Modeling and damping of high-frequency leakage currents in PWM inverter-fed AC motor drive systems

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This paper presents an equivalent circuit for high-frequency leakage currents in PWM inverter-fed ac motors, which forms a series resonant circuit. The analysis based on the equivalent circuit leads to such a conclusion that the connection of a conventional common-mode choke or reactor in series between the ac terminals of a PWM inverter and those of an ac motor is not effective to reduce the rms and average values of the leakage current, but effective to reduce the peak value.

Furthermore, this paper proposes a common-mode transformer which is different in damping principle from the conventional common-mode choke. It is shown theoretically and experimentally that the common-mode transformer is able to reduce the rms value of the leakage current to 25%, where the core used in the common-mode transformer is smaller than that of the conventional common-mode choke.

I. INTRODUCTION

According to increase in the carrier frequency of PWM inverters, a non-negligible amount of the high-frequency leakage current may cause a serious problem. It would flow through stray capacitors between stator windings and a motor frame due to a large step change of the common-mode voltage produced by a PWM inverter. The peak value may reach 10% of the rated current in the worst case. It may have an undesirable influence on the motor current control and may result in incorrect operation of residual current-operated circuit breakers. Furthermore, the leakage current may cause electromagnetic interference (EMI) to electronic equipment, e.g., AM radio receivers, because its oscillation has a frequency in a range from 100KHz to several MHz [1][2]. However, few papers on the leakage current have been reported.

This paper proposes an equivalent circuit for the leakage current, which forms a series resonant circuit. The validity of the equivalent circuit and the physical property of each component in the equivalent circuit are confirmed experimentally in detail. As a result, a motor model, including the stray capacitors, is also proposed, which is applicable to the analysis of both normal-mode and common-mode currents.

A common-mode choke has been used to reduce the undesirable leakage current, which is connected in series between the terminals of an inverter and those of a motor [3]-[5]. Analysis on the basis of the proposed equivalent circuit results in the following conclusion: The connection of the conventional common-mode choke is not effective to reduce the rms and average values of the leakage current, but effective to reduce the peak value. The analytical result is also verified by experiment.

Furthermore, this paper proposes a common-mode transformer capable of reducing the leakage current. The common-mode transformer is characterized by such a simple configuration that another isolated winding, the terminals of which are shorted by a damping resistor, is added to the common-mode choke. Thus, the authors named it the “common-mode transformer” which corresponds to the term of the conventional “common-mode choke”. The common-mode transformer does not play any role for the normal-mode voltage and current, while it acts as the damping resistor for the common-mode voltage and current. Therefore, it can damp the oscillation of the leakage current, dissipating a negligible amount of loss in the resistor. This is different in damping principle from the conventional common-mode choke, and already proposed suppression circuits [5][6]. The suppression circuits consist of four tightly coupled coils equipped with RC circuits. It is shown that a common-mode transformer, the core size of which is smaller than that of a conventional common-mode choke, is able to reduce the rms value of the leakage current to 25%. A design procedure of the common-mode transformer is also discussed in detail.

II. HIGH-FREQUENCY LEAKAGE CURRENTS

A. Common-Mode Voltage

Fig.1 shows a voltage-source inverter connected to a motor which is represented by three inductors and resistors. A set of voltage-current equations is given by

\[ \begin{align*}
    v_a - v_n &= R_m i_a + L_m \frac{di_a}{dt} \\
    v_b - v_n &= R_m i_b + L_m \frac{di_b}{dt} \\
    v_c - v_n &= R_m i_c + L_m \frac{di_c}{dt}
\end{align*} \]

(1)

where, \( v_a, v_b, v_c \): inverter phase voltages, 
\( i_a, i_b, i_c \): motor line currents, 
\( v_n \): neutral voltage.

The neutral voltage of the motor corresponds to the common-mode voltage in Fig.1. Adding the set of equations derives the following equation.

\[ v_a + v_b + v_c - 3v_n = \left( R_m + L_m \frac{d}{dt} \right) \left( i_a + i_b + i_c \right) \]

(2)

Since \( i_a + i_b + i_c = 0 \), the common-mode voltage in the motor is calculated by

\[ v_n = \frac{v_a + v_b + v_c}{3} \]
It is shown that only the switching state decides the common-mode voltage regardless of the motor impedance. The common-mode voltage changes by $E_{dc}/3$ every switching of the inverter. The common-mode voltage produced by the inverter forces the leakage current, which is discussed in this paper, to flow through stray capacitors between the motor windings and the motor frame.

**B. Modeling for High-Frequency Leakage Currents**

Fig. 2 shows an experimental system to measure the leakage current. An induction motor is driven by a voltage source PWM inverter, and the motor frame is grounded for safety. The leakage current flows from the motor frame through the grounding conductor. Table I shows the ratings of the tested inverter and induction motor.

Fig. 3 shows a leakage current waveform when a phase in the PWM inverter is switched. In this case, the switching gives a step-wise change to the common-mode voltage in the induction motor by $1/3$ of the dc link voltage, that is, $280/3V$. It is shown that a non-negligible amount of oscillatory leakage current flows through the stray capacitors between the stator windings and the motor frame.

A virtual grounding point is introduced in the experimental system to avoid the influence of an internal impedance between the earth terminal on the switch board and the actual grounding point. Three capacitors of $3 \mu F$ are connected to the three-phase input terminals of the inverter unit and the grounding conductor is connected to the neutral point of the capacitors, which is considered a virtual grounding point. By comparing the leakage current waveform with that of connecting the grounding conductor to the earth terminal, it has been confirmed that there is almost no difference in both the waveforms. The internal inductance upstream of the earth terminal has been estimated as $10 \mu H$ by the experiment.

The authors propose an equivalent circuit for the leakage current, which forms an LCR series resonant circuit shown in Fig. 4, because Fig. 3 is similar to the waveform of current after a step voltage is applied to the resonant circuit. If a step-wise voltage is applied to the LCR series resonant circuit, the damped and oscillatory current is given as follows:

$$i(t) = \frac{E}{\sqrt{1 - \zeta^2 \omega_n^2}} e^{-\zeta \omega_n t} \sin \sqrt{1 - \zeta^2} \omega_n t$$

where,

$$\omega_n = \frac{1}{\sqrt{LC}}, \quad \zeta = \frac{R}{2\sqrt{LC}}, \quad Z_0 = \sqrt{\frac{L}{C}}$$
and $\omega_n$, $\zeta$, and $Z_0$ mean the natural frequency, i.e., the resonant frequency, the damping factor, and the characteristic impedance, respectively. In the case of $1 \gg \zeta^2$, the current flowing in the resonant circuit is approximated to the following equation.

$$i(t) \approx \frac{E}{Z_0} e^{-\zeta \omega_n t} \sin \omega_n t$$

Therefore, the characteristic impedance $Z_0$ determines the peak value of the oscillatory current. The circuit constants described in the equivalent circuit in Fig. 4 have been estimated from the experimental waveform.

C. Discussion on Equivalent Circuit Constants

The validity of the equivalent circuit and the physical property of each component in the equivalent circuit will be discussed.

In order to take stray capacitors existing in the motor into consideration, two motor models shown in Fig. 5 are compared. In case of Fig. 5(a), the leakage current flowing through the stray capacitor corresponds to the zero-sequence current of the motor. Therefore, the leakage current depends on the zero-sequence impedance, i.e., the leakage inductance and the winding resistance. On the other hand, the zero-sequence impedance has no influence on the leakage current in the case of Fig. 5(b), because the high-frequency leakage current does not flow in the motor windings. The following experiment has been performed to investigate which motor model is adequate.

Fig. 6 shows a circuit diagram to measure the cable inductance between the voltage-source PWM inverter and the induction motor. The three-phase lines are shorted on both the inverter and motor sides, and the grounding conductor is moreover shorted on the motor side. An LCR meter (HP4263A) is connected to the shorted three-phase lines and the grounding conductor on the inverter side. As a result, the line inductance has been measured as 68 pF. This value approximates to the inductance which has been estimated from the experimental waveform, as shown in the equivalent circuit of Fig. 4. Therefore, it can be understood that the inductance of the equivalent circuit means the line inductance between the inverter and the motor side. As the leakage current. For this reason, Fig. 5(b) is a motor model suitable for analysis of the leakage current rather than Fig. 5(a). The proposed motor model is applicable to the analysis of both normal-mode and common-mode currents.

In addition, measurement of the impedance between the shorted stator windings and the motor frame has been performed. As a result, the capacitance and the series resistance almost coincide to their values calculated from the experimental waveform. The measurement mentioned above leads to the following conclusions:

- The equivalent circuit for the leakage current forms an LCR series resonant circuit.
- In the equivalent circuit, $C$ is the stray capacitance between the stator windings and the motor frame.
- Almost all resistive components are in the motor rather than in the cables.
- The zero-sequence motor impedance has no influence on the leakage current.

If the rise time of the step-wise common-mode voltage is longer than $1/2$ of the oscillation period, the leakage current decreases considerably. Limiting the $dv/dt$ on the inverter output would have a desirable effect on conducted and radiated EMI as well as machine insulation life [7][8].

III. EFFECT OF COMMON-MODE CHOKE ON LEAKAGE CURRENTS

A. Theoretical Analysis

A common-mode choke is connected between the inverter and the motor in order to suppress the leakage current. The insertion of the common-mode choke means an increase of inductance $L$ in the equivalent circuit. Furthermore, resistance $R$ also increases because of the additional loss in the common-mode choke. It is assumed that inductance $L$ and resistance $R$ including the common-mode choke are $n$ times and $m$ times as large as those excluding it, respectively. In this case, the natural frequency $\omega'_n$, the damping factor $\zeta'$, and the characteristic impedance $Z'_0$ are

$$\omega'_n = \frac{1}{\sqrt{nLC}} = \frac{1}{\sqrt{n}} \omega_n$$

$$\zeta' = \frac{mR}{2} \sqrt{\frac{C}{nL}} = \frac{m}{\sqrt{n}} \zeta$$

$$Z'_0 = \frac{nL}{C} = \sqrt{n}Z_0.$$
TABLE II

<table>
<thead>
<tr>
<th>Effect of Common-Mode Choke</th>
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<tbody>
<tr>
<td>Parameter</td>
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<tr>
<td>Inductance</td>
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<tr>
<td>Resistance</td>
</tr>
<tr>
<td>Natural Frequency</td>
</tr>
<tr>
<td>Damping Factor</td>
</tr>
<tr>
<td>Characteristic Impedance</td>
</tr>
<tr>
<td>Decay Time</td>
</tr>
<tr>
<td>Peak Value</td>
</tr>
<tr>
<td>RMS Value</td>
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<tr>
<td>Mean Value</td>
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</table>

Note: The right column shows the ratio of a parameter with a common-mode choke to the corresponding one without it, respectively.

Therefore, the leakage current is given by

$$i'(t) = \frac{E}{\sqrt{n-(m\zeta)^2}Z_0} \cdot e^{-\frac{m\zeta t}{n}} \cdot \sin \sqrt{n-(m\zeta)^2} \frac{\omega_n}{n} t. \quad (9)$$

If the damping factor is enough small, i.e., $n \gg (m\zeta)^2$, the leakage current is approximated by

$$i'(t) \approx \frac{E}{\sqrt{n}Z_0} \cdot e^{-\frac{m\zeta t}{n}} \cdot \sin \frac{\omega_n}{n} t. \quad (10)$$

It indicates that the amplitude, the decay time, and the resonant frequency are equal to $1/\sqrt{n}$, $n/m$, and $1/\sqrt{n}$ times due to the addition of the common-mode choke, respectively. Table II summarizes the effect of the common-mode choke on the leakage current. Since the amplitude and the decay time are equal to $1/\sqrt{n}$ and $n/m$ times, magnification of the rms value is calculated by

$$\sqrt{(1/\sqrt{n})^2 \times n/m} = 1/\sqrt{m}. \quad (11)$$

Similarly, magnification of the mean value is given by

$$1/\sqrt{n} \times n/m = \sqrt{n}/m. \quad (12)$$

If a common-mode choke has no loss, i.e., $m = 1$, no change occurs in the rms value, but the mean value increases, while the peak value decreases.

B. Experimental Investigation

Figs.7 and 8 show waveforms of the leakage current, when a step-wise common-mode voltage of 280V occurs at the output terminals of the PWM inverter. The experimental results are summarized in Table III. It is shown that a common-mode choke connected between the inverter and the motor reduces the peak value of the leakage current from 1900mA to 181mA. Fig.9 represents the equivalent circuit in the case of connecting the common-mode choke. Here, the common-mode choke makes the inductance $L$ and the resistance $R$ increase by 380 times and 9.2 times as large as those excluding it, respectively. The peak value and the rms value in the case of connecting the common-mode choke approximate to the theoretical values i.e., $1900mA/\sqrt{380} = 97.5mA$ and $181mA/\sqrt{9.2} = 59.6mA$ respectively.

IV. Effect of Common-Mode Transformer

Fig.10 shows an experimental system connecting a common-mode transformer proposed in this paper. The common-mode transformer connected between the inverter and the motor is the same as the common-mode choke except for an additional isolated winding, the terminal of which are shorted by a resistor $R_t$. The common-mode transformer acts as the damping resistor only for the common-mode current, i.e., the leakage current. Fig.11 shows the equivalent circuit for the leakage current. The common-mode transformer is represented by a T-type equivalent circuit. $L_t$ and $\ell_t$ mean exciting inductance and leakage inductance of the common-mode transformer, respectively.

<p>| TABLE III |</p>
<table>
<thead>
<tr>
<th>Peak and RMS Values of Leakage Current</th>
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<tbody>
<tr>
<td>Common-Mode Choke</td>
</tr>
<tr>
<td>-------------------</td>
</tr>
<tr>
<td>Disconnected</td>
</tr>
<tr>
<td>Connected</td>
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</table>
Fig. 9. Equivalent circuit for leakage current in case of connecting common-mode choke.

Fig. 10. Configuration of experimental system connecting common-mode transformer proposed in this paper.

Fig. 11. Equivalent circuit for leakage current in case of connecting common-mode transformer.

A. Root Locus and Time Response of Leakage Current

Eq. (13) shows the Laplace transform of the leakage current after a step-wise common-mode voltage of $E$ is applied to the inverter output terminals.

$$I(s) = \frac{C(sL_t + R_t)E}{s^3L_tLC + s^2(L_t + L)CR_t + sL_t + R_t} \quad (13)$$

Here, it is assumed that $C_t$ and $R$ are negligible in (13), because these are enough smaller than $L_t$ and $R_t$, respectively. Fig. 12 shows a root locus of the leakage current as a parameter of $R_t$.

If $R_t = 0 \sim 211\Omega$, there are one real root and two conjugate complex roots. The conjugate complex roots determine the waveform of the leakage current, because the real root near the origin is canceled by the zero of $I(s)$. Therefore, the leakage current becomes an oscillatory waveform. Fig. 13 shows the leakage current waveform in the case of $R_t = 0$. It is the same waveform as Fig. 3, because the common-mode transformer has no impedance for the leakage current.

If $R_t = 211 \sim 846\Omega$, three real roots exist. The second nearest real root to the origin mainly decides the waveform, because the nearest real root is canceled by the zero. Consequently, the leakage current has an aperiodic decayed waveform like a current in an RC series circuit to which a step-wise voltage is applied. Fig. 14 shows the leakage current waveform in case of $R_t = 510\Omega$. The peak and rms values of the leakage current are reduced to 1/3 and 1/4, respectively.

If $R_t = 846 \sim \infty\Omega$, $I(s)$ has one real root and two conjugate complex roots again. Since the real root exists far away from the origin, the conjugate complex roots determine the waveform. However, the oscillation frequency is much

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**Fig. 12. Root locus.**

**Fig. 13. Leakage current waveform ($L_t = 17\text{mH}, R_t = 0\Omega$).**
Fig. 14. Leakage current waveform ($L_t=17\text{mH}, R_t=5\text{k}\Omega$).

Fig. 15. Leakage current waveform ($L_t=17\text{mH}, R_t=20\text{k}\Omega$).

lower than that in the former case, because $L_t$ is much larger than $L$. Fig.15 shows the leakage current in the case of $R_t=20k\Omega$. Although the peak value is reduced to 1/9, the period and decay time of the oscillation are much longer than those in the case of Fig.13. As a result, the rms value becomes larger than that of Fig.14.

The analysis mentioned above results in the conclusion that the value of $R_t$ should be selected so that $I(s)$ has three real roots, in order to reduce both peak and rms values of the leakage current.

B. Breakaway Points

In the following, resistance $R_t$ is chosen so that the roots come onto a breakaway point. The characteristic equation corresponding to the denominator of (13) is shown by a 3rd-order equation:

$$s^3L_tLC + s^2(L_t + L)CB_t + sL_t + R_t = 0.$$  \hspace{1cm} (14)

From the discriminant $D$ in Cardano's method, the following equation should be satisfied.

$$-D = \frac{4}{9^3} \left[ \frac{3}{CL} - \left\{ \frac{(L_t + L)R_t}{L_tL} \right\}^2 \right]^3$$

$$+ \frac{1}{27^2} \left[ 2 \left\{ \frac{(L_t + L)R_t}{L_tL} \right\}^3 - \frac{9(L_t + L)R_t}{CL_tL^2} + \frac{27R_t}{CL_tL} \right]^2 = 0.$$  \hspace{1cm} (15)

Assuming that $L_t \gg L$, the above equation is simplified.

$$\frac{4}{L_tL}R_t^4 - \frac{1}{CL}R_t^2 + \frac{4}{C^2} = 0$$  \hspace{1cm} (16)

Therefore, $R_t$ at the breakaway points are solved as follows:

$$R_t = \frac{1}{2}Z_{\infty\infty} \text{ or } 2Z_{\infty\infty}.$$  \hspace{1cm} (17)

where,

$$Z_{\infty\infty} = \sqrt{\frac{L_t}{C}}$$  \hspace{1cm} (18)

$$Z_{\infty} = \sqrt{\frac{L}{C}}$$  \hspace{1cm} (19)

$Z_{\infty\infty}$ and $Z_{\infty}$ mean the characteristic impedances in the cases of $R_t=\infty$ and $R_t=0$, respectively. If (17) is satisfied, the roots of (13) are on the corresponding breakaway point. If $R_t$ satisfies the following condition, $I(s)$ has three real roots, so that both peak and rms values of the leakage current can be reduced.

$$2Z_{\infty\infty} \leq R_t \leq \frac{1}{2}Z_{\infty\infty}.$$  \hspace{1cm} (20)

V. DESIGN OF COMMON-MODE TRANSFORMER

A design procedure of common-mode transformers is discussed, assuming that stray capacitances are known. The equivalent circuit, having three real roots, can be approximated to an RC series circuit shown in Fig.16, because the leakage current flows mainly through $R_t$ rather than through $L_t$. In this case, the leakage current is approximated as follows:

$$i(t) = \frac{E}{R_t}e^{-t/RC_t}.$$  \hspace{1cm} (21)

where, $E$ means 1/3 of the dc link voltage. Assuming that the time constant is much smaller than the switching period, the rms value of the leakage current is given by

$$I_{\text{rms}} = \sqrt{\frac{6}{T}} \int_0^\infty i(t)^2 dt = E\sqrt{\frac{3C}{R_tT}}.$$  \hspace{1cm} (22)

Note that switching is performed six times every PWM period ($T=1/f_{sw}$) in the three-phase voltage-source PWM inverter.

The rms value of the leakage current should be specified in advance of the design. For example, sensitivity of a residual current-operated circuit breaker(3p3W, 200V, 30A) is rated as 30mA. If $I_{\text{rms}}$ is specified, the required resistance $R_t$ can be calculated backward:

$$R_t = \frac{3CE^2f_{sw}}{I_{\text{rms}}^2}.$$  \hspace{1cm} (23)

On the other hand, the power loss dissipated in resistor $R_t$ is easily given by the equivalent RC circuit:

$$P_{Rt} = 6 \cdot \frac{1}{2}CE^2f_{sw}.$$  \hspace{1cm} (24)
To minimize the size of the common-mode transformer, \( R_t \) should be equal to half of \( 20 \). Therefore, exciting inductance \( L_t \) is determined from the above condition:

\[
L_t = 4R_t^2C = \frac{36C^3E^4f_{sw}^2}{I_{rms}^2}.
\]

Furthermore, the following equation gives the maximum linkage flux \( \Phi_{max} \) in the secondary winding, because the same voltage as that across the resistor is applied to the exciting inductance:

\[
\Phi_{max} = 3CR_tE = \frac{9C^2E^3f_{sw}}{I_{rms}^2}.
\]

As mentioned above, \( L_t \) and \( \Phi_{max} \) are important parameters to design the core of the common-mode transformer.

VI. PROTOTYPE COMMON-MODE TRANSFORMER

A prototype common-mode transformer is constructed and tested in order to verify the effect on the leakage current. Fig.17 shows the shape of a ferrite core used in the prototype, and Table IV shows the specification of the ferrite core.

**TABLE IV**

<table>
<thead>
<tr>
<th>TYPE</th>
<th>Specification of toroidal ferrite core.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_e )</td>
<td>235 mm²</td>
</tr>
<tr>
<td>( A_s )</td>
<td>144 mm</td>
</tr>
<tr>
<td>( L_s )</td>
<td>13.2±25% ( \mu )H/N²</td>
</tr>
<tr>
<td>( W_b )</td>
<td>172 g</td>
</tr>
<tr>
<td>( B_s )</td>
<td>430(at 25°C) mT</td>
</tr>
<tr>
<td></td>
<td>260(at 100°C) mT</td>
</tr>
</tbody>
</table>

If the rms value of the leakage current, \( I_{rms} \), is specified as 27mA, \( R_t \), \( P_{RT} \), \( L_t \), and \( \Phi_{max} \) calculated by (23) to (26) are 516Ω, 0.38W, 6.4mH, and 866µWb, respectively. The number of turns \( N \) is given by the AL-value of the ferrite core:

\[
N = \sqrt{\frac{L_t}{AL}} = 22.
\]

Therefore, the effective sectional area \( A_e \) gives the maximum flux density,

\[
B_{max} = \frac{\Phi_{max}}{N} = 188\text{mT},
\]

and it is enough smaller than saturation flux density \( B_s \) of the core material(H1D). This result indicates that an optimal design of the core shape would make the common-mode transformer more compact. Fig.18 shows the photograph of the common-mode transformer. A damping resistor of 510Ω, 1/2W is connected to the terminals of the secondary winding.

Fig.19 shows the characteristics of the rms leakage current \( I_{rms} \), the maximum linkage flux \( \Phi_{max} \), and the power loss \( P_{RT} \) with respect to \( R_t \) at \( L_t=6.4\text{mH} \), respectively. Simulation for the equivalent circuit shown in Fig.11 is performed, using the PSpice circuit simulator. The simulation results are shown as the solid lines, while the experimental results are plotted. It shows the validity of the modeling for the leakage current as well as the effectiveness of the common-mode transformer. The measured values agree well with the simulated ones, and \( I_{rms} \) is reduced to 25%. Although the minimum point in \( I_{rms} \) exists around \( R_t=1\text{kΩ} \), \( \Phi_{max} \) is 1.5 times as large as that at \( R_t=510\text{Ω} \). \( \Phi_{max} \) at \( R_t=510\text{Ω} \) is smaller than the value calculated by (26), because \( R_t \) attenuates the exciting current. Finally, the leakage current waveform in case of connecting the prototype common-mode transformer is shown in Fig.20, which is the same waveform as Fig.14.

A common-mode transformer, the secondary winding of which is open, is equivalent to a conventional common-mode choke having the same inductance as the exciting
The equivalent circuit for the leakage current is represented as an LCR series resonant circuit.

The zero-sequence impedance of ac motors has no influence on the leakage current.

Conventional common-mode chokes are not effective to reduce the rms and average values of the leakage current, but effective to reduce the peak value.

Furthermore, a common-mode transformer has been proposed, which is able to reduce both the peak and rms values of the leakage current. The design procedure of the common-mode transformer has also been presented. A prototype common-mode transformer, dissipating a negligible amount of loss, i.e., about 0.38W, has been constructed and tested in a vector controlled induction motor of 3.7kW. It has been confirmed that the peak and rms values of the leakage current are reduced to 1/3 and 1/4, respectively, where the core size of the common-mode transformer is also reduced to 1/3 of that of the conventional common-mode choke. The authors believe that the common-mode transformer proposed in this paper is an effective alternative to the conventional common-mode chokes.

REFERENCES


