On operators related to p-stable measures in Banach spaces

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ON OPERATORS RELATED TO p-STABLE MEASURES IN BANACH SPACES

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1. Introduction and notations. Let $E$ be a Banach space with the dual space $E'$ and $p$ be a real number such that $0 < p \leq 2$. We say that $E$ is of stable type $p$ if for each sequence $|x_n|$ in $E$, $\sum_n \|x_n\|^p < \infty$ implies the series $\sum_n x_n \theta_n^p$ converges almost surely (a.s.) and $E$ is of stable cotype $p$ if for each sequence $|x_n|$ in $E$ such that the series $\sum_n x_n \theta_n^p$ converges a.s., there holds $\sum_n \|x_n\|^p < \infty$. Here $\theta_n^p$ denotes the sequence of independent identically distributed real random variables with the characteristic function (ch.f.) $\exp(-|t|^p)$. Let us denote by $\mathcal{L}(E, F)$ the set of all continuous linear operators from $E$ into a Banach space $F$. For an operator $T$ in $\mathcal{L}(E, F)$, we say that $T$ is $\mathcal{S}_p$-factorizable (resp. $\mathcal{SQ}_p$-factorizable) if it is factorizable through a subspace (resp. a subspace of a quotient) of some $L_p$. Let us recall that a sequence $|x_n|$ in $E$ is weakly $p$-summable if $\sum_n |<x_n, x'_n>|^p < \infty$ for all $x' \in E$. For an operator $T$ in $\mathcal{L}(E, F)$, we say that $T$ is of stable type $p$ if for each sequence $|x_n|$ in $E$, $\sum_n \|x_n\|^p < \infty$ implies the series $\sum_n T(x_n) \theta_n^p$ converges a.s. in $F$; $T$ is $\gamma_p$-summing if for each weakly $p$-summable sequence $|x_n|$ in $E$, the series $\sum_n T(x_n) \theta_n^p$ converges a.s. in $F$; and $T$ is $p$-summing if for each weakly $p$-summable sequence $|x_n|$ in $E$, $\sum_n \|T(x_n)\|^p < \infty$. We denote by $\Pi_{p}(E, F)$ (resp. $\Pi_{p}(E, F)$), the set of all $\gamma_p$-summing operators (resp. $p$-summing operators) from $E$ into $F$. Let $X$ be a Banach space and $1 < p \leq 2$. In the following we shall write with $X' \subset L_p$ if $X'$ is linearly isometric to a subspace of $L_p$. For such a space $X$, we say that an operator $T$ in $\mathcal{L}(X, E)$ is $\gamma_p$-Radonifying if $\exp(-\|T(x')\|^p)$, $x' \in E'$, is the ch.f. of a Radon measure on $E$, where $T'$ denotes the adjoint of $T$. The set of all $\gamma_p$-Radonifying operators from $X$ into $E$ will be denoted by $R_p(X, E)$. It is known that a symmetric Radon probability measure $\mu$ on $E$ is $p$-stable if and only if there exist a Banach space $X$ with $X' \subset L_p$ and an operator $T$ in $R_p(X, E)$ such that $\exp(-\|T(x')\|^p)$, $x' \in E'$, is the ch.f. of $\mu$ (see [4, Prop. 3]).

Then the main results of this paper are the following:

(1) Let $1 < p \leq 2$. Then the following properties of a Banach space $E$ are equivalent.

(1a) For each Banach space $X$ with $X' \subset l_p$, we have

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(1b) If $|x_n|$ and $|y_n|$ are two sequences in $E$ such that

$$\sum_n |\langle y_n, x' \rangle|^p \leq \sum_n |\langle x_n, x' \rangle|^p$$

for all $x' \in E'$,

and the series $\sum_n x_n \theta_n^\rho$ converges a.s. in $E$, then $\sum_n \|y_n\|^\rho < \infty$.

(2) Let $1 < p \leq 2$ and suppose that a Banach space $E$ has the property (1a). Then for each Banach space $F$, every operator of stable type $p$ from $F$ into $E$ is $S_\rho$-factorizable. In particular, if $E$ is of type $(B_\rho)$, $1 < p < 2$, in the sense of [1], then every operator of stable type $p$ from $F$ into $E$ is $S_\rho$-factorizable; and if $E$ is of stable cotype 2 and $F$ is of stable type 2, then every continuous linear operator from $F$ into $E$ is Hilbertian.

(3) Let $1 < p \leq 2$ and let $E$ be a Banach space satisfying the condition $R_\rho(l_\rho, E) \subset \Pi_\rho(l_\rho, E)$, where $1/p+1/p' = 1$. Then for each reflexive Banach space $F$, every operator of stable type $p$ from $F$ into $E$ is $S_{Q_\rho}$-factorizable. In particular, if $E$ belongs to the class $V_\rho(i)$ in the sense of [5], then every operator of stable type $p$, $1 < p < 2$, from a reflexive Banach space $F$ into $E$ is $S_{Q_\rho}$-factorizable.

(4) Let $E$ be a Banach space and $1 < p < 2$. Then $E$ is of stable type $p$ if and only if for each Banach space $F$, every $p$-summing operator from $F$ into $E$ is $\gamma_p$-summing; and $E$ is of finite dimension if and only if every operator of stable type $p$ from $l_\rho$ into $E$ is $\gamma_p$-Radonifying.

(5) Let $1 < p < \infty$ and let $T$ be a continuous linear operator from a Banach space $F$ into a Banach space $E$. Then $T$ is $S_\rho$-factorizable if and only if for each Banach space $X$ with $X' \subset l_\rho$ and each $S \in L(X, F)$ with $\sum_n \|S(f_n)\|^p < \infty$. $TS$ is $p$-summing. Here $f_n = J'(e_n)$, where $J$ is an isometric imbedding from $X'$ into $l_\rho$ and $e_n$ is the $n$-th unit vector of $l_\rho$. Furthermore, if we assume that $F$ is reflexive, then $T$ is $S_{Q_\rho}$-factorizable if and only if for each $S \in L(l_\rho, F)$ with $\sum_n \|S(e_n)\|^p < \infty$. $TS$ is $p$-summing.

Remark. In Section 2, the equivalence of (1a) and (1b) is proved, and some examples of Banach spaces $E$ having the property (1a) are given. For the case $p = 2$, it is well-known that $E$ has the property (1a) if and only if it is of stable cotype 2. and on the other hand, every Banach space is of stable cotype $p$ with $p < 2$ (see [6]). We also prove that a Banach space $E$ belongs to the class $V_\rho(i)$. $1 < p \leq 2$, if and only if $R_\rho(l_\rho, E) \subset \Pi_\rho(l_\rho, E)$. It is easy to see that every Banach space belongs to the class $V_2(i)$; and a
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Banach space of stable type \( p \), \( 1 < p < 2 \), belongs to the class \( V_\rho(i) \) if and only if it is of \( SQ_\rho \) type in the sense of [2], i.e., it is isomorphic to a subspace of a quotient of some \( L_\rho \). This extends a result of [5].

In Section 3, (4) is proved. We also prove that every \( r \)-summing operator is of stable type \( p \), where \( 0 < p \leq 2 \) and \( 0 < r < \infty \).

In Section 4, the results (2), (3) and (5) are proved. We note that (2) extends the results of [1], [5] and [7]; (3) extends the results of [4] and [5]; and (5) is an analogue of the results of [2] and [7]. Let us recall that a Banach space \( E \) is of \( S_\rho \) type if it is isomorphic to a subspace of some \( L_\rho \). As a consequence of (2), we obtain that a Banach space \( E \) is of stable type \( p \) and of \( S_\rho \) type, \( 1 < p < 2 \), if and only if for each Banach space \( X \) with \( X' \subset l_\rho \), there holds \( R_\rho(X, E) = \Pi_\rho(X, E) \). This extends a result of [1].

The paper is motivated from the works of [1], [2], [4], [5] and [7].

2. Banach spaces having the property (1a). We first prove the equivalence of (1a) and (1b) mentioned in Section 1.

Theorem 1. Let \( 1 < p \leq 2 \). Then the following properties of a Banach space \( E \) are equivalent.

(1) For each Banach space \( X \) with \( X' \subset l_\rho \), we have

\[
R_\rho(X, E) \subset \Pi_\rho(X, E).
\]

(2) If \( |x_n| \) and \( |y_n| \) are two sequences in \( E \) such that

\[
\sum_n |< y_n, x' >|^p \leq \sum_n |< x_n, x' >| ^p \text{ for all } x' \in E'.
\]

and the series \( \sum_n x_n \theta_n^x \) converges a.s. in \( E \), then \( \sum_n \|y_n\|^p < \infty \).

Proof. For the case \( p = 2 \), the equivalence of (1) and (2) easily follows from the fact that \( E \) is of stable cotype 2 if and only if \( R_\rho(l_2, E) \subset \Pi_\rho(l_2, E) \) (see [6]). Hence we may prove only the case \( 1 < p < 2 \).

(1) \( \Rightarrow \) (2): Let us assume that (1) is satisfied and let \( |x_n| \) and \( |y_n| \) be two sequences in \( E \) such that

\[
\sum_n |< y_n, x' >|^p \leq \sum_n |< x_n, x' >| ^p \text{ for all } x' \in E'.
\]

and the series \( \sum_n x_n \theta_n^x \) converges a.s. in \( E \). Since every Banach space is of stable cotype \( p \) with \( p < 2 \), we have \( \sum_n \|x_n\|^p < \infty \). Then there is a continuous linear operator \( S \) from \( l_\rho \) into \( E \) such that \( S(e_n) = x_n \) for all \( n \), where \( e_n \) is the \( n \)-th unit vector of \( l_\rho \) \( (1/p + 1/p' = 1) \). Evidently, \( S \) is
γ₁-Radonifying and there holds \( S'(x') = \left( \langle x_n', x' \rangle \right)_{n=1}^\infty \) for all \( x' \in E' \). Let \( X = Y' \), where \( Y = S'(E) \). Obviously, \( X \) is a closed subspace of \( l_p \), and the operator \( S \) can be factorized as follows;

\[
l_p \xrightarrow{J'} X \xrightarrow{T} E,
\]

where \( J \) denotes the natural injection from \( X' \) into \( l_p \). Then \( T \) is clearly \( γ₂ \)-Radonifying since \( S \) is so. From the assumption (1) it follows that \( T \) is \( p \)-summing. We note here that \( S'(E') \) is a dense subspace of \( X' \). Define the operator \( V : X' \to l_p \) by

\[
V : \left( \langle x_n, x' \rangle \right)_{n=1}^\infty \to \left( \langle y_n, x' \rangle \right)_{n=1}^\infty \text{ for all } x' \in E'.
\]

Then \( V \) is a continuous linear operator and there holds \( TV'(e_n) = y_n \) for all \( n \). Since \( TV' \) is \( p \)-summing, we have \( \sum_n \| y_n \| < \infty \).

(2) \( \Rightarrow \) (1) : Let us assume that (2) is satisfied and let \( X \) be a Banach space with \( X' \subset l_p \) and \( T \in R_p(X, E) \). To prove that \( T \) is \( p \)-summing, let \( \{x_n\} \) be a weakly \( p \)-summable sequence in \( X \). Then there is a continuous linear operator \( S \) from \( l_p \) into \( X \) such that \( S(e_n) = x_n \) for all \( n \). Evidently, we have

\[
\| S'T'(x') \|^p = \sum_n | \langle S'T'(x'), e_n \rangle |^p = \sum_n | \langle T(x_n), x' \rangle |^p, \quad x' \in E',
\]

and

\[
\| T'(x') \|^p = \| JT'(x') \|^p = \sum_n | \langle TJ'(e_n), x' \rangle |^p, \quad x' \in E',
\]

where \( J \) is an isometric embedding from \( X' \) into \( l_p \). Hence

\[
\sum_n | \langle T(x_n), x' \rangle |^p \leq \| S' \|^p \sum_n | \langle TJ'(e_n), x' \rangle |^p, \quad x' \in E'.
\]

Since \( TJ' : l_p \to E \) is clearly \( γ₂ \)-Radonifying, the series \( \sum_n TJ'(e_n) \theta_n^p \) converges a.s. in \( E \). From the assumption (2) it follows that \( \sum_n \| T(x_n) \|^p < \infty \) proving \( T \in \Pi_p(X, E) \). Thus the proof is completed.

Now we give some examples of Banach spaces \( E \) having the property (1a). Let us recall that for \( p = 2 \), \( E \) has the property (1a) if and only if it is of stable cotype 2.

Following [1], we say that a Banach space \( E \) is of type \( (B_p) \), \( 1 < p \leq 2 \), if for each Banach space \( X \) with \( X' \subset l_p \), there holds \( R_p(X, E) = \Pi_{\tau_p}(X, E) \). Of course every Banach space is of type \( (B_2) \) (see [1]). For \( 1 < p < 2 \), every Banach space of type \( (B_p) \) has the property (1a) since \( γ_p \)-summing operators are \( p \)-summing.
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Following [8], we say that a Banach space $E$ is of type $(S)$ if there exists an $S$-topology on $E'$. It is well-known that if $E$ has the approximation property, then $E$ is of type $(S)$ if and only if it is isomorphic to a subspace of some $L_0$. Obviously, every Banach space of type $(S)$ has the property (1a) for $1 < p \leq 2$.

Following [12], we say that a Banach space $E$ belongs to the class $(V_p)$, $1 \leq p \leq 2$, if for each $p$-stable Radon probability measure $\mu$ and each stable cylindrical measure $\nu$ on $E$, the inequality

$$|1 - \hat{\nu}(x')| \leq |1 - \hat{\mu}(x')|, \quad x' \in E',$$

implies $\nu$ is a Radon measure on $E$. Here $\hat{\mu}$ (resp. $\hat{\nu}$) denotes the ch. f. of $\mu$ (resp. $\nu$). It is easy to see that every Banach space belonging to the class $(V_p)$ has the property (1a) for $1 < p < 2$.

Finally, we give some examples of Banach space $E$ satisfying the condition $R_p(l_p, E) \subset \Pi_p(l_p, E)$, $1 < p \leq 2$. Evidently, every Banach space having the property (1a) satisfies this condition, but the converse is not true. It is known that every Banach space of $SQ_p$ type satisfies this condition (see [4]). Note that for $p = 2$, $E$ satisfies this condition if and only if it is of stable cotype 2. In the following we give another example of Banach spaces $E$ satisfying this condition.

Let $E$ be a Banach space and $1 < p \leq 2$. Denote by $\Lambda(p, E; l_p)$ the set of all continuous linear operators $T$ from $E$ into $l_p$ such that $\exp(-||T(x)||^p)$, $x' \in E'$, is the ch. f. of a Radon measure on $E$. It is known that for an operator $T$ in $L(E', l_p)$, $T \in \Lambda_p(E', l_p)$ if and only if there exists an operator $S$ in $R_p(l_p, E)$ such that $T = S$' (see [3, Th. 5]). Following [5], we say that $E$ belongs to the class $V_p(i)$ if $T \in \Lambda_p(E', l_p)$ implies $ST \in \Lambda_p(E', l_p)$ for all $S \in L(l_p, l_p)$.

Proposition 1. A Banach space $E$ belongs to the class $V_p(i)$, $1 < p \leq 2$, if and only if the inclusion $R_p(l_p, E) \subset \Pi_{r_p}(l_p, E)$ holds.

Proof. Let us first assume that $E$ belongs to the class $V_p(i)$ and let $T \in R_p(l_p, E)$. In order to prove that $T$ is $\gamma_p$-summing, take an weakly $p$-summing sequence $\{x_n\}$ in $l_p$. Then there is an operator $S$ in $L(l_p, l_p)$ such that $S(e_n) = x_n$ for all $n$. By the assumption, $T' \in \Lambda_p(E', l_p)$ implies $S'T' \in \Lambda_p(E', l_p)$. But this means that $TS$ is $\gamma_p$-Radonifying, and so the series $\sum_n TS(e_n) \theta_n^p = \sum_n T(x_n) \theta_n^p$ converges a. s. in $E$ (see [1] or [4]). Hence we get $T \in \Pi_{r_p}(l_p, E)$. On the other hand, suppose that the inclusion
$R_p(l_p', E) \subset \Pi_{\gamma_p}(l_p', E)$ holds. Let $T \in \Lambda_p(E', l_p)$. Then there is an operator $V$ in $R_p(l_p', E)$ such that $T = V'$. By the assumption, $V$ is $\gamma_p$-summing. Let $S$ be any operator in $L(l_p, l_p)$. Since $|S'(e_n)|$ is an weakly $p$-summable sequence in $l_p$, the series $\sum_n VS'(e_n)\theta_n^n$ converges a. s. in $E$. But this means that $VS'$ is $\gamma_p$-Radonifying, and so we get $ST = (VS')' \in \Lambda_p(E', l_p)$. Thus $E$ belongs to the class $V_p(i)$, and the proof is completed.

**Corollary 1.** Suppose that a Banach space $E$ is of stable type $p$, $1 < p < 2$. Then $E$ belongs to the class $V_p(i)$ if and only if it is of $SO_p$ type.

**Proof.** The assertion follows from Proposition 1 and [4, Th. 3].

**3. Operators of stable type $p$ and $\gamma_p$-summing operators.** Let us first remark that every operator factorizable through a Banach space of stable type $p$ is always of stable type $p$. It is well known that for $2 \leq r < \infty$, $L_r$ is of stable type 2, and in particular, it is of stable type $p$ for all $p \in (0, 2]$.

**Proposition 2.** For $0 < p \leq 2$ and $0 < r < \infty$, every $r$-summing operator is of stable type $p$.

**Proof.** The assertion easily follows from the facts that every $r$-summing operator is $s$-summing for $r < s$, and every $r$-summing operator is factorizable through a subspace of some $L_r$ (see [10]).

**Remark.** Every $\gamma_p$-summing operator, $0 < p < 2$, is always $p$-summing, but in general, the converse is not true. It is known that if a Banach space $E$ is of stable type $p$, $0 < p \leq 2$, then for each Banach space $F$, every $p$-summing operator from $F$ into $E$ is $\gamma_p$-summing (see [1]). The following result shows that the converse is true for $1 < p < 2$.

**Proposition 3.** Let $1 < p < 2$. Then the following properties of a Banach space $E$ are equivalent.

1. $E$ is of stable type $p$.
2. $\Pi_{\gamma_p}(l_p', E) = \Pi_{\gamma_p}(l_p', E)$.
3. For each Banach space $F$, we have $\Pi_{\gamma_p}(F, E) = \Pi_{\gamma_p}(F, E)$.

**Proof.** Since every $\gamma_p$-summing operator from $l_p$ into $E$ is $\gamma_p$-Radonifying, the assertion follows from [4, Th. 2].
Remark. Proposition 3 becomes false in the case \( p = 2 \). In this case, one of the properties (2) and (3) is equivalent to the fact that \( E \) is of stable cotype 2 (see [1], [6]).

Finally, we investigate the relationship among operators of stable type \( p \), \( \gamma_p \)-summing and \( \gamma_p \)-Radonifying operators.

**Theorem 2.** Let \( 1 < p < 2 \). Then the following properties of a Banach space \( E \) are equivalent.

1. \( E \) is of finite dimension.
2. Every operator of stable type \( p \) from \( l_p \) into \( E \) is \( \gamma_p \)-summing.
3. Every operator of stable type \( p \) from \( l_p \) into \( E \) is \( \gamma_p \)-Radonifying.

**Proof.** Of course, we only have to prove (3) \( \Rightarrow \) (1). Let us assume that (3) is satisfied. Then \( E \) is of stable type \( p \) (see Prop. 2 and [4, Th. 2]). Let \( \{x_n\} \) be an weakly \( p \)-sumnable sequence in \( E \). Then there is an operator \( T \) in \( L(l_p, E) \) such that \( T(e_n) = x_n \) for all \( n \). Since \( E \) is of stable type \( p \), \( T \) is of stable type \( p \). From the assumption (3) it follows that \( T \) is \( \gamma_p \)-Radonifying, and so the series \( \sum_n T(e_n) \theta_n^p = \sum_n x_n \theta_n^p \) converges a.s. in \( E \). Since every Banach space is of stable cotype \( p \) with \( p < 2 \) (see [6]), we have \( \sum_n \|x_n\|^p < \infty \). But this means that the identity map on \( E \) is \( p \)-summing, and so \( E \) is a nuclear Banach space (see [10]). Thus \( E \) is of finite dimension, and the proof is completed.

4. \( S_p \)-factorizable operators and \( SQ_p \)-factorizable operators. In this section, we prove the results (2), (3) and (5) stated in Section 1. The following two propositions are analogues of the results of [2] and [7].

**Proposition 4.** Let \( 1 < p < \infty \) and let \( T \) be a continuous linear operator from a Banach space \( F \) into a Banach space \( E \). Then the following are equivalent.

1. \( T \) is \( S_p \)-factorizable.
2. For each Banach space \( X \) with \( X' \subset l_p \) and each \( S \in L(X, E) \) with \( \sum_n \|S(f_n)\|^p < \infty \), \( TS \) is \( p \)-summing. Here \( f_n = J(e_n) \), where \( J \) is an isometric imbedding from \( X' \) into \( l_p \).

**Proof.** (1) \( \Rightarrow \) (2) easily follows from [11, Th. 3. 1]. On the other hand, let us assume that (2) is satisfied. In order to prove (1), we use the Maurey criterion [7] for the factorizability through a subspace of \( L_p \). Let
\[ |x_n| \text{ and } |y_n| \text{ be two sequences in } F \text{ such that}
\]
\[ \sum_n |< y_n, x' >|^p \leq \sum_n |< x_n, x' >|^p \text{ for all } x' \in F' \]

and
\[ \sum_n \| x_n \|^p < \infty. \]

Then by the same way as in the proof of Theorem 1, we can find an operator \( S \) in \( L(X, F') \) such that \( S(f_n) = x_n \) for all \( n \), where \( X \) is a Banach space with \( X' \subset l_\rho \) and \( \| ( < x_n, x' > )_{n=1}^\infty \| ; x' \in F' \| \) is a dense subspace of \( X' \). Define the operator \( V: X' \to l_\rho \) by
\[
V: ( < x_n, x' > )_{n=1}^\infty \to ( < y_n, x' > )_{n=1}^\infty \quad \text{for all } x' \in F'.
\]

Then \( V \) is a continuous linear operator and there holds \( SV'(e_n) = y_n \) for all \( n \). From the assumption (2) it follows that \( TS \) is \( p \)-summing, and so is \( TSV' \). Thus we get
\[ \sum_n \| T(y_n) \|^p = \sum_n \| TSV'(e_n) \|^p < \infty. \]

By the Maurey criterion [7], \( T \) is \( S_{\rho} \)-factorizable, and the proof is completed.

**Proposition 5.** Let \( 1 < \rho < \infty \) and let \( T \) be a continuous linear operator from a reflexive Banach space \( F \) into a Banach space \( E \). Then the following are equivalent.

1. \( T \) is \( SQ_{\rho} \)-factorizable.
2. For each \( S \in L(l_\rho, F) \) with \( \sum_n \| S(e_n) \|^p < \infty \), \( TS \) is \( p \)-summing.

**Proof.** (1) \( \Rightarrow \) (2) easily follows from [11, Th. 3. 1]. On the other hand, let us assume that (2) is satisfied. In order to prove (1), it is enough to show that \( T' \) is \( SQ_{\rho} \)-factorizable (see [11, Theorem 3. 1]). For the proof, we use the Kwapien criterion [2] for the factorizability through a subspace of a quotient of \( L_{\rho} \). Let \( V \) be a \( p \)-integral operator from \( F' \) into a Banach space \( G \). Since \( F \) is reflexive, by [9, Cor. 1], \( V \) is \( p \)-nuclear, and so it is factorized by the bounded linear operators \( U: F' \to l_\omega, D: l_\omega \to l_\rho \) and \( W: l_\rho \to G \), where \( D \) is a diagonal operator. Evidently, \( U'D' \) is a continuous linear operator from \( l_\rho \) into \( F \), and there holds \( \sum_n \| U'D'(e_n) \|^p \leq \infty \). From the assumption (2) it follows that \( TU'D' \) is \( p \)-summing, and so we get \( (VT')' \in \Pi_{\rho}(G', E') \). Thus by Kwapien [2, Cor. 7], \( T' \) is \( SQ_{\rho} \)-factorizable, and the proof is completed.
Now we prove the following main theorem extending the results of [1] and [7].

**Theorem 3.** Let $1 < p \leq 2$ and suppose that a Banach space $E$ has the property (1a). Then for each Banach space $F$, every operator of stable type $p$ from $F$ into $E$ is $S_p$-factorizable.

**Proof.** Let $T$ be an operator of stable type $p$ from $F$ into $E$. Then for each Banach space $X$ with $X' \subset l_p$ and each $S \in L(X, F)$ such that $\sum_n \|S(f_n)\|^p < \infty$, the series $\sum_n TS(f_n) \theta_n^p$ converges a.s. in $E$, where $f_n$ is the same as in (2) of Proposition 4. But this means that $TS$ is $\gamma_p$-Radonifying (see [1]), and so $TS$ must be $p$-summing because $E$ has the property (1a). By Proposition 4, it follows that $T$ is $S_p$-factorizable, and the proof is completed.

**Corollary 2.** Let $E$ be a Banach space having the property (1a) and $F$ be a Banach space of stable type $p$, $1 < p \leq 2$. Then every continuous linear operator from $F$ into $E$ is $S_p$-factorizable.

**Corollary 3.** Let $E$ be a Banach space of type $(B_p)$ and $F$ be a Banach space of stable type $p$, $1 < p < 2$. Then every continuous linear operator from $F$ into $E$ is $S_p$-factorizable.

**Corollary 4 (Maurey [7]).** Let $E$ be a Banach space of stable cotype 2 and $F$ be a Banach space of stable type 2. Then every continuous linear operator from $F$ into $E$ is Hilbertian. In particular, if $E$ is both of stable type 2 and of stable cotype 2, then $E$ is isomorphic to a Hilbert space.

**Corollary 5.** Let $E$ be a Banach space of stable type $p$ with $1 < p < 2$. Then the following are equivalent.

1. $E$ is of $S_p$ type.
2. $E$ has the property (1a).
3. $E$ is of type $(S)$.
4. $E$ is of type $(B_p)$.
5. $E$ belongs to the class $(V_p)$.

**Theorem 4.** Let $1 < p < 2$. Then the following properties of a Banach space $E$ are equivalent.

1. $E$ is of stable type $p$ and of $S_p$ type.
(2) For each Banach space $X$ with $X \subset l_p$, we have

$$R_p(X, E) = \Pi_p(X, E).$$

Proof. The assertion follows from Corollary 5 and [4, Th. 2].

Remark. Corollaries 3, 5 and Theorem 4 become false in the case $p = 2$. It is known that $E$ has the property (2) of Theorem 4 for $p = 2$ if and only if $E$ is of stable cotype 2 (see [6]).

Finally, we prove the following theorem extending the results of [4] and [5].

Theorem 5. Let $1 < p \leq 2$ and suppose that a Banach space $E$ satisfies the condition $R_p(l_p, E) \subset \Pi_p(l_p, E)$. Then for each reflexive Banach space $F$, every operator of stable type $p$ from $F$ into $E$ is SQ$_p$-factorizable.

Proof. Let $T$ be an operator of stable type $p$ from $F$ into $E$. Then for each $S \in L(l_p, F)$ with $\sum_n \|S(e_n)\|_p < \infty$, the series $\sum_n TS(e_n) \theta^n$ converges a.s. in $E$. But this means that $TS$ is $\gamma_p$-Radonifying, and so by the assumption, $TS$ is $p$-summing. By Proposition 5, it follows that $T$ is SQ$_p$-factorizable, and the proof is completed.

Corollary 6. Let $1 < p < 2$ and let $E$ be a Banach space belonging to the class $V_p(i)$. Then for each reflexive Banach space $F$, every operator of stable type $p$ from $F$ into $E$ is SQ$_p$-factorizable.

REFERENCES

[6] B. MAUREY: Espaces de cotype $p$, $0 < p \leq 2$, Séminaire Maurey-Schwartz 1972–73, Exposé VII.
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