On Pervin’s quasi uniformity

Norman Levine*

*The Ohio State University
ON PERVIN'S QUASI UNIFORMITY

NORMAN LEVINE

1. Introduction

In [1], Pervin introduced a quasi uniformity $\mathcal{U}(X)$ determined by a topology $\mathcal{I}$ on the set $X$ by taking sets of the form $O \times O \cup \mathcal{O} O \times X$, $O \in \mathcal{I}$ as subbase. He proved that the topology induced by $\mathcal{H}(X)$ is in fact the original topology $\mathcal{I}$. Thus every topological space is quasi uniformizable.

It is the purpose of this paper to explore more fully the relationships that exist between $\mathcal{I}$ and $\mathcal{H}(X)$. In §2, the following topological properties are characterized in terms of $\mathcal{H}(X)$: $T_0$, $T_1$, $T_2=\mathcal{I}$ (denoting the class of all closed sets), $\mathcal{I}$ is discrete, $\mathcal{I}$ is indiscrete, $\mathcal{I}$ has three or less elements, $(X, \mathcal{I})$ is disconnected, $X$ is finite and $\mathcal{I}$ is discrete. In §4, relationships between continuity and uniform continuity are determined and compactness is characterized in terms of $\mathcal{H}(X)$. In §5, we give an example to show that $\mathcal{H}(X) \neq \mathcal{H}(\mathcal{I})$.

2. Topological properties

Theorem 2.1. $(X, \mathcal{I})$ is a (i) $T_\varnothing$ space iff $\Delta = \bigcap\{U \cap U^{-1}: U \in \mathcal{H}(X)\}$ (ii) $T_1$ space iff $\Delta = \bigcap\{U: U \in \mathcal{H}(X)\}$ and (iii) $T_2$ space iff $\Delta = \bigcap\{c U: U \in \mathcal{H}(X)\}$.

Proof of (i). Suppose that $(X, \mathcal{I})$ is a $T_\varnothing$ space and that $(x, y) \in U \cap U^{-1}$ for each $U \in \mathcal{H}(X)$. We must show that $x = y$. Suppose on the contrary that $x \neq y$. Case 1. There exists an $O^* \in \mathcal{I}$ such that $x \in O^*$ and $y \notin O^*$. Then $(x, y) \notin O^* \times O^* \cup \mathcal{O} O^* \times X = U^*$. Thus $(x, y) \notin U^* \cap U^*$. Case 2. There exists an $O^\varnothing \in \mathcal{I}$ such that $x \notin O^\varnothing$ and $y \in O^\varnothing$. Then $(y, x) \notin O^\varnothing \times O^\varnothing \cup \mathcal{O} O^\varnothing \times X = U^\varnothing$. Hence $(x, y) \notin U^\varnothing \cap U^\varnothing$.

Conversely, suppose that $\Delta = \bigcap\{U \cap U^{-1}: U \in \mathcal{H}(X)\}$ and suppose that $x \neq y$. Then $(x, y) \notin U \cap U^{-1}$ for some $U \in \mathcal{H}(X)$. Case 1. $(x, y) \notin U$. Then $(x, y) \notin O \times O \cup \mathcal{O} O \times X$ for some $O \in \mathcal{I}$ and it follows that $x \in O$ and $y \notin O$. Case 2. $(x, y) \notin U^{-1}$. Then $(y, x) \notin U$ and case 1 may be applied.

Proof of (ii). Let $(X, \mathcal{I})$ be a $T_1$ space and suppose that $x \neq y$. We will show that $(x, y) \notin U$ for some $U \in \mathcal{H}(X)$. In fact, we may take $U = \mathcal{O} \{y\} \times \mathcal{O} \{y\} \cup \mathcal{O} \mathcal{O} \{y\} \times X$. 

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Conversely, let \( x \neq y \) in \( X \). Then \((x, y) \notin U^*\) for some \( U^* \in \mathcal{U}(\mathcal{I})\) and hence \((x, y) \notin O^* \times O^* \cup \mathcal{O} O^* \times X\) for some \( O^* \in \mathcal{I} \). Hence \( x \in O^*\) and \( y \notin O^*\).

**Proof of (iii).** If \( \triangle = \cap \{ cU : U \in \mathcal{U}(\mathcal{I}) \} \), then \( \triangle \) is closed in \( X \times X \) and \((X, \mathcal{I})\) is a \( T_\alpha \)-space.

Conversely, suppose that \((X, \mathcal{I})\) is a \( T_\alpha \)-space and that \( x \neq y \). There exist disjoint open sets \( O_x \) and \( O_y \) such that \( x \in O_x \) and \( y \in O_y \). Hence \((x, y) \notin cU\) where \( U = O_x \times O_y \cup \mathcal{O} O_x \times X\).

**Theorem 2.2.** Let \( \mathcal{F} \) denote the family of closed sets in \((X, \mathcal{I})\). Then \( \mathcal{I} = \mathcal{F} \) iff \( U \) is a neighborhood of \( \triangle \) whenever \( U \in \mathcal{U}(\mathcal{I}) \).

**Proof.** If \( \mathcal{I} = \mathcal{F} \), then \( O \times O \cup \mathcal{O} O \times X \) is an open neighborhood of the diagonal for each \( O \in \mathcal{I} \). Thus \( U \) in \( \mathcal{U}(\mathcal{I}) \) implies that \( U \) is a neighborhood of the diagonal.

Conversely, suppose that \( U \) in \( \mathcal{U}(\mathcal{I}) \) implies that \( U \) is a neighborhood of the diagonal. Let \( O \in \mathcal{I} \). Then \( O \times O \cup \mathcal{O} O \times X \in \mathcal{U}(\mathcal{I}) \) and hence there exists a \( G \in \mathcal{I} \times \mathcal{I} \) such that \( O \times O \cup \mathcal{O} O \times X \supseteq G \supseteq \Delta \). Then \( O \times O \cup \mathcal{O} O \times \mathcal{O} O \supseteq G \cap G^{-1} \supseteq \Delta \) and \( G \cap G^{-1} \in \mathcal{I} \times \mathcal{I} \). Let \( x \in \mathcal{O} O \). Then \( x \in G \cap G^{-1}[x] \subseteq (O \times O \cup \mathcal{O} O \times \mathcal{O} O)[x] = \mathcal{O} O \) and \( \mathcal{O} O \) is open. It follows that \( \mathcal{I} = \mathcal{F} \).

**Corollary 2.3.** \( \mathcal{U}(\mathcal{I}) \) is a uniformity iff \( \mathcal{I} = \mathcal{F} \).

**Proof.** If \( \mathcal{U}(\mathcal{I}) \) is a uniformity, then \( U \in \mathcal{U}(\mathcal{I}) \) implies that \( U \) is a neighborhood of \( \Delta \) and hence by Theorem 2.2, \( \mathcal{I} = \mathcal{F} \).

Conversely, suppose that \( \mathcal{I} = \mathcal{F} \). It suffices to show that \((O \times O \cup \mathcal{O} O \times X)^{-1} \in \mathcal{U}(\mathcal{I})\) when \( O \in \mathcal{I} \). But \((O \times O \cup \mathcal{O} O \times X)^{-1} \supseteq O \times O \cup \mathcal{O} O \times \mathcal{O} O = (O \times O \cup \mathcal{O} O \times X) \cap (\mathcal{O} O \times \mathcal{O} O \times O \times X) \in \mathcal{U}(\mathcal{I}) \).

**Corollary 2.4.** The following are equivalent:

(i) \((X, \mathcal{I})\) is discrete (ii) \( \mathcal{I} = \mathcal{F} \) and \((X, \mathcal{I})\) is a \( T_\alpha \)-space (iii) \( \mathcal{U}(\mathcal{I}) \) is a uniformity and \( \Delta = \cap \{ U \cap U^{-1} : U \in \mathcal{U}(\mathcal{I}) \} \).

**Proof.** (i) clearly implies (ii) and (ii) is equivalent to (iii) by corollary 2.3 and (i) of theorem 2.1. To show that (ii) implies (i), it suffices to show that \{\{x\} \} is closed for each \( x \in X \). But \( \mathcal{I} = \mathcal{F} \) and \((X, \mathcal{I})\) a \( T_\alpha \)-space clearly implies that \((X, \mathcal{I})\) is a \( T_\alpha \)-space and hence a \( T_\gamma \)-space.

**Theorem 2.5.** \((X, \mathcal{I})\) is trivial iff \((X, \mathcal{U}(\mathcal{I}))\) is trivial.

**Proof.** Exercise for the reader.
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Lemma 2.6. Let \( B \subseteq X, B \neq X \). If \( A \times A \cup \mathcal{C} A \times X \subseteq B \times B \cup \mathcal{C} B \times X \), then \( A \supseteq B \).

Proof. Let \( b \in B \) and suppose that \( b \notin A \). Take \( q \notin B \). Then \( (b, q) \in A \times A \cup \mathcal{C} A \times X \), but \( (b, q) \notin B \times B \cup \mathcal{C} B \times X \), a contradiction.

Lemma 2.7. Let \( \emptyset \neq B \subseteq X \) and suppose that \( A \times A \cup \mathcal{C} A \times X \subseteq B \times B \cup \mathcal{C} B \times X \). Then \( A \subseteq B \).

Proof. Case 1. \( B = X \). Then \( A \subseteq B \). Case 2. \( B \neq X \). Then by Lemma 2.6, \( A \supseteq B \). Now suppose that \( A \notin B \). Take \( a \in A, a \notin B \) and \( b \in B \). Then \( (b, a) \in A \times A \subseteq A \times A \cup \mathcal{C} A \times X \), but \( (b, a) \notin B \times B \cup \mathcal{C} B \times X \), a contradiction.

Corollary 2.8. If \( \emptyset \neq B \subseteq X \) and \( A \times A \cup \mathcal{C} A \times X \subseteq B \times B \cup \mathcal{C} B \times X \), then \( A = B \).

Theorem 2.9. \( \{ O \times O \cup \mathcal{C} O \times X : O \in \mathcal{X} \} \) is a base for \( \mathcal{V}(\mathcal{X}) \) iff \( \mathcal{X} \) consists of at most three sets.

Proof. If \( \mathcal{X} = \{ \emptyset, X \} \) or if \( \mathcal{X} = \{ \emptyset, O, X \} \), then \( \{ X \times X \} \) or \( \{ O \times O \cup \mathcal{C} O \times X, X \times X \} \) is a base for \( \mathcal{V}(\mathcal{X}) \).

Conversely, suppose that \( \emptyset \neq O_i \neq X \) for \( i = 1, 2 \) and that \( \{ O_i \times O_i \cup \mathcal{C} O_i \times X : O_i \in \mathcal{X} \} \) is a base for \( \mathcal{V}(\mathcal{X}) \). Then \( (O_1 \times O_1 \cup \mathcal{C} O_1 \times X) \cap (O_2 \times O_2 \cup \mathcal{C} O_2 \times X) \supseteq O \times O \cup \mathcal{C} O \times X \) for some \( O \in \mathcal{X} \). By Corollary 2.8, \( O = O_2 \) and hence \( \mathcal{X} \) consists of at most three sets.

Theorem 2.10. \((X, \mathcal{X})\) is disconnected iff there exists an \( A \) such that \( \emptyset \neq A \neq X \) and \( A \times A \cup \mathcal{C} A \times \mathcal{C} A \in \mathcal{V}(\mathcal{X}) \).

Proof. If \((X, \mathcal{X})\) is disconnected, let \( A \) be both open and closed and \( \emptyset \neq A \neq X \). Then \( A \times A \cup \mathcal{C} A \times \mathcal{C} A = (A \times A \cup \mathcal{C} A \times X) \cap (\mathcal{C} A \times \mathcal{C} A \cup A \times X) \in \mathcal{V}(\mathcal{X}) \).

Conversely, suppose that \( \emptyset \neq A \neq X \) and that \( A \times A \cup \mathcal{C} A \times \mathcal{C} A \in \mathcal{V}(\mathcal{X}) \). We will show that \( A \) is open (and by symmetry, \( \mathcal{C} A \) is open). Let \( a \in A \). Then \((A \times A \cup \mathcal{C} A \times \mathcal{C} A)[a] = A\).

Theorem 2.11. \((X, \mathcal{V}(\mathcal{X}))\) is totally bounded (\( U \subseteq \mathcal{V}(\mathcal{X}) \) implies that \( U[A] = X \) for some finite set \( A \)).

Proof. Let \( U \subseteq \mathcal{V}(\mathcal{X}) \). Then \( U \supseteq \bigcap \{ O_i \times O_i \cup \mathcal{C} O_i \times X : 1 \leq i \leq n \} \). Consider the \( 2^n \) sets of the form \( A_1 \cap \cdots \cap A_n \) where \( A_i = O_i \) or \( A_i = \mathcal{C} O_i \). Pick \( q \in A_1 \cap \cdots \cap A_n \) whenever \( A_1 \cap \cdots \cap A_n \neq \emptyset \) and let \( A \) be the set of \( q \)-points thus picked. Clearly, \( A \) is finite and we show now that \( U[A] \)
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\[ x \in X \] Let \( x \in X \). Let \( A_i = O_i \) if \( x \in O_i \) and let \( A_i = \emptyset \) if \( x \notin O_i \). Then \( \bigcap A_i \neq \emptyset \). There exists a \( q \) in \( A \) such that \( q \in \bigcap A_i \). Then \( (q, x) \in U \) or \( x \in U[A] \). If \( (q, x) \notin U \), then \( (q, x) \notin O_j \times O_j \cup O_i \times X \) for some \( j \) and hence \( q \in O_j \) and \( x \notin O_j \). Then \( A_j = O_j \), and \( q \in A_j = O_j \), a contradiction.

**Corollary 2.12.** \( \Delta \in \mathcal{H}(\mathcal{X}) \) iff (i) \( X \) is finite and (ii) \( \mathcal{X} \) is discrete.

**Proof.** Let \( \Delta \in \mathcal{H}(\mathcal{X}) \). By Theorem 2.11, there exists a finite set \( A \) such that \( X = \Delta[A] = A \). Thus, \( X \) is finite and (i) holds. (ii) follows from the fact that \( \mathcal{H}(\mathcal{X}) \) is a discrete uniform space when \( \Delta \in \mathcal{H}(\mathcal{X}) \).

Conversely, suppose that (i) and (ii) hold. Then \( \Delta = \bigcap \{ \{ x \} \times \{ x \} \cup \mathcal{C}[x] \times X : x \in X \} \in \mathcal{H}(\mathcal{X}) \).

**Theorem 2.13.** If \( \mathcal{X} \) is countable, then \( \mathcal{H}(\mathcal{X}) \) has a countable base. If \( \mathcal{H}(\mathcal{X}) \) has a countable base, then \( (X, \mathcal{X}) \) is a second axiom space.

**Proof.** If \( \mathcal{X} = \{ O_i : i \in P \} \), then \( \{ O_i \times O_i \cup O_i \times X : i \in P \} \) is a countable subbase for \( \mathcal{H}(\mathcal{X}) \) and hence \( \mathcal{H}(\mathcal{X}) \) has a countable base.

Suppose \( \mathcal{H}(\mathcal{X}) \) has a countable base \( \{ U_i : i \in P \} \). Now \( U_i \supseteq \bigcap \{ O_{ij} \times O_{ij} \cup O_{ij} \times X : 1 \leq j \leq n \} \) for each \( i \in P \). We will show that the \( \{ O_{ij} \} \) forms a subbase for \( \mathcal{X} \). Let \( x \in O \in \mathcal{X} \). Then \( U[x] \subseteq O \) for some \( U \in \mathcal{H}(\mathcal{X}) \). But \( U \supseteq U_i \supseteq \bigcap \{ O_{ij} \times O_{ij} \cup O_{ij} \times X : 1 \leq j \leq n \} \) and hence \( \bigcap \{ O_{ij} \times O_{ij} \cup O_{ij} \times X \} [x] \subseteq O \). But \( (O_{ij} \times O_{ij} \cup O_{ij} \times X) [x] = O_{ij} \) or \( X \). Thus \( x \in O^* \subseteq O \) where \( O^* \) is an intersection of sets from the collection \( \{ O_{ij} : 1 \leq j \leq n \} \).

**Theorem 2.14.** (i) If \( (X, \mathcal{X}) \) is regular, then \( c(\Delta) \subseteq O \times O \cup \mathcal{C} O \times X \) for each \( O \in \mathcal{X} \). (ii) If \( c(\Delta) \subseteq O \times O \cup \mathcal{C} O \times X \) for each \( O \in \mathcal{X} \), then \( (X, \mathcal{X}) \) is an \( R^s \)-space \( (x \in O \in \mathcal{X} \) implies that \( c(x) \subseteq O \)). (iii) If \( (X, \mathcal{X}) \) is \( T_s \) then \( c(\Delta) \subseteq O \times O \cup \mathcal{C} O \times X \) for each \( O \in \mathcal{X} \).

**Proof.** (i) Suppose \( (x, y) \notin O \times O \cup \mathcal{C} O \times X \) for some \( O \in \mathcal{X} \). Then \( x \in O \) and \( y \notin O \). But \( x \in O^* \subseteq O^* \subseteq O \) for some \( O^* \in \mathcal{X} \) since \( (X, \mathcal{X}) \) is regular. Hence \( (x, y) \in O^* \times \mathcal{C} O^* \) and \( O^* \times \mathcal{C} O^* \cap \Delta = \emptyset \). Thus \( (x, y) \notin c(\Delta) \).

(ii) Let \( x \in O \in \mathcal{X} \) and suppose that \( c(x) \subseteq O \). Then take \( y \in c(x) \cap \mathcal{C} O \). Thus \( (x, y) \in c(x) \times c(y) \subseteq c(x) \times c(x) \subseteq c(\Delta) \subseteq O \times O \cup \mathcal{C} O \times X \). Hence \( (x, y) \in O \times O \cup \mathcal{C} O \times X \). But \( x \in O \) and \( y \notin \mathcal{C} O \), a contradiction.

(iii) If \( (X, \mathcal{X}) \) is \( T_s \), then \( c(\Delta) = \Delta \subseteq O \times O \cup \mathcal{C} O \times X \) for each \( O \in \mathcal{X} \).

The converse of (i) is false; take \( (X, \mathcal{X}) \) any \( T_s \)-space that is not regular. The converse of (iii) is false; take any regular space that is not
The converse of (ii) is false; take \((X, \mathcal{X})\) an infinite space with the cofinite topology.

3. Subspaces

**Theorem 3.1.** Let \((X', \mathcal{X}')\) be a subspace of \((X, \mathcal{X})\). Then \(\mathcal{H}(\mathcal{X}') = X' \times X' \cap \mathcal{H}(\mathcal{X})\).

**Proof.** If \(O' = O \cap X'\) where \(O \in \mathcal{X}\), then \(O' \times O' \cup \mathcal{O}' O' \times X' = X' \times X' \cap (O \times O \cup \mathcal{O} O \times X)\).

4. Transformations

**Theorem 4.1.** Let \((X, \mathcal{X})\) and \((Y, \mathcal{Y})\) be topological spaces and \(f: X \to Y\) a transformation. Then \(f\) is continuous relative to \(\mathcal{X}\) and \(\mathcal{Y}\) iff \(f\) is uniformly continuous relative to \(\mathcal{H}(\mathcal{X})\) and \(\mathcal{H}(\mathcal{Y})\).

**Proof.** Only the necessity requires proof. Let \(O' \times O' \cup \mathcal{O}' O' \times Y\) be subbasic in \(\mathcal{H}(\mathcal{X}')\). Then \((f \times f)^{-1}(O' \times O' \cup \mathcal{O}' O' \times Y) \supseteq (f^{-1} O' \times f^{-1} O') \cup \mathcal{O} f^{-1} O' \times X\). Since \(f^{-1} O' \subseteq \mathcal{X}\), it follows that \((f \times f)^{-1}(O' \times O' \cup \mathcal{O}' O' \times X) \subseteq \mathcal{H}(\mathcal{X})\).

**Theorem 4.2.** A net \(S: D \to X\) is \(\mathcal{H}(\mathcal{X})\)-Cauchy iff \(O \in \mathcal{X}\) implies that \(S\) is eventually in \(O\) or \(S\) is eventually in \(\mathcal{O}\).

**Proof.** Let \(S: D \to X\) be a \(\mathcal{H}(\mathcal{X})\)-cauchy net and suppose that \(O \in \mathcal{X}\). Then there exists an \(N\) in \(D\) such that \(m, n \geq N\) implies that \((S(m), S(n)) \in O \times O \cup \mathcal{O} O \times X\). Suppose \(S\) is not eventually in \(O\) nor eventually in \(\mathcal{O}\). Take \(m^* \geq N\) and \(S(m^*) \notin O\). Take \(n^* \geq N\) and \(S(n^*) \notin \mathcal{O}\). Then \(m^*, n^* \geq N\), but \((S(n^*), S(m^*)) \notin O \times O \cup \mathcal{O} O \times X\), a contradiction.

Conversely, suppose \(S: D \to X\) is a net with the property that \(S\) is eventually in \(O\) or eventually in \(\mathcal{O}\) for each \(O \in \mathcal{X}\). We will show that \(S\) is then \(\mathcal{H}(\mathcal{X})\)-cauchy. Let \(O \times O \cup \mathcal{O} O \times X\) be subbasic in \(\mathcal{H}(\mathcal{X})\). If \(S\) is eventually in \(O\), then \(S \times S\) is eventually in \(O \times O \cup \mathcal{O} O \times X\). If \(S\) is eventually in \(\mathcal{O}\), then \(S \times S\) is eventually in \(\mathcal{O} O \times X \subseteq O \times O \cup \mathcal{O} O \times X\).

**Corollary 4.3.** Let \(S: D \to X\) be a net. Then \(S\) is \(\mathcal{H}(\mathcal{X})\)-cauchy iff \(S\) frequently in \(O \in \mathcal{X}\) implies that \(S\) is eventually in \(O\).

In a space \((X, \mathcal{X})\), a net \(S: D \to X\) is called an \(O\)-net iff for \(O \in \mathcal{X}\), \(S\) frequently in \(O\) implies that \(S\) is eventually in \(O\). In [2], the following theorem is proved.

**Theorem 4.4.** \((X, \mathcal{X})\) is compact iff every \(O\)-net in \(X\) converges.
Theorem 4.5. \((X, \mathcal{X})\) is compact iff \((X, \mathcal{U}(\mathcal{X}))\) is complete.

Proof. \((X, \mathcal{U}(\mathcal{X}))\) is complete iff every \(\mathcal{U}(\mathcal{X})\)-cauchy net converges iff every \(O\)-net converges (Corollary 4.3) iff \((X, \mathcal{X})\) is compact (Theorem 4.4).

Theorem 4.6. Let \(f : X \rightarrow Y\) be a transformation and \(\mathcal{X}'\) a topology for \(Y\). Let \(\mathcal{X}\) be the weak topology for \(X\) determined by \(f\) and \(\mathcal{X}'\). Let \(\mathcal{U}\) be the weak quasi uniformity for \(X\) induced by \(f\) and \(\mathcal{U}(\mathcal{X}')\). Then \(\mathcal{U} = \mathcal{U}(\mathcal{X})\).

Proof. \(f : X \rightarrow Y\) is \(\mathcal{X}\mathcal{X}'\) continuous and by Theorem 4.1, \(f : X \rightarrow Y\) is \(\mathcal{U}(\mathcal{X}) - \mathcal{U}(\mathcal{X}')\) uniformly continuous. Thus \(\mathcal{U} \subseteq \mathcal{U}(\mathcal{X})\). We show now that \(\mathcal{U}(\mathcal{X}) \subseteq \mathcal{U}\). Let \(O' \in \mathcal{X}'\). Then \(f^{-1}O' \times f^{-1}0' \cup \mathcal{U} f^{-1}0' \times X\) is subbasic in \(\mathcal{U}(\mathcal{X})\). But \(f^{-1}0' \times f^{-1}0' \cup \mathcal{U} f^{-1}0' \times X \supseteq (f \times f)^{-1}(O' \times O' \cup \mathcal{U} O' \times X) \in \mathcal{U}\).

5. Products

Example 5.1. For each positive integer \(i\), let \((X_i, \mathcal{X}_i)\) be the two point space \(\{0, 1\}\) with the discrete topology and let \((X, \mathcal{X}) = \times \{(X_i, \mathcal{X}_i) : i \in P\}\). Then \(\mathcal{U}(\mathcal{X}) \neq \times \{\mathcal{U}(\mathcal{X}_i) : i \in P\}\). For, let \(O = \cup \{P^{-1} \circ [\circ] : i \in P\}\). Then \(O \in \mathcal{X}\) and \(O \cup \mathcal{U} O \times X \in \mathcal{U}(\mathcal{X})\). But \(O \cup \mathcal{U} O \times X \supseteq (P_1 \times P_1)^{-1} \Delta \cap \cdots \cap (P_n \times P_n)^{-1} \Delta\) for every integer \(n\) and hence \(O \cup \mathcal{U} O \times X \supseteq \times \{\mathcal{U}(\mathcal{X}_i) : i \in P\}\).

REFERENCES


DEPARTMENT OF MATHEMATICS,
THE OHIO STATE UNIVERSITY

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