On a result of Ribenboim

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ON A RESULT OF RIBENBOIM

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Throughout the present note, $A$ will represent a ring with 1, $B$ a
subring of $A$ with the same 1, $\sigma$ a $B$-ring automorphism of $A$, and $T$
the subset $\{y \in A; y \sigma \neq y\}$. If $\sigma \neq 1$ then it is easy to see that $A = T \cup
\{T \sigma - T\} = B[T]$. The extension $A/B$ is called left locally finite if $B[F]$ is
left finite over $B$ (finitely generated as a left $B$-module) for every finite
subset $F$ of $A$. In below, we shall give a generalization of the result

**Proposition.** Let $B$ be left Noetherian, and $A/B$ a left locally finite
extension that is not left finite. If $\sigma \neq 1$ then there exists a subset $Q$ of
$A$ such that $Q \cap Q \sigma = \emptyset$ (empty set) and $B[Q, Q \sigma]$ is not left finite over
$B$.

**Proof.** Let $Q'$ be an arbitrary finite subset of $T$ with $Q' \cap Q' \sigma = \emptyset$,
and set $B' = B[Q', Q' \sigma^{-1}]$, $B^* = B[Q', Q' \sigma, Q' \sigma^{-1}]$, $T' = T \setminus B'$ (complement
of $B'$ in $T$) and $T^* = T \setminus B^*$. Since $A = B[T] = B'[T']$, we readily see
that $B[T']$ is not left finite. Accordingly, $B^*[T^*] = B^*[T'] \supseteq B[T']$
implies that $B[T^*] / B$ is not left finite. In particular $T^*$ contains an
element $x$. Since $x \notin B^*$, $B^* \sigma^{-1} \subseteq B^*$ implies $x \sigma \notin B'$, and then we can
easily see that $(x) \cup Q' \cap (x) \cup Q' \sigma = \emptyset$. Repeating the above argument,
we obtain an infinite ascending chain $Q_1 \subseteq Q_2 \subseteq \ldots$ of finite subsets $Q_n$ of
$T$ such that $Q_n \cap Q_n \sigma = \emptyset$ and $B \subseteq B[Q_n, Q_n \sigma] \subseteq B(Q_n, Q_n \sigma) \subseteq \ldots$. If we set
$Q = \bigcup_{n=1}^{\infty} Q_n$ then $Q \cap Q \sigma = \emptyset$ and $B[Q, Q \sigma] = \bigcup_{n=1}^{\infty} B[Q_n, Q_n \sigma]$, that is
not left finite over $B$.

**REFERENCE**

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