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EXTERNAL CONDITIONS FOR THE VECTOR POTENTIAL
IN THE BOUNDARY ELEMENT METHOD

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Abstract: The boundary element method using electric field
and magnetic flux density has been applied to three-
dimensional electromagnetic field problems. We can also
use magnetic vector potential as unknown vector variable.
In this paper, formulae for the boundary element method
using vector potential were derived. Furthermore, the
external conditions for the vector potential in the boundary
element method were discussed by using a three-
dimensional magnetostatic problem.

Introduction

Many useful numerical methods have been developed
for the analysis of three-dimensional electrostatic,
magnetostatic and eddy current problems. The boundary
integral equations for electric field and magnetic field were
formulated for electromagnetic field problems by Stratton[1] at first, and the boundary element method using
electric field and magnetic flux density as unknown vector
variables was applied to three-dimensional eddy current
problems[2]. On the other hand, in order to take account of
the impressed voltages which are applied to the electrical
machinery, we can use magnetic vector potential and scalar
potential for boundary element method in three-
dimensional eddy current problems[3].

In this paper, authors propose a boundary element
method for magnetostatic problems, and present a formula
to obtain magnetic flux density at arbitrary point from the
discretized values of the vector potential and the tangential
components of magnetic flux density on the boundary
surfaces in magnetostatic problems. The formula is formed
only by using integrations and does not need numerical
differentiations[3]. The vector potential is determined by
the governing equation which is obtained from the
Maxwell’s equations, but the ambiguity in the vector
potential has to be removed by a gauge condition.
Furthermore, when a impressed magnetic flux density is
given by the Maxwell’s equations as follows:
\[
V \times H = J
\]
\[
V \cdot B = 0
\]
where \( H \) is the magnetic field, \( J \) is the current density and \( B \)
is the magnetic flux density. \( \mu \) is the permeability.

We can introduce a magnetic vector potential \( A \)
for magnetostatic problems. The vector potential \( A \) is defined
by
\[
B = V \times A
\]
and \( A \) satisfies the following gauge condition.
\[
V \cdot A = 0
\]
From Eqs. (1) and (2), a governing equation for the vector
potential is obtained as follows:
\[
V \times V \times A = \mu J
\]
By using the vector Green's theorem[1],[5], the vector
potential \( A_i \) at the computation point \( i \) on the boundary
surface \( S \) is obtained with the aid of Eqs. (4) and (5).
\[
A_i = \int_V \mu J \delta v d v - \int_S \left( (A \times n') V V' \phi - \left(A \times n'\right) \times V' \phi \right)
- (V' \times A) \delta n' d s + A_0
\]
where \( \phi \) is the fundamental solution, \( n' \) is the unit normal
vector at the source point on the boundary surface \( S \) which
is the boundary surface of the region \( V \), \( J \) is the current
density in the region \( V \), and \( A_0 \) is the external vector
potential which is defined by impressed magnetic flux
density. In Eq. (6), primes are used to indicate vector
operations in source coordinates. The fundamental solution
\( \phi \) is given by
\[
\phi = \frac{1}{4 \pi r}
\]
Furthermore, the magnetic flux density \( B_i \) at
arbitrary computation point \( i \) in the region \( V \) is introduced
by the curl of \( A_i \) as follows:
\[
B_i = V \times A_i
\]
\[
= \int_V \mu J \delta v d v + \int_S \left[ \left( (A \times n') V V' \phi \right) \right] d s + V \times A_0
\]
where \( V \times A_0 \) expresses the impressed magnetic flux density.
Equation (8) is expressed only by integrals and does not
need numerical differentiations[3].

When there are two magnetic materials, \( A \times n' \) and
\( V \times A \), which are unknowns on the boundary surface \( S \)
between the materials 1 and 2, are solved by introducing the
following boundary conditions:
\[
(A \times n_1) = (A \times n_2)
\]
\[
(A \times n_1) = (A \times n_2)
\]
\[
\left( (V' \times A) \times n' \right) / \mu_1 = \left( (V' \times A) \times n' \right) / \mu_2
\]
where \( \mu_1 \) and \( \mu_2 \) are the permeabilities of the materials 1
and 2, respectively.

The external vector potential \( A_0 \) is defined by the
impressed magnetic flux density \( B_0 \). Therefore, \( A_0 \) has to
satisfy the following equations:

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\[ \nabla \times A_0 = B_0 \]  
\[ \nabla \cdot A_0 = 0 \]

But, \( A_0 \) has still some freedoms: for example, when \( A_1 \) satisfies Eqs. (12) and (13), \( A_2 = A_1 + C \) also satisfies them where \( C \) is a constant vector.

**Computation Results**

In order to investigate the external condition of the magnetic vector potential, we chose a magnetic sphere model in an external uniform field \( B_0 = (0, 0, 1) \). Figure 1 shows the magnetic sphere model and its arrangement of triangular elements. In this case, the number of the triangular elements is 288 and unknown variables are defined to be constant on the triangular element.

Table 1 shows the external conditions: impressed current of a large circular loop or impressed magnetic flux density. The external vector potential \( A_0 \) satisfies Eqs. (12) and (13). In addition, the external vector potential of case B almost coincides that of case A because the vector potential of case B is a approximation of the vector potential which is induced by the circular loop current with large radius.

External vector potentials and computed vector potentials are shown in Fig. 2 through 4. The computed vector potentials of the case B, C and D give same

<table>
<thead>
<tr>
<th>J</th>
<th>( A_0 )</th>
<th>( A_{0x} )</th>
<th>( A_{0y} )</th>
<th>( A_{0z} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case A</td>
<td>Circular loop radius = 1 (m) ( I = 1.29 \times 10^6 ) (A)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Case B</td>
<td>0</td>
<td>( y )</td>
<td>( x )</td>
<td>0</td>
</tr>
<tr>
<td>Case C</td>
<td>0</td>
<td>( y - 0.1 )</td>
<td>( x - 0.1 )</td>
<td>0</td>
</tr>
<tr>
<td>Case D</td>
<td>0</td>
<td>0</td>
<td>( x )</td>
<td>0</td>
</tr>
</tbody>
</table>

**Fig. 1** A magnetic sphere model in an external uniform field.

**Fig. 2** Distributions of the computed vector potentials for case B.

**Fig. 3** Distributions of the computed vector potentials for case C.

**Fig. 4** Distributions of the computed vector potentials for case D.
Fig. 5 Distribution of the computed magnetic flux density for case B, C and D.

Fig. 6 Computed magnetic flux density along z-axis for the external conditions of Table 1.

Fig. 7 The convergence of computation results of magnetic flux density for the number of elements.

**Conclusion**

The boundary element method using vector potential for magnetostatic problems was proposed and the formulation was shown. Furthermore, the formula of the magnetic flux density expressed by unknown variables on the boundary surface was introduced.

From the computation results of the magnetic sphere model, it became clear that the external vector potential which gives impressed magnetic field and satisfies the gauge condition has some freedoms but same computation results of magnetic flux density are obtained from different external vector potentials in the proposed boundary element method. In addition, the convergence characteristic of the proposed boundary element method for the number of elements was verified.

**References**