involuntarily transferred to their rival. Thus, if firm \( j \) succeeds in reducing marginal cost by \( x_j \), then its transfer enables the rival to lower its marginal cost by \( \beta x_j \), where \( \beta > 0 \) refers to the spillover rate of the latter, i.e., “the free-ride effect” (Kamien et al., 1992). For example, \( \beta = 0 \) implies that firm \( j \) perfectly appropriates the outcome of R&D investment, while \( \beta = 1 \) corresponds to either the case in which a research joint venture (RJV) between the firms is formed so as to internalize the spillover effects or the case in which all of the outcome perfectly flows out from each firm. The formation of the RJV has two advantages to its participants: one is that they can share mutually their information about R&D activities, and the second is that they can eliminate the duplication of their R&D activities (internalization of the externality, so called) (Kamien et al.)'. Except for the RJV formation, an example with \( \beta = 1 \) is the development of financial and insurance commodities. With respect to these commodities, not only a financial institution (e.g. one insurance company), which developed them, but also other institutions (insurance companies) have been allowed to freely sell them to customers. Firm \( i \)'s marginal cost is thus reduced by \( x_i \) in all if it and its rival can reduce their marginal costs as a result of R&D investments, \( x_i \) and \( x_j \), respectively. It is assumed that \( c > x_i + \beta x_j \), \( i, j = 1, 2, i \neq j \).

We use a two-stage game model like d’Aspremont and Jacquemin (1988) and Kamien et al. (1992), in which each of the firms determines its investment level in the first stage and then chooses its output in the second stage, given \((x_1, x_2)\). In the second stage each firm chooses output so as to maximize its production profits net of its first-stage R&D expenditures:

\[
\pi_i = [p_i - (c - x_i - \beta x_j)]q_i - c^i(x_i) = (p_i - A_i)q_i - c^i(x_i), \quad i \neq j;
\]

where \( A_i = c - x_i - \beta x_j > 0 \). For the following discussion it is assumed that \( a > A_i \). Then the first-order conditions for maximization are given by\(^6\)

\[
\frac{\partial \pi_i}{\partial q_i} = a - A_i - 2bq_i - dq_j = 0, \quad i \neq j.
\]

Note that the reaction curves in ouptut space slope downwardly (upwardly) when \( d \) is positive (negative), so that outputs are substitutes (complements) as \( d > (\leq) 0 \). Solving these equations, we obtain Cournot–Nash equilibrium outputs \((q_1^C, q_2^C)\) such as

---

2 Goto and Nagata (1997) conclude that spillovers of technology are fairly high in Japanese companies because their appropriability is low.

3 According to Goto and Nagata (1997), main resources for firms to acquire information on the R&D and innovation activities of a rival are due to patents, publications, and informal conversations in Japan and the U. S.

4 As another advantage of its formation, it is asserted that economies of scale concerning R&D investment are yielded.

5 Our treatment of R&D investment is the same as d’Aspremont and Jacquemin (1988), but a little different from that of Kamien et al. (1992).

6 The second-order conditions are satisfied, and the second-stage equilibrium is locally stable.
\[ q_i^C = \frac{2b(a - A_i) - d(a - A_i)}{4b^2 - d^2}, \quad i \neq j, \]  

(4)

where superscript C denotes output in the Cournot–quantity competition. The effects of \( x_i \) and \( x_j \) on output are derived as follows:

\[
\frac{\partial q_i^C}{\partial x_i} = \frac{2b - d \beta}{4b^2 - d^2} > 0 \quad \text{for any } \beta \text{ and } d
\]

\[
\frac{\partial q_j^C}{\partial x_j} = \frac{2b \beta - d}{4b^2 - d^2} \geq 0 \quad \text{as } \beta \leq \frac{d}{2b} \quad \text{for } d > 0, \quad \text{and} \quad \frac{\partial q_i^C}{\partial x_j} > 0 \quad \text{for } d < 0, \quad i \neq j.
\]

Let us turn to the decision of R&D investment in the first stage. Differentiating (2) with respect to \( x_i \) and using (3), we have the first-order conditions with respect to R&D:

\[
\frac{\partial \pi_i}{\partial x_i} = \frac{\partial \pi_j}{\partial q_j} \frac{\partial q_j^C}{\partial x_i} + q_i^c - c^b(x_i) = 0, \quad i \neq j,
\]

(5)

where \( \frac{\partial \pi_j}{\partial q_j} = -d q_j^c \) for \( d (>) 0 \). From (5) we obtain the Cournot–Nash equilibrium R&D investments, \((x_1^{\text{CNC}}, x_2^{\text{CNC}})\), where NC of superscript CNC denotes variables in R&D competition.\(^7\) Firm i’s R&D reaction curve is upward-sloping. For the following discussion it is required that the game has a subgame-perfect Cournot–Nash equilibrium, \([(x_1^{\text{CNC}}, x_2^{\text{CNC}}), (q_1^{\text{CNC}}, q_2^{\text{CNC}})]\) where \( q_i^{\text{CNC}} = q_i(x_1^{\text{CNC}}, x_2^{\text{CNC}}), i = 1, 2, \) and is also assumed that the first-stage equilibrium is locally stable.

It has been assumed in the above discussion that each firm noncooperatively determines R&D investment. We next consider the case in which both firms cooperatively behave in the R&D decision, namely, form an R&D cartel, and choose the levels of R&D investments so as to maximize the sum of combined profits, maintaining quantity competition in the product markets. In the first stage of R&D choice each firm internalizes its rival’s behavior like the case of a merger by forming an R&D cartel, while in the second stage of quantity choice they keep competition. It is assumed for the time being that each firm attempts to appropriate performances acquired by R&D investment without exchanging them with each other. The maximization problem of firm \( i \) joining in the R&D cartel is expressed as

\[
\max_{x_i} \pi = \pi_i + \pi_j, \quad i \neq j.
\]

Then the first-order condition for firm \( i \) is given by\(^8\)

---

\(^7\) The second-order conditions require that \( \frac{\partial^2 \pi_i}{\partial x_i^2} = (2b - d \beta)^2 \frac{1}{(4b^2 - d^2)} c^b(x_i) < 0.\)

\(^8\) It is assumed that the second-order conditions are satisfied.
\[
\frac{\partial \pi_i}{\partial x_i} + \frac{2b (2b \beta - d) q_j^C}{4b^2 - d^2} = 0, \quad i \neq j. \tag{6}
\]

The second term on the right-hand side of this condition is the combined profit externality given by firm \(i\)'s R&D investment on the profit of the rival. Whether or not this externality is positive depends on the magnitude of \(\beta\) and \(d\). Incidentally, the optimal conditions in the second stage do not change as quantity competition is kept.

Since preparations for comparison have completed, we compare R&D investments in such two modes as R&D cartelization (cooperation) (Case CC), and R&D competition (Case CNC), where first symbol C of each case stands for Cournot competition, and NC and C after its C stand for R&D competition and its cartelization, respectively. Let us introduce a research joint venture (RJV). RJV competition (case NJ) and RJV cartelization (case CJ) of Kamien et al. (1992) correspond to our cases CNC with \(\beta = 1\) (say case CNCJ) and CC with it (say case CCJ), respectively, where RJV stands for research joint venture. Then we obtain the following proposition:

**Proposition 1.** (a) Suppose that outputs are substitutes, i.e. \(d > 0\). Then the equilibrium R&D investments, \(x^k\), \(k = \text{CNC, CNCJ, CC and CCJ}\), satisfy

\[
x^\text{CNC} > x^\text{CC} \text{ if } 0 \leq \beta < d/2b; \quad x^\text{CNC} \leq x^\text{CC} \text{ if } d/2b \leq \beta \leq 1 \text{ (equity if } \beta = d/2b) ;
\]

and \(x^\text{CNC} \geq x^\text{CNCJ} \text{ if } 2b/d - 2 \leq \beta \leq 1\); and \(x^\text{CC} \leq x^\text{CCJ}\).

(b) Suppose that outputs are complements, i.e. \(d < 0\). Then the equilibrium R&D investments \(x^k\) also satisfy

\[
x^\text{CNC} < x^\text{CC} ; \quad x^\text{CNC} \leq x^\text{CNCJ} \text{ and } x^\text{CC} \leq x^\text{CCJ} .
\]

Proof. First, consider the case with \(d > 0\). Since \(\partial \pi_j / \partial x_i\) is negative, zero, or positive according as \(0 \leq \beta < d/2b\), \(\beta = d/2b\), or \(d/2b < \beta \leq 1\), \(\partial \pi_i / \partial x_i\) is less than, equal to, or larger than \(\partial \pi_i / \partial x_i\), respectively. Combining these results and (6), we obtain that \(x^\text{CNC} > x^\text{CC} \text{ if } 0 \leq \beta < d/2b \text{ and } x^\text{CNC} \leq x^\text{CC} \text{ if } d/2b \leq \beta \leq 1 \text{ (equity holding if } \beta = d/2b)\).

The first-order conditions for both R&D and RJV competitions are given from (5) as follows, respectively:

\[
\left[ \frac{a - c + (1 + \beta)x}{2b + d} \right] \frac{(4b^2 - 2bd \beta)}{(4b^2 - d^2)} = c^x(x) \tag{5}'
\]

\[
\left[ \frac{a - c + 2x}{2b + d} \right] \frac{(4b^2 - 2bd)}{(4b^2 - d^2)} = c^x(x). \tag{5}''
\]

9 When expenditure function \(c^1(x)\) is more (less) convex, the amount of R&D investment is larger (smaller) in RJV competition than in R&D competition, i.e. \(x^\text{CNC} \leq (> ) x^\text{CNCJ}\).
Making a comparison of the left-hand sides of both equations yields

\[
\frac{(a - c + (1 + \beta)x)(4b^2 - 2bd\beta)}{(2b + d)(4b^2 - d^2)} > \frac{(a - c + 2x)(4b^2 - 2bd)}{(2b + d)(4b^2 - d^2)} \quad \text{if} \quad \frac{2b}{d} - 2 \leq \beta \leq 1.
\]

Then we obtain \(x^\text{CNC} \geq x^\text{CNCJ}\) whenever \(2b/d - 2 \leq \beta \leq 1\). Otherwise, this may or may not hold. Similarly, the first-order conditions for R&D and RJV cartelizations are also given from (6) as follows, respectively:

\[
\frac{2b(1 + \beta)[a - c + (1 + \beta)x]}{2b + d} = c^\nu(x) \quad (6)'
\]

\[
\frac{4b(a - c) + 8bx}{2b + d} = c^\nu(x) \quad (6)''
\]

As the left-hand side of (6)'' is always larger than that of (6)', \(x^\text{CC} \leq x^\text{CCJ}\) holds.

Second, consider the case with \(d < 0\). The second term on the right-hand side of (6), \(\partial \tau_j / \partial x_i\), is positive, irrespective of \(\beta\), so that \(x^\text{CNC} < x^\text{CC}\) holds. Compare the investments in R&D and RJV competitions. Then we obtain \(x^\text{CNC} \leq x^\text{CNCJ}\) since the left-hand side of (6)' is smaller than that of (6)'' for any \(\beta\). Meanwhile, as for the comparison of the investment levels under R&D and RJV cartelizations the same result as in the case with \(d > 0\) is obtained.

Given \(d > 0\), the result concerning RJV competition is unambiguously different from the result of Kamien et al. (1992) that the amount of R&D investment is always larger in R&D competition than in RJV competition. However, as shown in the proposition, whether R&D investment in R&D competition exceeds that in RJV competition depends on a spillover rate. The intuitive explanation for this is given as follows. Given \(\beta \geq 2b/d - 2\), the marginal profitability (revenue) of R&D investment decreases as spillover increases, and its curve goes down to the lowest level in the case of RJV competition. This is because each firm free rides on its rival’s R&D investment. Hence R&D investment is minimized in the RJV cartel. On the other hand, given \(\beta < 2b/d - 2\), the comparison of investments under CNC and CNCJ is impossible, because the marginal profitability curve is not much shifted upwardly by an increase in the spillover rate as the free-rider effect is not great. Figure 1 illustrates the relationship between the marginal profitability curves of R&D investment in R&D and RJV competitions when \(d\) is positive. In short, the difference between our and Kamien et al.’s results seems to result in a difference between the assumptions concerning the R&D production functions (or the R&D expenditure functions). Note that the product differentiation between goods 1 and 2 has a strong effect on the relationship between R&D investments in R&D competition and RJV competition. Namely, \(4/9 \leq d^2/b^2 \leq 1\) must hold in order for \(b\) and \(d\) to satisfy both \(2b/d - 2 \leq \beta \leq 1\) and \(b \geq d\). For example, if \(b = d\), then R&D investment is larger in R&D competition than in RJV competition for any spillover rate, but if \(b = 1.5d\), then this holds if and only if \(\beta = 1\). These imply that the range of spillover for which R&D investment is larger in R
&D competition than in RJV competition reduces as product differentiation makes progress: there is no spillover such that $x^{\text{CNC}} > x^{\text{CNCJ}}$ as long as $0 < d^2/b^2 < 4/9 \approx 0.444$. Furthermore, when we make a comparison between the R&D investments under $d > 0$ and $d < 0$, its level is reduced to be less in the former than in the latter in the two cases of R&D and RJV competitions.

Given $d > 0$, the result concerning the comparison of the R&D investments under both R&D and RJV cartelizations is the same as Kamien et al. As spillover rises, costs are curtailed and outputs increase, so that the marginal profitability curve is shifted upwardly. In particular, the amount of R&D investment is maximized in the RJV cartel. This is because the free-rider effect is eliminated in the R&D cartel in that its participants coordinate their R&D investments, as explained by Kamien et al. In the case with $d < 0$, the same result is obtained. We note that both R&D investments under R&D and RJV cartelizations are larger in the case with $d < 0$ than in the case with $d > 0$, respectively, as the outputs are complements in the former case.

Let us turn to the comparisons of the equilibrium prices in the four models. Then the corollary to Proposition 1 is derived as follows.

**Corollary 1.** The equilibrium prices, $p^k$, $k = \text{CNC, CNCJ, CC, and CCJ}$, satisfy the following classifications:

(a) given $d > 0$, $p^{\text{CNC}} < p^{\text{CC}}$ if $0 \leq \beta < d/2b$, and $p^{\text{CNC}} \geq p^{\text{CC}}$ if $d/2b \leq \beta \leq 1$ (equity if $\beta = d/2b$), and $p^{\text{CNC}} \leq p^{\text{CNCJ}}$ if $2b/d - 2 \leq \beta \leq 1$, and $p^{\text{CC}} \geq p^{\text{CCJ}}$; and (b) given $d < 0$, $p^{\text{CNC}} > p^{\text{CC}}$, $p^{\text{CNC}} \geq p^{\text{CNCJ}}$, and $p^{\text{CC}} \geq p^{\text{CCJ}}$.

Proof. Outputs are an increasing function of R&D investment. Consider the case with $d > 0$. From Proposition 1 we immediately get $q^{\text{CNC}} > q^{\text{CC}}$ if $0 \leq \beta < d/2b$, and $q^{\text{CNC}} \leq q^{\text{CC}}$ if $d/2b \leq \beta \leq 1$ (equity if $\beta = d/2b$). We also get both $q^{\text{CNC}} \geq q^{\text{CNCJ}}$ if $2b/d - 2 \leq \beta \leq 1$ and $q^{\text{CC}} \leq q^{\text{CCJ}}$ from the comparison of the outputs under CNC and CNCJ, and CC and CCJ. In the case with $d < 0$, it follows from Proposition 1 that $q^{\text{CNC}} < q^{\text{CC}}$, $q^{\text{CNC}} \leq q^{\text{CNCJ}}$, and $q^{\text{CC}} \leq q^{\text{CCJ}}$. Now the following relationship about output and price is derived from the inverse demand functions:
where $X$ and $XX$ denote CC and CNC. By the assumption that the firms are symmetric, the classifications on the prices are the reverse of those on the outputs.

Conventionally, it is well known that less output and high price are lead by the formation of an output cartel whenever outputs are substitutes and there is no spillover. Even if firms form an R&D investment cartel instead of the output cartel, the similar conclusion to the conventional one is derived as long as spillover is small. However, when spillover is large, this is not the case. The reason why the different result from the conventional one is derived is due to the positive externality. Specifically, when the spillover effect is large, firms in the R&D cartel intend to invest in (produce) more R&D (output) than in R&D competition since the R&D expenditure of one firm leads to an increase in the combined profits.

3. The Bertrand–price competition

There is the duality structure between the demand functions in quantity-setting models and in price-setting models (Sonnenschein, 1968). So when applying this duality to (1), we have

$$q_1 = \alpha - \delta p_1 + \gamma p_2 \tag{1}$$

$$q_2 = \alpha + \gamma p_1 - \delta p_2,$$

where $\alpha = (ab - bd)/(b^2 - d^2) > 0$, $\delta = b/(b^2 - d^2) > 0$, and $\gamma = d/(b^2 - d^2)$. It is assumed that $p_i > A_i$, $i = 1, 2$.

In the strategic two-stage game each firm chooses its level of R&D investment in the first stage and then its price in the second stage. The first-order conditions for maximization in the second stage are obtained by differentiating (2) with respect to $p_i$ and using (1):

$$\frac{\partial \pi_i}{\partial p_i} = \alpha + \delta A_i - 2 \delta p_j + \gamma p_j = 0, \ i \neq j. \tag{7}$$

This shows that the reaction curve of each firm slopes upwardly (downwardly) in price space when $d > (<) 0$. Bertrand competition with complements is the dual of Cournot competition with substitutes. From (7) Bertrand–Nash equilibrium prices in the second stage are lead as

$$p_i^B = \frac{2\delta(\alpha + \delta A_i) + \gamma(\alpha + \delta A_j)}{4\delta^2 - \gamma^2}, \ i \neq j, \tag{8}$$

where superscript $B$ stands for variables in Bertrand–price competition of the second stage. Then the effects of
x_i and x_j on prices are also lead as
\[ \frac{\partial x_i}{\partial x_i} = -\frac{b(2b + d\beta)}{4b^2 - d^2} < 0 \quad \text{and} \quad \frac{\partial x_i}{\partial x_j} = -\frac{b(2b + d\beta)}{4b^2 - d^2} < 0, \quad i \neq j. \] (9)

Incidentally, the output corresponding to price p_i^B is given as
\[ q_i^B = \frac{b[(2b^2 - d^2)(a - A_i) - bd(a - A_j)]}{(4b^2 - d^2)(b^2 - d^2)}, \quad i \neq j. \]

In the first stage the first–order condition for firm i is given by
\[ \frac{\partial \pi_i}{\partial x_i} = \frac{\partial \pi_i}{\partial p_i^B} \frac{\partial p_i^B}{\partial x_i} + q_i^B - c^v(x_i) = 0, \] (10)

where \( \frac{\partial \pi_i}{\partial p_i^B} = \gamma(p_i^B - A_i) = d(p_i^B - A_i)/(b^2 - d^2) > 0. \) The second–order condition requires \( \frac{\partial^2 \pi_i}{\partial x_i^2} < 0. \) When the game has a subgame–perfect Bertrand–Nash equilibrium, \( (\lambda_i^{BNC}, \lambda_j^{BNC}, (p_i^{BNC}, p_j^{BNC})) \), where \( p_i^{BNC} = p_i(x_i^{BNC}, x_j^{BNC}), \) \( i = 1, 2, \) there are the strategic equilibrium R&D investments, \( (\lambda_i^{BNC}, \lambda_j^{BNC}) \), where NC of superscript BNC denotes variables in R&D competition of the first stage. The first term on the right–hand side of (10) is the strategic term. As mentioned in the previous section, firms involving R&D competition strive to appropriate the outcome of R&D investment except for the formation of an RJV.

Consider R&D investment under R&D cartelization (cooperation). The two firms form an R&D cartel and determine R&D investments so as to maximize the sum of combined profits, \( \pi = \pi_i + \pi_j, i \neq j, \) but noncooperatively determining prices in the second stage. Then the first–order condition for firm i in the R&D cartel is\(^{11}\)
\[ \frac{\partial \pi}{\partial x_i} = \frac{\partial \pi_i}{\partial x_i} + \frac{2[(2b^2 - d^2)\beta - bd]}{4b^2 - d^2} q_i^B = 0, \quad i \neq j. \] (11)

When the R&D investments satisfy this equation, the strategic R&D cartel equilibrium, \( (x_i^{BC}, x_j^{BC}) \), is given. The second term, \( \frac{\partial \pi_i}{\partial x_i}, i \neq j, \) of (11) is the externality that R&D investment of firm i confers its rival’ profits, but this is internalized for the former by maximizing the combined profits.

When \( \beta = 1, \) R&D competition and R&D cartelization are converted to RJV competition (BNCJ) and RJV cartelization (BCJ), as in Kamien et. al (1992). Compare the R&D investments in both R&D competition, R&D cartelization, RJV competition, and RJV cartelization. As the counterpart of Proposition 1 the following

---

10 The reaction curves in R&D investment space have a positive slope. The stability condition in the first stage is \[ [\partial^2 \pi_i/\partial x_i^2] > 0, \quad \text{and} \quad \frac{\partial^2 \pi_j/\partial x_i \partial x_j}{\partial x_i} > 0, \quad i \neq j. \] As \( \frac{\partial^2 \pi_i/\partial x_i^2}{\partial x_i} < 0 \) and \( \frac{\partial^2 \pi_j/\partial x_i \partial x_j}{\partial x_i} > 0, \) the condition is finally reduced to \( 1 + \beta(2b^2 - bd)\beta - d^2 < (2b^2 - d^2)(b + d)e^v(x_i), \) if \( e^v(x_i) = 0, \) then it is not explicitly satisfied. For the equilibrium to be stable, \( e^v(x_i) \) must be at least negative.

11 The second–order conditions are assumed to be satisfied.
Proposition 2. (a) Suppose that outputs are complements, i.e. \( d > 0 \). Then the equilibrium R&D investments, \( x^k, k = \text{BNC, BNCJ, BC}, \) and \( \text{BCJ} \), satisfy
\[
x^{\text{BNC}} > x^{\text{BC}} \quad \text{if} \quad 0 \leq \beta < \frac{bd}{(2b^2 - d^2)}; \quad x^{\text{BNC}} \leq x^{\text{BC}} \quad \text{if} \quad \frac{bd}{(2b^2 - d^2)} \leq \beta \leq 1 \quad \text{(equity holding if} \ \beta = \frac{bd}{(2b^2 - d^2)}; \quad x^{\text{BNC}} \geq x^{\text{BNCJ}} \quad \text{if} \quad \frac{(2b^2 - d^2)}{bd} - 2 \leq \beta \leq 1; \) and \( x^{\text{BC}} \leq x^{\text{BCJ}} \).
\]
(b) Suppose that outputs are substitutes, i.e. \( d < 0 \). Then those R&D investments satisfy
\[
x^{\text{BNC}} < x^{\text{BC}}, \quad x^{\text{BNC}} \leq x^{\text{BNCJ}} \quad \text{and} \quad x^{\text{BC}} \leq x^{\text{BCJ}}.
\]
Proof. First, let us take the case with \( d > 0 \). When combining (10) and (11), we obtain that \( x^{\text{BNC}} > x^{\text{BC}} \) if \( 0 \leq \beta < \frac{bd}{(2b^2 - d^2)} \) and \( x^{\text{BNC}} \leq x^{\text{BC}} \) if \( \frac{bd}{(2b^2 - d^2)} \leq \beta \leq 1 \) that \( \partial \pi_i / \partial x_i < \partial \pi_i / \partial x_i \) if \( 0 \leq \beta < \frac{bd}{(2b^2 - d^2)} \), and \( \partial \pi_i / \partial x_i \geq \partial \pi_i / \partial x_i \) if \( \frac{bd}{(2b^2 - d^2)} \leq \beta \leq 1 \) (equity if \( \beta = \frac{bd}{(2b^2 - d^2)} \)).

The first–order conditions for BNC and BNCJ are obtained from (10) as follows, respectively:
\[
\frac{2b(2b^2 - d^2 - bd\beta)(a - c + (1 + \beta)x_i)}{(b + d)(2b - d)(4b^2 - d^2)} = c^\nu(x);
\]
\[
\frac{2b(b - d)(a - c + 4bx)}{(b + d)(2b - d)^2} = c^\nu(x).
\]
Making a comparison of the left–hand sides of these equations yields
\[
\frac{2b(2b^2 - d^2 - bd\beta)(a - c + (1 + \beta)x_i)}{(b + d)(2b - d)(4b^2 - d^2)} > \frac{2b(b - d)(a - c + 4bx)}{(b + d)(2b - d)^2} \quad \text{if} \quad \frac{2b^2 - d^2}{bd} - 2 \leq \beta \leq 1.
\]
It now follows that \( x^{\text{BNC}} \geq x^{\text{BNCJ}} \) if \( \frac{(2b^2 - d^2)}{bd} - 2 \leq \beta \leq 1 \).

Similarly, the following first–order conditions for BC and BCJ are obtained from (11):
\[
\frac{2b(b - d)(1 + \beta)(a - c + (1 + \beta)x_i)}{(b + d)(2b - d)^2} = c^\nu(x);
\]
\[
\frac{4b(b - d)(a - c + 2x)}{(b + d)(2b - d)^2} = c^\nu(x).
\]
Since the left–hand side of (11)’ is obviously greater than that of (11)”, we have \( x^{\text{BC}} \leq x^{\text{BCJ}} \) for any \( \beta \). This result holds for the case with \( d < 0 \) as well.

\[1\] Poyago–Theotoky (1999) shows that each of two firms has an incentive to disclose its R&D knowledge to the partner when spillover is considered to be as an exogenous, not endogenous, variable. This provides a theoretical foundation for why firms intend to form an R&D cooperation.
Interestingly, the relationship between $x^{BC}$ and $x^{BCJ}$ holds independent of spillover. As for the comparison between the R&D investments in R&D and RJV competitions we note that, given $d < 0$, R&D investment is larger in R&D competition than in RJV competition for any spillover. On the other hand, given $d > 0$, the range of $\beta$ for which $x^{BNC} \geq x^{BNCJ}$ holds reduces as the product differentiation between them makes progress. In fact, there is no $\beta$ such that the inequality holds as long as $b > (3 + \sqrt{17})d / 4$, i.e., $0 < d^2 / b^2 \leq 8 / (13 + \sqrt{17}) \approx 0.467$.

Since price $p^B$ is a decreasing function of R&D investment, we immediately can obtain from the following corollary.

**Corollary 2.** (a) Given $d > 0$, the prices, $p^k$, $k = BNC, BNCJ, BC,$ and $BCJ$, satisfy $p^{BNC} < p^{BC}$ if $0 \leq \beta < bd / (2b^2 - d^2)$; $p^{BNC} \geq p^{BC}$ if $bd / (2b^2 - d^2) < \beta \leq 1$ (equity holding if $\beta = bd / (2b^2 - d^2)$); and $p^{BNC} \leq p^{BNCJ}$ if $(2b^2 - d^2) / bd - 2 \leq \beta \leq 1$, and $p^{BC} \geq p^{BCL}$.
(b) Given $d < 0$, those prices satisfy $p^{BNC} > p^{BC}, p^{BNC} \geq p^{BNCJ},$ and $p^{BC} \geq p^{BCJ}$.

This corollary is the counterpart of Corollary 1. From Corollaries 1 and 2 we point out that the classification between the prices under R&D cartelization and R&D competition in Bertrand–price competition is similar to that in Cournot–quantity competition, as suggested by Kamien et al. (1992), so their suggestion is verified\(^\text{13}\). However, we state that there are some differences between our and their classifications on detail points.

Note that price under BCJ (BNCJ) is the lowest (highest) among the four modes, BNC, BNCJ, BC and BCJ, if spillovers are large. We, moreover, find that the formation of an RJV has opposite effects on consumer’s surplus by whether firms operate in either R&D competition or R&D cartelization.

The results obtained in Cournot quantity–setting and Bertrand price–setting models are almost the same, but there still exists some difference between them. To put it concretely, the range of $\beta$ such that $x^{CNC} \leq x^{CC}$ holds is greater than the one such that $x^{BNC} \leq x^{BC}$ holds, while its range such that $x^{CNC} \geq x^{CNCJ}$ is smaller than the one such that $x^{BNC} \geq x^{BNCJ}$ holds.

From the comparisons of the results in both the quantity–setting and price–setting models we see that both classifications on output are hardly affected by whether the mode of competition in the product markets is *a la* Cournot or *a la* Bertrand. We conclude that a difference in strategy in product markets does not have a great effect on their R&D behavior.

As in Cournot–quantity competition, the existence of precommitment such as R&D investment and advertising is beneficial to consumers if spillover effects are small, while it is detrimental if large. In the presence of large spillover rates there also exists a strong incentive for firms to make either R&D investment or advertising before the choice of quantity. This shows that they have a ground for making such precommitment.

\(^{13}\) Their suggestion is limited to the cases in which spillover rates are relatively large, so does not refer to all cases.
4. Comparison of the Cournot–quantity and the Bertrand–price competition models

It is commonly recognized that output (price) is less (higher) in the quantity competition than in the price competition. However, it seems that there are no papers which investigate whether or not this conventional outcome is valid in the case where both R&D investment and its spillover exist. So let us investigate it. By utilizing the results in Sections 2 and 3, we compare the market performances in both Cournot–quantity and Bertrand–price competitions.

Thus we can conduct the comparison of the R&D investment revels in the Cournot quantity-setting and in the Bertrand price-setting models as follows:

**Proposition 3.** (i) Suppose that two firms engage in R&D competition in the first stage. Then, (a) if \( d > 0 \), the equilibrium R&D investment is larger in Cournot–quantity competition than in Bertrand–price competition, i.e., \( x_{\text{CNC}} > x_{\text{BNC}} \); and (b) if \( d < 0 \), the reverse result holds, i.e., \( x_{\text{CNC}} < x_{\text{BNC}} \). (ii) Suppose that they form an R&D cartel in the first stage. Then, (a) if \( d > 0 \), the equilibrium R&D investment is larger in Cournot–quantity competition than in Bertrand–price competition, i.e., \( x_{\text{CC}} > x_{\text{BC}} \); and (b) if \( d < 0 \), the reverse result holds, i.e., \( x_{\text{CC}} < x_{\text{BC}} \).

Proof. We can rewrite (5) and (10) in the Cournot–strategic and in the Bertrand–strategic games, respectively, as follows:

\[
\left( \frac{\partial \pi}{\partial x_i} \right)_{\text{CNC}} = \frac{2b(2b - d\beta)}{4b^2 - d^2} q_i^C - c^y(x_i) = 0 \tag{12}
\]

\[
\left( \frac{\partial \pi}{\partial x_i} \right)_{\text{BNC}} = \frac{2b(2b^2 - bd\beta - d^2)}{(b + d)(2b - d)^2} q_i^B - c^y(x_i) = 0. \tag{13}
\]

where \( q_i^C = (b + d)(2b - d)q^B/b(2b + d) \). The first terms on the right-hand sides of these equations stand for the marginal revenues of R&D investment in Cournot–quantity and Bertrand–price competitions. As for the coefficients of \( q_i^C \) of both terms we obtain

\[
\frac{2b(2b - d\beta)}{4b^2 - d^2} > (\text{<}) \frac{2b(2b^2 - bd\beta - d^2)}{(b + d)(2b - d)^2} \text{ for } d > (\text{<}) 0.
\]

This demonstrates that the marginal revenue of R&D investment is greater or less in Cournot–quantity competition than in Bertrand–price competition according as \( d \) is positive or negative. It follows that \( x_{\text{CNC}} > (\text{<}) x_{\text{BNC}} \) as \( d > (\text{<}) 0 \).

Compare the R&D investments under the R&D cartels in both Cournot–quantity and Bertrand–price competitions. Rewriting (6) and (11) yields
Let us make a comparison of the outputs in the quantity-setting and price-setting models. Then we have the following proposition:

**Proposition 4.** (a) Given \( d > 0 \), the equilibrium output (price) can be greater (lower) in Cournot–quantity competition than in Bertrand–price competition if \( x^{Ck} \) is relatively large enough in comparison with \( x^{Bk} \). \( q^{Ck} > q^{Bk} \) and \( p^{Ck} < p^{Bk} \), \( k = NC, C \). (b) Given \( d < 0 \), the equilibrium output (price) is greater (lower) in Bertrand–price competition than in Cournot–quantity competition: \( q^{Ck} < q^{Bk} \) and \( p^{Ck} > p^{Bk} \).

Proof. Each of the outputs in the quantity-setting and price-setting models is given by \( q^{Ck} = (a - A^{Ck})/(2b + d) \) and \( q^{Bk} = b(a - A^{Bk})/(b + d)(2b - d) \), where \( A^{ik} = c - (1 + \beta)x^{ik} \), \( i = C, B \). Let us take the case with \( d > 0 \). Comparing both outputs yields

\[
q^{Ck} - q^{Bk} = -\frac{1}{(4b^2 - d^2)(b + d)}\left\{ad^2 + b(2b + d)[c - (1 + \beta)x^{Ck}]/\left[b(2b + d) - c - (1 + \beta)x^{Bk}\right]\right\}.
\]

As shown in Proposition 3, when \( d \) is positive, \( x^{Ck} > x^{Bk}, k = NC, C \), so the term of the braces can be negative, i.e. \( q^{Ck} > q^{Bk} \), if \( x^{Ck} \) is relatively large compared with \( x^{Bk} \). Next take the case with \( d < 0 \). In this case since \( a - A^{Ck} < a - A^{Bk} \) as \( x^{Ck} < x^{Bk} \), \( k = NC, C \), we obtain \( q^{Ck} < q^{Bk} \).

Furthermore, under the assumption of symmetry we have \( p^{Bk} - p^{Ck} = (b + d)(q^{Ck} - q^{Bk}) \) from the inverse demand functions, where \( p^{Ck} = [ab + (b + d)A^{Ck}]/(2b + d) \) and \( p^{Bk} = [a(b - d) + bA^{Bk}]/(2b - d) \). Then it follows from the results above that if \( d > 0 \), then \( p^{Ck} < p^{Bk} \) can hold for relatively large \( x^{Ck} \) compared with \( x^{Bk} \), while if \( d < 0 \), then \( p^{Ck} > p^{Bk} \) holds for any R&D investment.

To intuitively explain how Result (a) of the proposition holds, let us look at such a case with \( d = 0.99b \). When comparing the outputs in Cournot–quantity and Bertrand–price competitions without R&D investment, we get \( q^{CNC} - q^{BNC} = 0.163(a - c)/b \). This shows that the difference between them is not so large, rather
small. Furthermore, when turning to the comparison of both outputs with R&D investment, we have \( q^{CNC} > q^{BNC} \) as long as \( x^{CNC} > 1.489x^{BNC} + 0.488(a - c)/(1 + \beta) \). The latter inequality seems not too implausible in view of the fact that, given \( d > 0 \), \( x^{CNC} > x^{BNC} \), as shown in Proposition 3. The reason why the conventional result may be invalid when the products are substitutes \( (d > 0) \) and R&D investments are chosen prior to quantity or price choices is that the effect of R&D investment on output is larger in Cournot competition than in Bertrand one. It follows that in terms of consumer surplus and welfare the Cournot equilibrium may dominate the Bertrand one in this case, but except for this case the former equilibrium unambiguously dominate the latter one.

The proposition says that the market performance in strategic two-stage game models is the same as the conventional one without strategic behavior.

When combining the results of Propositions 3 and 4, (10), and (22), we obtain the following corollary:

**Corollary 3.** (a) The R&D investments and outputs under CNC, CNCJ, BNC, and BNCJ satisfy

\[
x^{BNCJ} \leq x^{BNC} < x^{CNCJ} < x^{CNC} \quad \text{and} \quad q^{CNCJ} \leq q^{CNC} < q^{BNCJ} < q^{BNC}.
\]

(b) The R&D investments and outputs under CC, CCJ, BC, and BCJ satisfy

\[
x^{BC} \leq x^{BCJ} < x^{CC} < x^{CCJ} \quad \text{and} \quad q^{CCJ} \leq q^{CC} < q^{BCJ} < q^{BC}.
\]

This states that organization of an RJV under R&D cartelization leads to an increase in output as long as firms engage in Bertrand–price competition, so output in RJV competition is the greatest among the four modes. Meanwhile, the formation of an RJV in R&D competition in Cournot–quantity competition leads to a decrease in output, and its output is the smallest among them. More interestingly, in price–setting models larger amounts of output are yielded although the amounts of investment are smaller, regardless of whether firms behave competitively in R&D or form an R&D cartel. That is, the efficiency of R&D investment is, in general, greater in Bertrand–price competition than in Cournot–quantity competition.

Furthermore, the comparison on price among the RJV cases (CNCJ, CCJ, BNCJ, and BCJ) derives the following corollary from Proposition 4, Corollaries 1 and 3, and other previous results:

**Corollary 4.** (a) The prices under R&D and RJV competitions in Cournot–quantity and Bertrand–price competitions satisfy \( p^{BNC} \leq p^{BNCJ} \leq p^{CNC} \leq p^{CNDJ} \) for all values of \( \beta \); and (b) the prices under R&D and RJV cartelizations in Cournot–quantity and Bertrand–price competitions satisfy \( p^{BCJ} \leq p^{BC} \leq p^{CCJ} \leq p^{CC} \) for all values of \( \beta \).

In the comparison of R&D and RJV competitions price under BNC is the lowest, and one under CNCJ is the highest, so that consumer’s surplus is the greatest (lowest) under BNC (CNCJ). On the other hand, in the
comparison of cartelization the lowest price is obtained under BCJ, and the highest price under CC, so the 
surplus is maximized (minimized) in BCJ (CC). From the viewpoint of industrial policy, if output markets are in 
price competition, then the government could lead to the lowest price by promoting the policy of creation of a 
research joint venture (RJV). On the whole, the creation of the RJV will lead to low prices in comparison with 
the case without such creation. This is, however, not the case in quantity competition.

The lower price is and the less R&D expenditure is, the more welfare increases. Then, taking use of 
Propositions 3 and 4 into consideration, we establish the following proposition:

**Proposition 5.** Welfare is greater in Bertrand–price competition than in Cournot–price competition, irrespective 
of whether firms engage in R&D competition or form an R&D cartel in the first stage\(^{14}\).

This proposition holds for any spillover rate. The conventional outcome as to the comparison of welfare, 
therefore, seems to be robust. The implication of the proposition is that it is beneficial to promote price 
competition among firms even if they use advertising except for R&D as a strategic method.

It appears that the results obtained in this section are also maintained even if the number of firms is extended 
from 2 to \( n \).

## 5. Conclusion

We have considered the behavior of strategic R&D investment with its spillover under four organizational 
modes (e.g., R&D competition, RJV competition, R&D cartelization, and RJV cartelization) in Cournot– 
quantity and in Bertrand–price competitions and its effects on prices and outputs when assuming product 
differentiation. Then the R&D investments in the four modes within each of Cournot–quantity and Bertrand– 
price competitions are mutually compared. Furthermore, we make a comparison of the four market 
performances between the two competitions.

We have derived the classifications with respect to R&D investment, output, and price not only in Cournot– 
quantity competition but also in Bertrand–price one. With respect to the comparisons of the four R&D 
investments under R&D competition, R&D cartelization, RJV competition, and RJV cartelization in the 
quantity–setting model, the same outcomes as Kamien et al. (1992) are obtained. We point out that their 
classifications in fact rely heavily on the assumption of symmetry, in other words, if there is no symmetry 
assumption, the magnitudes of spillover rates, in which their outcomes are valid, are very restricted. On the 
other hand, in the price–setting model, if products are complements, then the classifications of the four R&D 
investments are the similar to those in the quantity–setting model. Concretely, given large spillovers (roughly

\(^{14}\) From the classifications above and Corollaries 3 and 4 we obtain the following results: \( x^{CNC} \geq x^{CNCJ} > x^{BNC} \geq x^{BNCJ} \) and 
\( p^{CNCJ} \geq p^{CNC} > p^{BNCJ} \geq p^{BNC} \), and \( x^{CCJ} \geq x^{CC} > x^{BCJ} \geq x^{BC} \) and \( p^{CC} \geq p^{CCJ} > p^{BC} \geq p^{BCJ} \). Thus welfare is greater in BC and 
BCJ than in CC and CCJ, but it is difficult to compare it between BC and BCJ, and CC and CCJ.
speaking, great product differentiation), the formation of an RJV under R&D cartelization leads to the lowest price among the four modes, while its formation under R&D competition, conversely, leads to the highest price. Thus it follows that the formation of RJV under R&D cartelization causes consumer’s surplus to increase, and its formation under R&D competition conversely causes it to decrease.

Secondly, we have taken a step forward and compared between the R&D investments and prices in both Cournot—quantity and Bertrand—price competitions. This gives an answer to a question of whether or not the conventional conclusion that price is lower in the latter than in the former holds in strategic two—stage games as well. When making a comparison between the prices (outputs), we obtain the result that prices (outputs) are lower (greater) in Bertrand—price competition than in Cournot—quantity one. Consequently, we conclude that the conventional conclusion is obviously robust, because it is kept unchanged not only even if such a strategic variable as R&D investment and a spillover effect are introduced but also even if such a spillover effect changes. More interestingly, output is greater in Bertrand—price competition than in Cournot—quantity one although R&D investment under R&D competition is less in the former than in the latter.

Thirdly, when comparing welfare in Cournot—quantity and Bertrand—price competitions, we note that welfare is greater in the former than in the latter. This implies that the conventional conclusion concerning welfare comparison carries over to the case where firms have strategic commitment prior to output or price choice.

From the viewpoint of industrial policy, we may be able to provide some guideline with respect to the choice of optimal R&D policy.

References

Sonnenschein, Hugo (1968), The Dual of Duopolist is Complementary Monopoly: or, Two of Cournot’s Theories are One, *Journal of Political Economy*, March/April 76 (2), 316 – 318.

Cooperative and Noncooperative R&D in Cournot and Bertrand Duopolies with Spillovers, and Their Comparison

Shoji Haruna

We consider the choices of R&D investment, output, and price in Cournot–quantity and Bertrand–price competitions by using strategic two–stage game models. When comparing the market performances in both competitions, we see that although the amount of R&D investment is larger in Cournot–quantity than in Bertrand–price one, the conventional outcome that output (price) is larger (higher) in the former competition than in the latter one carries over to the strategic game model with R&D investment and spillover effects. It is shown that strategic R&D behavior is beneficial to consumers when the spillover effect is small, and firms have a strong incentive to make such commitment as R&D investment and advertising when they are large. Whether in Cournot–quantity competition firms have an incentive to use more or less R&D investment depends on the magnitude of spillover, while in Bertrand–price competition they always have an incentive to use less R&D investment.