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H. Tsuboi
Okayama University

M. Tanaka
Okayama University

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H. Tsuboi and M. Tanaka
Department of Electrical and Electronic Engineering,
Okayama University,
Tsushima, Okayama 700, Japan

Abstract: The boundary element method using electric field and magnetic flux density has been applied to three-dimensional electromagnetic field problems. We can also use magnetic vector potential as unknown vector variable. In this paper, formulae for the boundary element method using vector potential were derived. Furthermore, the external conditions for the vector potential in the boundary element method were discussed by using a three-dimensional magnetostatic problem.

Introduction

Many useful numerical methods have been developed for the analysis of three-dimensional electrostatic, magnetostatic and eddy current problems. The boundary integral equations for electric field and magnetic field were formulated for electromagnetic field problems by Stratton[1] at first, and the boundary element method using electric field and magnetic flux density as unknown vector variables was applied to three-dimensional eddy current problems[2]. On the other hand, in order to take account of the impressed voltages which are applied to the electrical machinery, we can use magnetic vector potential and scalar potential for boundary element method in three-dimensional eddy current problems[3].

In this paper, authors propose a boundary element method for magnetostatic problems, and present a formula to obtain magnetic flux density at arbitrary point from the discretized values of the vector potential and the tangential components of magnetic flux density on the boundary surfaces in magnetostatic problems. The formula is formed only by using integrations and does not need numerical differentiations[3]. The vector potential is determined by the governing equation which is obtained from the Maxwell's equations, but the ambiguity in the vector potential has to be removed by a gauge condition. Furthermore, when a impressed magnetic flux density is given, the external condition for vector potential has some freedoms because there are some vector potential distributions which give the same magnetic flux density. However, the external conditions of the magnetic vector potential have not discussed. In this paper, the external conditions for vector potential in the boundary element method are investigated by using a three-dimensional magnetostatic problem.

Formulation

The basic equations for magnetostatic problem are given by the Maxwell's equations as follows:

$$\nabla \times \mathbf{H} = \mathbf{J} \tag{1}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{2}$$

where $\mathbf{H}$ is the magnetic field, $\mathbf{J}$ is the current density and $\mathbf{B}$ is the magnetic flux density. $\mathbf{B}$ is represent $\mu\mathbf{H}$ where μ is the permeability.

We can introduce a magnetic vector potential $\mathbf{A}$ for magnetostatic problems. The vector potential $\mathbf{A}$ is defined by

$$\mathbf{B} = \nabla \times \mathbf{A} \tag{3}$$

and $\mathbf{A}$ satisfies the following gauge condition.

$$\nabla \cdot \mathbf{A} = 0 \tag{4}$$

From Eqs. (1) and (2), a governing equation for the vector potential is obtained as follows:

$$\nabla\times \nabla\times\mathbf{A} = \mu\mathbf{J} \tag{5}$$

By using the vector Green's theorem[1],[5], the vector potential $\mathbf{A}_i$ at the computation point i on the boundary surface S is obtained with the aid of Eqs. (4) and (5).

$$\frac{\mathbf{A}_i}{2} = \int_V \mu\mathbf{J}\phi dv - \int_S \{(\mathbf{A}\cdot\mathbf{n}')\nabla'\phi - (\mathbf{A}\times\mathbf{n}')\times\nabla'\phi - (\nabla'\times\mathbf{A})\times\mathbf{n}'\phi\}\, ds + \mathbf{A}_0 \tag{6}$$

where ϕ is the fundamental solution, $\mathbf{n}'$ is the unit normal vector at the source point on the boundary surface S which is the boundary surface of the region V, $\mathbf{J}$ is the current density in the region V, and $\mathbf{A}_0$ is the external vector potential which is defined by impressed magnetic flux density. In Eq. (6), primes are used to indicate vector operations in source coordinates. The fundamental solution ϕ is given by

$$\phi = \frac{1}{4\pi r} \tag{7}$$

Furthermore, the magnetic flux density $\mathbf{B}_i$ at arbitrary computation point i in the region V is introduced by the curl of $\mathbf{A}_i$ as follows:

$$\mathbf{B}_i = \nabla\times\mathbf{A}_i = \int_V \mu\mathbf{J}\times\nabla\phi dv + \int_S [\{(\mathbf{n}'\times\mathbf{A})\cdot\nabla\}\nabla'\phi + \nabla\phi\times\{(\nabla'\times\mathbf{A})\times\mathbf{n}'\}]\, ds + \nabla\times\mathbf{A}_0 \tag{8}$$

where $\nabla\times\mathbf{A}_0$ expresses the impressed magnetic flux density. Equation (8) is expressed only by integrals and does not need numerical differentiations[3].

When there are two magnetic materials, $\mathbf{A}\cdot\mathbf{n}'$, $\mathbf{A}\times\mathbf{n}'$ and $\nabla'\times\mathbf{A}$, which are unknowns on the boundary surface S between the materials 1 and 2, are solved by introducing the following boundary conditions:

$$(\mathbf{A}\cdot\mathbf{n}')_1=(\mathbf{A}\cdot\mathbf{n}')_2 \tag{9}$$

$$(\mathbf{A}\times\mathbf{n}')_1=(\mathbf{A}\times\mathbf{n}')_2 \tag{10}$$

$$\{(\nabla'\times\mathbf{A})\times\mathbf{n}'\}_1/\mu_1=\{(\nabla'\times\mathbf{A})\times\mathbf{n}'\}_2/\mu_2 \tag{11}$$

where μ_1 and μ_2 are the permeabilities of the materials 1 and 2, respectively.

The external vector potential $\mathbf{A}_0$ is defined by the impressed magnetic flux density $\mathbf{B}_0$. Therefore, $\mathbf{A}_0$ has to satisfy the following equations:

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$$\nabla\times\mathbf{A}_0=\mathbf{B}_0 \tag{12}$$

$$\nabla\cdot\mathbf{A}_0=0 \tag{13}$$

But, $\mathbf{A}_0$ has still some freedoms: for example, when $\mathbf{A}_1$ satisfies Eqs. (12) and (13), $\mathbf{A}_2=\mathbf{A}_1+\mathbf{C}$ also satisfies them where $\mathbf{C}$ is a constant vector.

Computation Results

In order to investigate the external condition of the magnetic vector potential, we chose a magnetic sphere model in an external uniform field $\mathbf{B}_0=(0,0,1)$. Figure 1 shows the magnetic sphere model and its arrangement of triangular elements. In this case, the number of the triangular elements is 288 and unknown variables are defined to be constant on the triangular element.

Table 1 shows the external conditions: impressed current of a large circular loop or impressed magnetic flux density. The external vector potential $\mathbf{A}_0$ satisfies Eqs. (12) and (13). In addition, the external vector potential of case B almost coincides that of case A because the vector potential of case B is a approximation of the vector potential which is induced by the circular loop current with large radius.

External vector potentials and computed vector potentials are shown in Fig. 2 through 4. The computed vector potentials of the case B, C and D give same

Table 1 External conditions for the impressed magnetic flux density $\mathbf{B}_0=(0,0,1)$.

	J	$\mathbf{A}_0$		
		$\mathbf{A}_{0x}$	$\mathbf{A}_{0y}$	$\mathbf{A}_{0z}$
Case A	Circular loop radius = 1 (m) I = 1.59×10^6 (A)	0	0	0
Case B	0	$-\frac{y}{2}$	$\frac{x}{2}$	0
Case C	0	$-\frac{y-0.1}{2}$	$\frac{x-0.1}{2}$	0
Case D	0	0	x	0

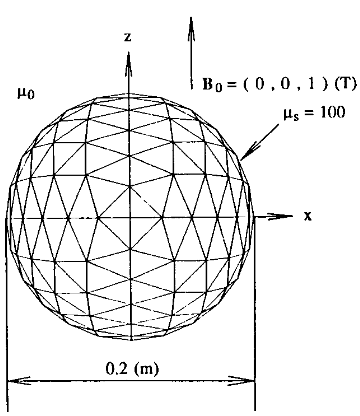


Fig. 1 A magnetic sphere model in an external uniform field.

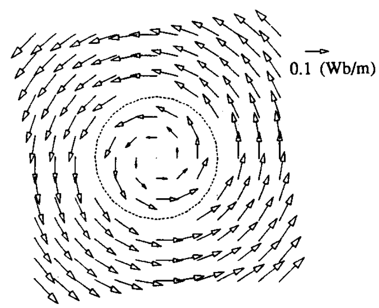


Fig. 2 Distributions of the computed vector potentials for case B.

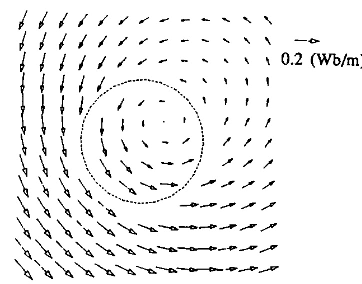


Fig. 3 Distributions of the computed vector potentials for case C.

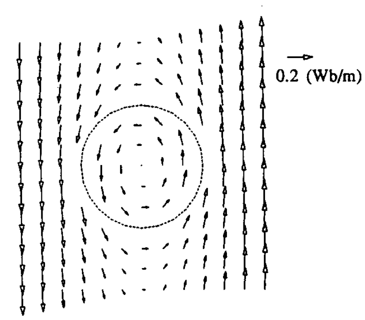


Fig. 4 Distributions of the computed vector potentials for case D.

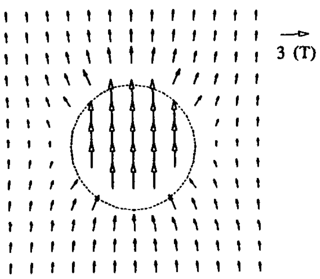


Fig. 5 Distribution of the computed magnetic flux density for case B, C and D.

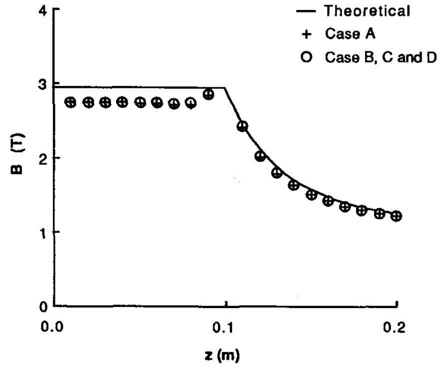
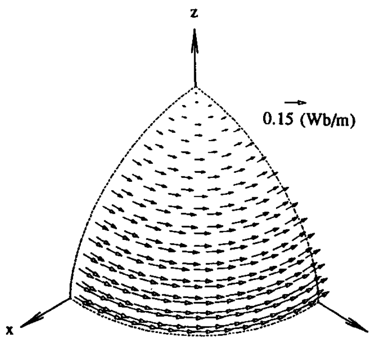


Fig. 6 Computed magnetic flux density along z-axis for the external conditions of Table 1.



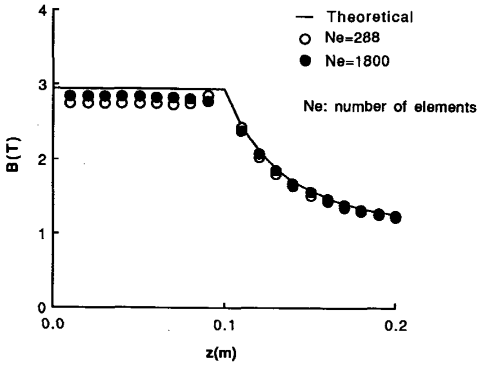


Fig. 7 The convergence of computation results of magnetic flux density for the number of elements.

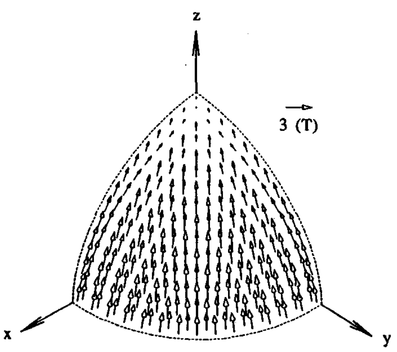


Fig. 8 Distributions of the unknown variables on the boundary surface, (a) vector potential, (b) tangential component of magnetic flux density.

distributions of magnetic flux density shown in Fig.5. The comparison between the magnetic flux density distribution of case A and that of the case B, C and D is shown in Fig.6. The computed results of the two external conditions almost agree with theoretical values.

Figure 7 shows the convergence of the boundary element method for the number of elements. In this case, the external condition is the case B and the number of elements are 288 and 1800. Figure 8 shows the distributions of vector potential and tangential component of magnetic flux density which are unknown variables on the sphere surface for 1800 elements case.

Conclusion

The boundary element method using vector potential for magnetostatic problems was proposed and the formulation was shown. Furthermore, the formula of the magnetic flux density expressed by unknown variables on the boundary surface was introduced.

From the computation results of the magnetic sphere model, it became clear that the external vector potential which gives impressed magnetic field and satisfies the gauge condition has some freedoms but same computation results of magnetic flux density are obtained from different external vector potentials in the proposed boundary element method. In addition, the convergence characteristic of the proposed boundary element method for the number of elements was verified.

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