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Analysis of Resonance Characteristics of a Power Bus with Rectangle and Triangle Elements in Multilayer PCBs

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Abstract- One of the major source of radiated EMI is attributed to power bus resonance in a printed circuit board (PCB). A fast algorithm, combined with the segmentation method, is applied for calculating resonance characteristics of a power bus whose pattern consisting of several segments of rectangles and/or right-angled triangles. Good agreements between the calculated and measured results demonstrate the usefulness and accuracy of the fast algorithm and the segmentation method.

1. Introduction

Resonances of the power/ground planes in multilayer printed circuit boards (PCB) not only cause radiated emission as EM interference, but also affect ground bounce due to switching noise in a digital system. Power bus resonance characteristics have been investigated with various models, such as a full cavity-mode resonator model [1], a distributed lumped-element equivalent circuit model [2], a partial element equivalent circuit (PEEC) approach [3] and its extension [4], and numerical models based on a finite-difference time-domain method [5]. Based on the full cavity-mode resonator model, a fast algorithm can be applied for accurately calculating power bus impedance characteristics. The fast algorithm is attributed to a closed-form expression for the impedance Z-matrix of the power/ground planes, which is in the form of a singly infinite series [6,7]. This is assuming that the pattern of the power/ground planes is rectangular.

In actual PCBs, the pattern of the power/ground planes is usually complicated.
In many cases, however, the pattern consists of several segments, which have simpler shapes, such as rectangles and triangles. In addition, a slight difference in the circumference of the pattern does not significantly affect the resonance characteristics, so that the shape of the pattern may be approximated with only rectangles and/or triangles. In previously reported work [8], the fast algorithm was extended for a pattern consisting of several segments of rectangles, using the segmentation method [9]-[11] that was proposed many years ago for analyzing two-dimensional microwave planar circuits. In this paper, the fast algorithm is extended to the case of the pattern consisting of several segments of rectangles and right-angled triangles as shown in Fig.1. Such complicated power bus geometry is often encountered in practical multilayer PCBs.

2. Full Cavity-Mode Model

The full cavity-mode resonator model is an analytical description of the impedance matrix (Z-parameters) of an unloaded power/ground plane structure (a bare board). For a rectangular power/ground plane structure with length $a$ and width $b$, an expression for fast calculation of the transfer impedance between two ports on the power/ground planes was developed as follows [6]:

$$Z_{ij}^{(rec)} = \sum_{n=0}^{\infty} \frac{\omega \mu_d h a}{j 2b} C_n \cos(k_y n) \cos(k_x n)$$

$$\times \sin^2(k_x w) \left[ \cos(\alpha_n x_-) + \cos(\alpha_n x_+) \right] \alpha_n \sin \alpha_n$$

(1)

where $\sin(x) = \sin(x)/x$, $k_{ym} = n \pi / b$, $\alpha_n = a \sqrt{k^2 - k_{ym}^2}$, $x_\pm = 1 - (x_i \mp x_j) / a$, $(x_i, y_i)$ and $(x_j, y_j)$ are the coordinates of the center of the $i$th and $j$th ports in the $x$- and $y$-directions, respectively, $w$ is much less than the wavelengths of interest and represents the port half width, $h$ is the dielectric thickness between the power/ground planes, $\omega$ is radian frequency, $\mu_d$ is the permeability of the dielectric, and $j = \sqrt{-1}$. For simplicity it is assumed here that the port sizes in the $x$- and $y$-directions for the

![Fig.2: A equilateral right-angled triangular board.](image)

![Fig.3: Comparisons between the calculated and measured $|S_{11}|$ or $|S_{21}|$ for the board of Fig.1.](image)
ith and jth ports are the same. The constant \( C_n \) is assigned as \( C_n = 1 \) if \( n = 0 \), and \( C_n = 2 \) if \( n \neq 0 \). The complex transverse wavenumber \( \kappa \) is obtained as \( \kappa^2 = \omega^2 \mu_d \epsilon_d - j 2 \omega \epsilon_d Z_s / h \), where \( Z_s \) represents the surface impedance of the power/ground conductors. To obtain sufficiently accurate resonance characteristics, both the dielectric loss and the conductor loss of the copper layers must be taken into account. The dielectric loss naturally appears in the imaginary part of the dielectric constant \( \epsilon_d \), while the conductor loss is incorporated into the surface impedance \( Z_s \) of the conductors.

For a equilateral right-angled triangular power/ground plane structure with length \( a \), making use of the relation between the Green's functions of a right-angled triangle and a square [12], the transfer impedance between two ports on a right-angled triangle can be expressed in terms of the corresponding ones on a square (a rectangle with the same length and width \( a = b \)) as

\[
Z_{ij}^{(tri)}(x_i, y_i, x_j, y_j) = Z_{ij}^{(rec)}(x_i, y_i, x_j, y_j) + Z_{ij}^{(rec)}(a - x_i, a - y_i, x_j, y_j)
\]  

(2)

Using the segmentation method, the expressions given above for a rectangle and a right-angled triangle can be easily applied to those geometries which result from the connection of rectangles and right-angled triangles. The details for the segmentation method can be found in a previous work [8]. When the board is loaded with passive components or active devices, the impedance characteristics of the loaded board can be analyzed by considering the loaded board as a multi-port circuit network interconnected by the Z-matrix elements of the corresponding bare board [6].

3. Results and Comparisons

The overall scattering S-matrix can easily be obtained from the overall Z-matrix. To check the validity of the expression (2), the calculated and measured results for \( S_{11} \) at the port 1 and \( S_{21} \) from the port 1 to the port 2 are plotted in Fig.3, for the equilateral right-angled triangular board (the thickness \( h = 1.6 \)mm, the dielectric constant \( \epsilon_r = 4.3 - j 0.086 \)) shown in Fig.2. Very good agreements between the calculated and measured ones can be observed.

In practical PCBs, more complicated power bus geometries are often encountered. The pattern in Fig.1 is an example of such complicated ones. Using the fast algorithm and combining with the segmentation method as the segments (seven rectangles and three right-angled triangles) shown in Fig.1, the calculated \( S_{11} \) and \( S_{21} \) are compared with the measured ones in Fig.4. The calculated results agree well with the mea-
sured ones. The computation time is about 3 minutes in a computer with dual 933MHz CPU, for 1000 frequency points.

4. Conclusions

Resonance characteristics of a power bus whose pattern consists of several segments of rectangles and right-angled triangles can be analyzed by using the fast algorithm based on the full cavity-mode resonator model and the segmentation method. Good agreements between the calculated and measured results have demonstrated the usefulness and accuracy of the fast algorithm and the segmentation method.

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References


