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Exchange Rate Cycles in the Mundell-Fleming Model when the Marshall-Lerner Condition is Violated

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Abstract

In many standard undergraduate textbooks of macroeconomics, open economies are discussed by means of the Mundell–Fleming model, an open macroeconomic version of the IS–LM model. This short paper develops a simple differential–equation version of the dynamic Mundell–Fleming model, taking account of two key assumptions : (i) the Marshall–Lerner condition is globally violated and (ii) the investment function depends nonlinearly on the current output level. Under our settings, we demonstrate that the exchange rate and the other relevant variables can display persistent fluctuations due to the occurrence of a stable limit cycle. We also discuss a paradox that the resulting dynamics may well be consistent with the J–curve effect.

1 Introduction

In many standard undergraduate macroeconomic textbooks such as Mankiw (2002) and Blanchard (2002), the IS–LM model still plays a central role for macroeconomic analysis, although such an IS–LM approach seems to have largely disappeared in the recent academic literature. The Mundell–Fleming model, an extension of the IS–LM model for the open economy, also attracts much popularity in many classrooms of economics, probably because of its clarity and tractability. Some might argue that such IS–LM type models have been well examined so far and their policy implications are well known for every economist and that, therefore, nothing is left for further study. However, in *dynamic* settings rather than in static settings, many problems seem to be left still unsolved.

This paper will attempt to shed some light on one of such problems whether the dynamic Mundell–Fleming model is viewed as an *endogenous* business (or exchange rate) cycle model. To do this, we dynamitize the usual open IS–LM model in such a way that output, say, is adjusted over time according to its excess demand, exploiting the interest rate parity condition to incorporate the time change in the exchange rate into the model. To obtain an ‘endogenous business cycle model’, we introduce some Kaldor type nonlinearities (see Kaldor 1940, Goodwin 1951, Chang and Smyth 1971) into the investment function (or saving function). Furthermore, we focus on the case where the Marshall–Lerner condition (in some wider sense) is globally violated. The last part of our specification seems somewhat problematic from the empirical viewpoint, but this is, at the very least theoretically, not inconsistent with the relation between the net export and exchange rate. Taking the importance

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of the Mundell–Fleming model into consideration, it would be still worthwhile doing such a 'hairsplitting' work.

Our proposed model will be shown to be reducible to the well–known Liénard differential equation. It will be, consequently, demonstrated that due to the existence of a (unique) stable limit cycle, the exchange rate and the other variables keep fluctuating without any external shocks.

In the existing literature, there have been, of course, some attempts to modify the IS–LM variants into endogenous business cycle models. For instance, Schinasi (1981, 1982) demonstrated, using the Poincaré–Bendixson Theorem and the Liénard's Theorem, that the dynamic IS–LM model augmented by a government constraint is capable of limit cycles when a Kaldor type investment function is introduced. From the viewpoint of 'Keynesian' endogenous business cyles, the present research is along that line.

2 The Dynamic Mundell–Fleming Model

The dynamic Mundell–Fleming system is given as follows :

$$\dot{Y} = \alpha_1 \left[I(Y,R) + NX(E,Y) - S(Y) \right], \tag{1}$$

$$\dot{R} = \alpha_2 \left[L(Y,R) - M \right], \tag{2}$$

$$\dot{E} = R + \beta - R_f. \tag{3}$$

Eq. (1) represents the adjustment equation in the good market with I, Y, R, NX, E, and S representing the investment function, output, the interest rate, net export, the log of the exchange rate (price of one unit of the foreign currency in terms of the home currency), and the saving function, respectively. Eq. (2) is the adjustment equation in the money market with L and M representing the liquidity preference function and the money supply. We assume that home and foreign prices are both fixed at unity and that the expected inflation rates in the both countries are zero for simplicity. Eq. (3) represents the (approximated) interest rate parity condition for which R_f denotes the foreign interest rate and β the risk preminum. Since the home country is assumed to be small relative to the foreign country, R_f is given exogenously. As usual, we assume that capital is perfectly mobile between the two countries. We assume that the investors have perfect foresight so that the expected rate of change in the exchange rate equals the actual one. The last assumption is reminiscent of that in Dornbusch (1976).

All functions appeared in the above model are assumed sufficiently smooth. As usual, we impose the following conditions on the derivatives :

$$\frac{\partial I(Y,R)}{\partial Y} \geq 0, \quad \frac{\partial I(Y,R)}{\partial R} < 0, \quad \frac{\partial NX(Y,E)}{\partial Y} < 0,$$

$$\frac{dS(Y)}{dY} \in (0,1), \quad \frac{\partial L(Y,R)}{\partial Y} > 0, \quad \frac{\partial L(Y,R)}{\partial R} < 0.$$

Furthermore, we assume that the net export always decreases when the home currency depreciates, that is,

$$\frac{\partial NX(Y,E)}{\partial E} < 0 \tag{4}$$

for each (Y,E). In other words, the above condition implies that the Marshall–Lerner condition is globally violated. In order to reduce the dimensionality of the model, we suppose that the money market adjustment is sufficiently fast relative to the good market adjustment so that the money market is cleared at every point of time, that is,

$$L(Y,R) = M. \tag{5}$$

Eq. (5) implicitly defines the LM relation that can be expressed as

$$R = \varphi(Y) \tag{6}$$

with $\varphi'(Y) > 0$. Putting eq. (6) into eq. (1) and eq. (3), the Mundell–Fleming model reduces to the following two–dimensional system :

$$\begin{aligned} \dot{Y} &= \alpha D(Y,E), \\ \dot{E} &= \varphi(Y) + \beta - R_f, \end{aligned} \tag{7}$$

where $D(Y,E)$ denotes the excess demand function for the good market, which is defined by

$$D(Y,E) := I(Y,\varphi(Y)) + NX(Y,E) - S(Y). \tag{8}$$

Here, we have set $\alpha_1 = \alpha$ just for notational simplicity.

3 Further Specifications

Note first that by condition (4) we have

$$\frac{\partial D(Y,E)}{\partial E} = \frac{\partial NX(Y,E)}{\partial E} < 0.$$

The partial derivative of $D(Y,E)$ with respect to Y is given by

$$\frac{\partial D(Y,E)}{\partial Y} = \frac{\partial I(Y,\varphi(Y))}{\partial Y} + \left[\frac{\partial I(Y,\varphi(Y))}{\partial R}\varphi'(Y) + \frac{\partial NX(Y,E)}{\partial Y} - S'(Y)\right]. \tag{9}$$

As is readily seen, the sign of the expression in (9) is ambiguous ; the sign of the first term on the r. h. s. of (9) is nonnegative, whereas the sign of the terms in the bracket is definitely negative.

For simplicity of analysis, we further assume that the excess demand function $D(Y,E)$ in (8) is separable in the arguments, that is,

$$D(Y,E) := f(Y) - g(E), \tag{10}$$

where $g'(E) > 0$. We will impose some Kaldor type nonlinearity (see Kaldor 1940 ; Chang and Smyth 1971) on the function $f(Y)$; that is, there exist two values of output, Y^* and Y^{**} with $Y^* < Y^{**}$, such that

$$\begin{aligned} f'(Y) &< 0 \quad \text{for} \quad Y < Y^*, \\ f'(Y) &> 0 \quad \text{for} \quad Y^* < Y < Y^{**}, \quad \text{and} \\ f'(Y) &< 0 \quad \text{for} \quad Y^{**} < Y. \end{aligned}$$

The intuition behind the conditions is as follows. See Goodwin (1951) for a similar argument for his nonlinear accelerator. When the economy is in depression $(Y < Y^*)$, firm holds unemployed excess capital stock, which cannot get unmade. As a result, the marginal investment with respect to output, I_Y, is close to zero in depression, so that f' is negative. There is, however, some intermediate range of output level $(Y^* < Y < Y^{**})$ where firm's willingness for investment is so high that I_Y becomes large so as to make f' positive. Because of the presence of production capacity, however, firm can no longer extend its investment level even when the economy is in boom $(Y > Y^{**})$, so f' again turns to negative.

In order to obtain the simplest possible form of the function f that embodies the above features, we assume that f is of a cubic polynomial form :

$$f(Y) := \mu(Y - Y^3), \tag{11}$$

where μ is a positive constant. Here, we have shifted the function so that f is symmetric around the origin and $f(0) = 0$, without changing notations. Note that $f'(0) = \mu > 0$.

Now in order to emphasize the nonlinearity in $f(Y)$, let other functions be of simplest possible linear forms. So we set

$$\varphi(Y) := Y \quad \text{and} \quad g(E) := E. \tag{12}$$

Substituting (11) and (12) into (7), we obtain the following specific system :

$$\begin{aligned} \dot{Y} &= \alpha\left[\mu(Y - Y^3) - E\right], \\ \dot{E} &= Y - b, \end{aligned} \tag{13}$$

where $b = R_f - \beta$.

4 The Occurrence of Periodic Behavior

4.1 Case of Fast Adjustment Speed

One easy way of detecting a periodic solution for the system (13) might be to take a look at the phase portrait for the case where the adjustment speed in the good market is very fast ; in other words, the adjustment parameter $\alpha (> 0)$ in (13) is very large.

Fig. 1 depicts the phase portrait for the system (13) in the (Y, E)–plane when α is suffciently large (i.e., ideally $\alpha \to +\infty$). The $\dot{E} = 0$ curve is a vertical line, i. e., $Y = b$. The $\dot{Y} = 0$ curve is given by $E = \mu(Y - Y^3)$

with $\mu > 0$, which is a cubic curve passing through the origin. Note that the $\dot{Y} = 0$ curve is downward sloping on the right and left branches, whereas it is upward sloping in the vicinity of the origin. The system has a unique equilibrium point $C = (b, \mu(b - b^3))$. It is easily seen that the equilibrium point C is an attractor (repellor) when the $\dot{E} = 0$ line cuts the upward–sloping (downward–sloping, respectively) branch of the $\dot{Y} = 0$ curve. Note that for b close to zero, the equilibrium point is a repellor.

Now let b be so small that the equilibrium point C is a repellor. For $\alpha = \infty$, every trajectory that does not start from the equilibrium point C must move along the $\dot{Y} = 0$ curve. It will, however, undergo a discontinuous jump when it reaches a critical point in the sense of the local maximum or minimum point of the $\dot{Y} = 0$ curve. As a result, each trajectory (except for C) eventually displays a persistent cyclical business cycle with discontinuous jumps in the output level as depicted in Fig. 1. This type of cycle is usually referred to as relaxation oscillation. It is interesting to notice that over the course of the business cycle for $\alpha = \infty$, the exchange rate moves smoothly whereas the output level as well as the interest rate undergo sudden jumps and that the cycle is consistent with perfect foresight.

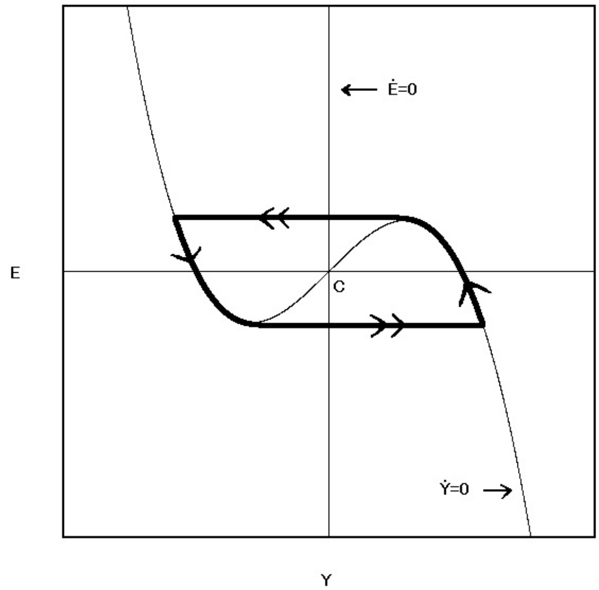

Fig.1 : Phase portrait on the (Y, E)–plane for α large ($\alpha = 300$), $\mu = 1$, $b = 0$. The case of (approximated) relaxation oscillation around the equilibrium point $C = (0, 0)$.

4. 2 Case of Slow Adjustment Speed

In the previous subsection, we have seen that cyclical behavior of the exchange rate will occur when the adjustment speed in the good market for the system (13) is very fast ($\alpha = \infty$). Even when the adjustment speed is slow ($0 < \alpha < \infty$), cyclical behavior can be seen to arise. For $b = 0$, the system (13) is equivalent to the Liénard system, which is known to have a unique limit cycle, which is asymptotically stable. See Appendix for the Liénard's Theorem. Thus, by structural stability, the system (13) for suffciently small $|b|$ exhibits exactly one limit cycle, which is stable. See Fig.2 for a typical phase portrait of the (Y, E)–plane.

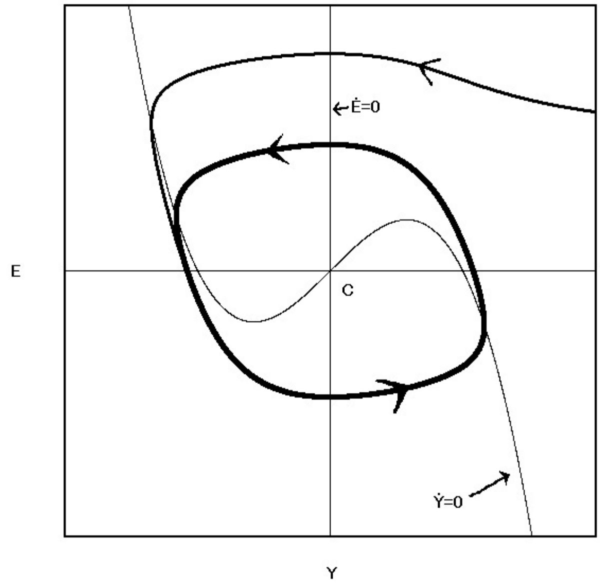

Fig.2 : Phase portrait on the (Y, E)–plane for $\alpha = 2$, $\mu = 1$, $b = 0$. A typical stable limit cycle around the equilibrium $C = (0, 0)$.

5 Discussion

We have built a dynamic Mundell–Fleming model under assumptions that the Marshall–Lerner condition is globally violated and that the investment function has a Kaldor type nonlinearity. For our polynomial version of the model, we have demonstrated that the system exhibits a stable limit cycle around the unique equilibrium point. This shows that the exchange rate will converge to a cyclical motion irrespective of initial conditions. Our result may explain how business cycles in the open economic situation can appear even when the economy is not subject to exogenous shocks.

Furthermore, it is interesting to observe that the trajectory along the limit cycle for α sufficiently large may well be consistent with the so–called J–curve effect. Take a look at Fig.1 and Fig.2 again. At the point where the home currency begins to depreciate on the limit cycle, output begins to decrease. Since output decreases slowly relative to the change in the exchange rate for the initial time interval, the home current account may well be worst off first because of the violation of the Marshall–Lerner condition. The rate of changes in the exchange rate will slow down at the corner of the (approximated) 'discontinuous jump point' and then the rate of (negative) growth in output becomes large relative to the change in the exchange rate so that the effect of the decrease in imports becomes dominant, and it follows that the current account may well be improved for a while. Since we have assumed that the Marshall–Lerner condition is globally violated, this result seems paradoxical at the first glance, but it is, as we have just seen, plausible in dynamic settings.

Some might, however, reasonably argue whether strong regularity generated by a limit cyle as shown in the present paper is really consistent with the interest rate parity condition, as it is generally thought that strong regularity will create opportunities to arbitrage in the foreign exchange market. This point is controversial, which we will leave for our future research. The author have recently examined some different variants of the Mundell–Fleming model with and without periodic perturbations of the foreign interest rate and found that the resulting dynamics can be much richer than the limit cycles that the present paper has demonstrated. See Yokoo (2005a, b).

6 Appendix

In this appendix, we state the Liénard's Theorem for the reader's convenience : see Perko (1991) for the proof of the theorem. Let us consider the following system :

$$\begin{aligned} \dot{x} &= y - F(x), \\ \dot{y} &= -\,g(x). \end{aligned} \tag{14}$$

Liénard's Theorem (e.g. Perko 1991, p.234) Under the assumptions that F and g are of C^1, F and g are odd functions of x, $gx(x) > 0$ for $x \neq 0$, $F(0) = 0$, $F'(0) < 0$, F has a single positive zero at $x = a$, and F increases monotonically to infinity for $x \geq a$ as $x \to \infty$, it follows that the Liénard system (14) has exactly one limit cycle and it is stable.

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