3-D eddy current analysis in moving conductors of permanent magnet type retarders using moving coordinate system

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***Abstract* - A 3-D dc steady state eddy current analysis of a permanent magnet type retarder which rotates at high speed is carried out using a moving coordinate system. The method of dc steady state analysis using a moving coordinate system is described. The calculated results, braking torque etc., are compared with measured ones.**

I. Introduction

In heavy vehicles, auxiliary braking systems such as permanent magnet type retarders, which produce a braking torque by flux and eddy currents, are used sometimes [1]. In order to improve the performance of the retarder, the dc steady state flux and eddy current distributions should be analyzed. However, the results obtained are oscillating due to the high Peclet number [2] when the ordinary Galerkin FEM using a fixed coordinate system is used. In order to overcome this difficulty, the upwind FEM [2], the Petrov-Galerkin FEM [1], and the method of dc steady state eddy current analysis using a moving coordinate system [3] have been proposed.

In this paper, the method of 3-D dc steady state eddy current analysis using a moving coordinate system is applied to the analysis of the retarder. The flux distribution and braking torque of retarder at high speed can be obtained without spurious oscillations.

II. Method of Analysis

A. Fundamental Equations

In the moving conductor region of the retarder, eddy currents are induced, The fundamental equations of the $\boldsymbol{A}-\phi$ method ($\boldsymbol{A}$: magnetic vector potential, ϕ: electric scalar potential) using a moving coordinate system [3] are given by

$$\mathrm{rot}(\nu \mathrm{rot}\boldsymbol{A}) = -\sigma\left(\frac{\partial \boldsymbol{A}}{\partial t} + \mathrm{grad}\phi\right) \quad (1)$$

$$\mathrm{div}\left\{-\sigma\left(\frac{\partial \boldsymbol{A}}{\partial t} + \mathrm{grad}\phi\right)\right\} = 0 \quad (2)$$

where ν and σ are the reluctivity and the conductivity, respectively. In the permanent magnetic region (standstill) without eddy currents, the fundamental equation is given by

$$\mathrm{rot}(\nu_0 \mathrm{rot}\boldsymbol{A}) = \nu_0 \mathrm{rot}\boldsymbol{M} \quad (3)$$

where $\boldsymbol{M}$ is the magnetization vector. ν_0 is the reluctivity in vacuum.

B. Discretization of Eddy Current Term

The eddy current term $\boldsymbol{J_e}$ ($=-\sigma$ ($\partial \boldsymbol{A}/\partial t$+grad$\phi$) in (1) and (2) can be discretized by the backward difference method as follows [3]:

$$\boldsymbol{J}_e(p_2) = -\sigma\left\{\frac{\boldsymbol{A}(p_2)-\boldsymbol{A}(p_1)}{\Delta t} + \mathrm{grad}\phi(p_2)\right\} \quad (4)$$

In (4), it is assumed that the point p_1 is moved to the point p_2 during the time interval Δt. $\boldsymbol{A}(p_1)$ is interpolated using the potential at each node in an element e which contains the point p_1 as follows:

$$\boldsymbol{A}(p_1) = \sum_{i=1}^{n^{(e)}} N_i^{(e)} \boldsymbol{A}_i^{(e)} \quad (5)$$

where $n^{(e)}$ is the number of nodes in the element e, $N_i^{(e)}$ is the interpolation function.

As the distribution of $\boldsymbol{A}$ does not change with time, both $\boldsymbol{A}(p_1)$ and $\boldsymbol{A}(p_2)$ in (4) should be treated as unknown variables. In this case, the coefficient matrix becomes unsymmetric. The ILUBCGSTAB method [4] is used to solve the linear equations. The dc steady state flux and eddy current distributions can be obtained without time iteration.

III Analysis of Retarder

A. Description of Model

Fig. 1 shows a model of permanent magnet type retarder. The outer rotor rotates with a constant speed. The outer rotor and yoke are made of carbon steel (S10C). The permanent magnet is assumed to be magnetized in parallel direction and the

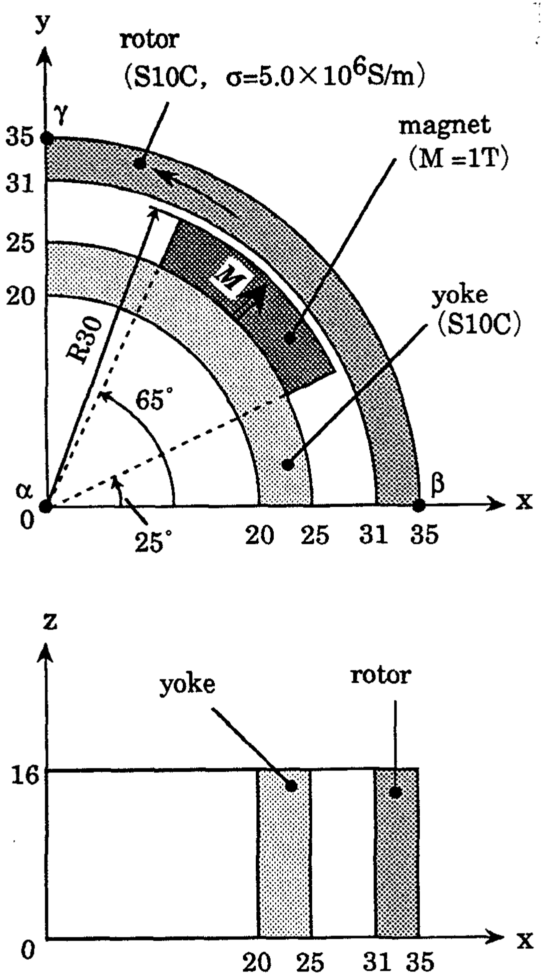


Fig. 1. Analyzed model.

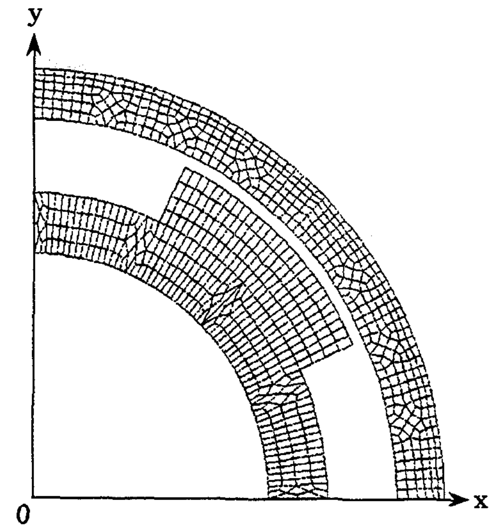


Fig. 2. Mesh without air region.

magnetization is 1T. In the dc steady state analysis, eddy currents flow only in the outer rotor.

B. Method of Analysis

The analyzed region can be reduced to 1/8 of the whole region as shown in Fig. 1, when the following periodic boundary condition is applied on the boundary surfaces α-β and α-γ:

$$Ax_{\alpha-\beta} = -Ay_{\alpha-\gamma} \quad (6)$$

$$Ay_{\alpha-\beta} = Ax_{\alpha-\gamma} \quad (7)$$

$$Az_{\alpha-\beta} = -Az_{\alpha-\gamma} \quad (8)$$

$$\phi_{\alpha-\beta} = -\phi_{\alpha-\gamma} \quad (9)$$

where, for example, $Ax_{\alpha-\beta}$ is the x-component of magnetic vector potential on the boundary surface α–β.

1/8 region is subdivided into 1st-order hexahedral nodal elements. The mesh is shown in Fig. 2. The maximum value of the Peclet number ($=\mu\sigma v L/2$, v: velocity in the rotating direction, L: length of the element in the rotating direction) of the outer rotor is about 200 (the relative permeability and the rotational speed are assumed to be 1,000 and 10,000 rpm, respectively).

The time interval Δt in (4) is chosen to be 1.67×10^{-5} s at 10,000 rpm. When the rotational speed is changed, only the time interval Δt is changed (the same mesh is used).

C. Results and Discussion

Fig. 3 shows the flux distributions at 0, 1,000 and 10,000 rpm. The flux distributions in the rotor change due to the rotation of the rotor. Stable flux distributions without spurious oscillations can be obtained by using a moving coordinate system.

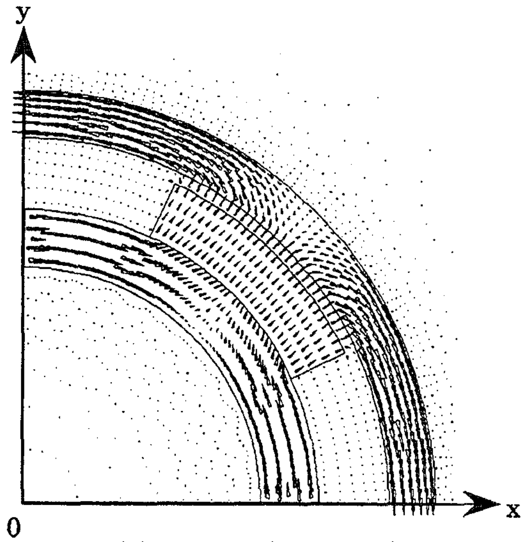
The braking torque is calculated using the nodal force method [5]. The relationship between rotational speed and braking torque is shown in Fig. 4. The braking torque increases with the rotational speed.

Table I shows the discretization data. The increase of the memory and CPU time requirements for the analysis at high speeds can be limited to 3 to 7 times of those at standstill.

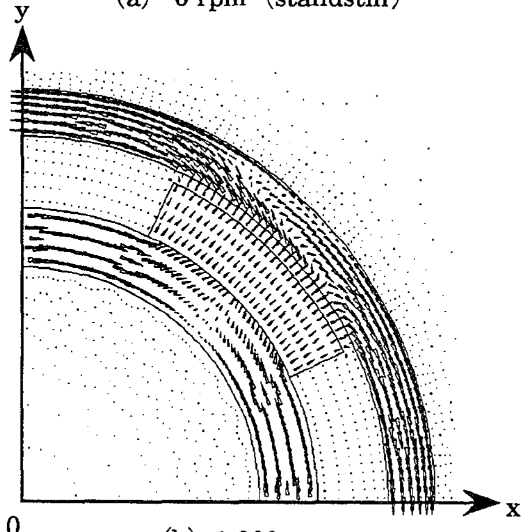
IV. Conclusions

The dc steady state eddy current analysis of a permanent magnet type retarder is carried out using a moving coordinate system. The results obtained are stable at high speed (10,000 rpm).

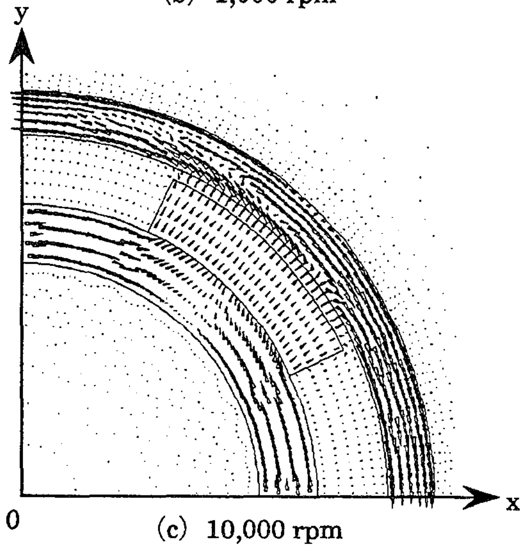
The influence of the shape of the retarder on the performance of retarder and the comparison between the calculated and measured results will be presented in the full paper.



(a) 0 rpm (standstill)



(b) 1,000 rpm



(c) 10,000 rpm

Fig. 3. Flux Distributions.

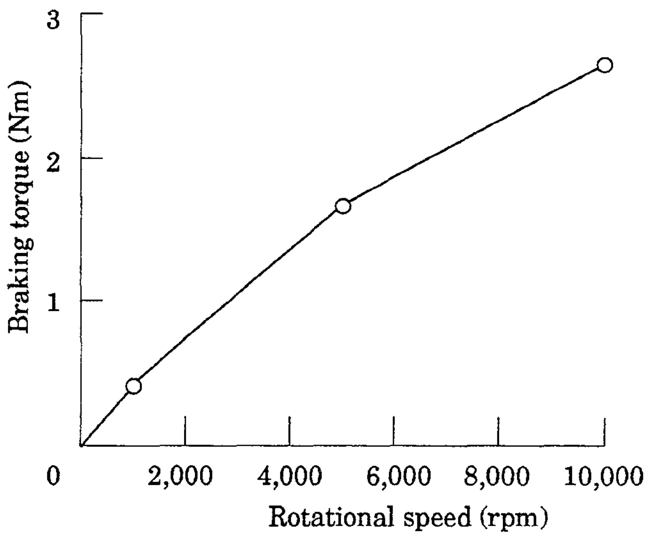


Fig. 4. Effects of speed on braking torque.

Table I Discretization data and CPU time

speed (rpm)	0	1,000	5,000	10,000
number of elements	13,160			
number of nodes	15,432			
number of unknowns	41,536	43,328		
number of non-zeros	1,526,648	4,763,622		
memory requirement (MB)	35.9	93.7		
number of nonlinear iterations	9	11	15	13
total CPU time (h)	3.1	11.2	20.7	20.7

Convergence criterion for Newton–Raphson method : 0.01(T)
Convergence criterion for ICCG method (standstill) : 10^{-7}
Convergence criterion for ILUBCGSTAB method (rotation) : 10^{-7}
Computer used : IBM 3AT workstation (49.7 MFLOPS)

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