New approximate method for calculating three-dimensional magnetic fields in laminated cores

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NEW APPROXIMATE METHOD FOR CALCULATING THREE-DIMENSIONAL MAGNETIC FIELDS IN LAMINATED CORES

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ABSTRACT

A new approximate method for calculating three-dimensional magnetic fields in laminated cores has been developed by modifying the two-dimensional finite element method. Using this new method, both computer storage and computing time can be considerably reduced compared with a three-dimensional analysis. The flux distribution ratios defined in 2.2. An economical analysis of the flux distribution in each layer has become possible using the new method.

1. INTRODUCTION

At the T-joint of the so-called scrap-less type three-phase transformer core made of grain-oriented silicon steel, the laminations are alternately turned over so that the angle between the rolling directions of overlapping sheets is 90° [1]. In such laminations, the flux distribution in one layer is different from that in the adjacent one. Thus a three-dimensional analysis is required. If an approximate analysis of such three-dimensional fields is possible by modifying the two-dimensional finite element method, both computer storage and computing time can be considerably reduced.

A new approximate method for calculating three-dimensional magnetic fields in laminated cores has been developed by introducing the "flux distribution ratio [2]" defined in 2.2. An economical analysis of the flux distribution in each layer has become possible using the new method.

2. ANALYSIS

2.1 Analyzed Model

Figure 1 shows the so-called scrap-less type three-phase transformer core. Laminations are alternately turned over. The solid arrows denote the rolling directions of the grain-oriented silicon steel sheets in the first layer and the dashed arrows denote those in the adjacent layer. In the hatched parts of Fig.1, the angle between the rolling directions of the adjacent sheets is 90°.

2.2 Flux Density in Each Sheet

Figure 2 shows the cross-section of the core along the line α-α' in Fig.1. As the laminations are alternately turned over, it is sufficient to discuss the behaviour of fluxes in only the two layers in Fig.2. The magnetic reluctance in the x-direction of the first layer is different from that of the second layer. Therefore, the flux $\phi_x$ in the first layer is different from the flux $\phi_x$ in the second layer.

The x-components $B_x$ and $B_x$ of the flux densities in the first and second layers shown in Fig.2 are denoted by

$$B_x = \frac{\phi_x}{l}t \cdot \phi_x/t,$$

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where $\phi_x$, $\phi_x$, and $\phi_x$ are the x-components of the total flux in the two layers, the flux in the first layer and the flux in the second layer, respectively. These are the fluxes per 1(m) in the y-direction. $t$ is the effective thickness of one sheet in the z-direction. The flux distribution ratios $F_{x1}$ and $F_{x2}$ in each element are defined as follows:

$$F_{x1} = \frac{\phi_x}{\phi_x},$$

$$F_{x2} = \frac{\phi_x}{\phi_x} = 1 - F_{x1}.$$

The total flux $\phi_x$ can be written in terms of the vector potential $A$, as follows [3]:

$$\phi_x = 2t3A/3y.$$

By substituting Eqs. (4) and (5) into Eqs. (1) and (2), the flux densities $B_{x1}$ and $B_{x2}$ can be rewritten in terms of $A$ and $F_{x1}$ as follows:

$$B_{x1} = 2F_{x1}3A/3y,$$

$$B_{x2} = 2(1 - F_{x1})3A/3y.$$

The y-components $B_y$ and $B_y$ of the flux densities in the first and second layers can also be derived in the same way and written as follows:

$$B_{y1} = -2F_{y1}3A/3x,$$

$$B_{y2} = -2(1 - F_{y1})3A/3x,$$

where the flux distribution ratio $F_{y1}$ is defined by

$$F_{y1} = \phi_{y1}/\phi_y.$$

$\phi_{y1}$ and $\phi_y$ are the y-components of the flux in the first layer and of the total flux respectively.

2.3 Calculation of Energy, Vector Potentials and Flux Distribution Ratios

In our new approximate method, the vector potentials (A) and the flux distribution ratios ($F_{x1}$ and $F_{y1}$) are calculated under the minimum energy principle. The total energy $X$ of the two layers in Fig.2 can be expressed as

$$X = \frac{1}{2} \int \left( B_{x1}^2 + B_{x2}^2 + B_{y1}^2 + B_{y2}^2 \right) dL,$$

$$= \frac{1}{2} \int \left( F_{x1}B_{x1}^2 + (1 - F_{x1})B_{x2}^2 + F_{y1}B_{y1}^2 + (1 - F_{y1})B_{y2}^2 \right) dL.$$

where $F_{x1}$ and $F_{y1}$ are the fluxes per 1(m) in the y-direction. The energy of the total flux in the two layers is given by

$$E = \frac{1}{2} \int \left( B_{x1}^2 + B_{x2}^2 + B_{y1}^2 + B_{y2}^2 \right) dL,$$

$$= \frac{1}{2} \int \left( F_{x1}B_{x1}^2 + (1 - F_{x1})B_{x2}^2 + F_{y1}B_{y1}^2 + (1 - F_{y1})B_{y2}^2 \right) dL.$$
where \( X_1 \) and \( X_2 \) are the energies in the first and second layers, and these are given by

\[
X_1 = x_{11} + x_{12},
\]
(11)

\[
X_2 = x_{21} + x_{22},
\]
(12)

\[
x_{11} = t \int_0^1 \left( \sum_{i=1}^{B_{X_1}} \sum_{j=1}^{B_{Y_1}} dB_{X_1} \right) dx dy,
\]
(13)

\[
x_{12} = t \int_0^1 \left( \sum_{i=1}^{B_{X_1}} \sum_{j=1}^{B_{Y_1}} dB_{Y_1} \right) dx dy,
\]
(14)

\[
x_{21} = t \int_0^1 \left( \sum_{i=1}^{B_{X_2}} \sum_{j=1}^{B_{Y_2}} dB_{X_2} \right) dx dy,
\]
(15)

\[
x_{22} = t \int_0^1 \left( \sum_{i=1}^{B_{X_2}} \sum_{j=1}^{B_{Y_2}} dB_{Y_2} \right) dx dy,
\]
(16)

\[
x_{11} = t \int_0^1 \left( \sum_{i=1}^{B_{X_1}} \sum_{j=1}^{B_{Y_1}} dB_{X_1} \right) dx dy,
\]
(17)

In Eqs. (14)-(17), it is assumed that the \( z \)-components of the fluxes and eddy currents are neglected, and the energy in each layer can be calculated using the \( x \)- and \( y \)-components of reluctivities. \( S \) denotes the region to be analyzed, and \( v_{x1}, v_{x2}, v_{y1}, \) and \( v_{y2} \) are the \( x \)- and \( y \)-components of the reluctivities in the first and second layers, respectively.

In a laminated core, as the \( z \)-component of the flux density is very small compared with the \( x \)- or \( y \)-component, the above-mentioned assumption is acceptable, when the interlaminar air gap between the sheets is very small compared with the thickness of the sheet.

In order to calculate both the flux distributions in the first and second layers, not only the vector potentials \( A \) but also the flux distribution ratios \( \{F_{x1}\} \) and \( \{F_{y1}\} \) are treated as independent variables.

The following equations are obtained under the minimum energy principle:

\[
\frac{\delta X}{\delta A_1} = 0 \quad (i = 1, \ldots, n),
\]
(18)

\[
\frac{\delta X}{\delta F_{x1}} = 0 \quad (i = 1, \ldots, n_e),
\]
(19)

\[
\frac{\delta X}{\delta F_{y1}} = 0 \quad (i = 1, \ldots, n_e),
\]
(20)

where \( A_1 \) is the vector potential at node \( i \). The suffix \( e \) denotes the element \( e \). \( n \) is the number of nodes at which the vector potential is unknown, and \( n_e \) is the number of elements in the region to be analyzed.

Although, only the case of two kinds of layers with different magnetic reluctances is discussed here, the analysis of the case of \( m \) kinds of layers is possible by introducing \( 2(m-1) \) kinds of distribution factors.

### 2.4 Finite Element Formulation

The energy \( X \) is a nonlinear function of the vector potentials \( A \) and the flux distribution ratios \( \{F_{x1}\} \) and \( \{F_{y1}\} \). Therefore, it is difficult to solve Eqs. (18), (19) and (20) directly and so an iterative technique is introduced.

(a) Discretization of Eq. (18)

The derivative of \( X \) in Eq. (18) with respect to the vector potential \( A_1 \) is given as follows from Eqs. (11)-(13):

\[
\frac{\partial X}{\partial A_1} = t \int_0^1 \left[ \frac{3B_{X_1}}{A_1} v_{x1} B_{X_1} dx dy, \right.
\]
(21)

From Eq. (14), \( \frac{\partial x_{11}}{\partial A_1} \) is given by

\[
\frac{\partial x_{11}}{\partial A_1} = \frac{3B_{X_1}}{2B_{X_1}} \frac{3B_{Y_1}}{2B_{Y_1}}
\]

where \( \{F_{x1}\} \) and \( \{F_{y1}\} \) are assumed to be known.

The derivative of \( X_{11}, X_{12} \) and \( X_{12} \) with respect to \( A_i \) and \( A_j \) can be calculated in the same way, and the final \( \frac{\partial^2 X}{\partial A_i \partial A_j} \) is given by
(b) Discretization of Eqs.(19) and (20)

\[ \frac{\partial x}{\partial Fx_1} \text{ and } \frac{\partial x}{\partial Fy_1} \text{ are obtained from Eqs. (6)-(9) and (11)-(17) under the assumption that the vector potentials } \{A\} \text{ are known as follows:} \]

\[ \frac{\partial x}{\partial Fx_1} = \sum_{n=1}^{3} \left( \frac{\partial h_k}{\partial x} \right) \delta A_k \frac{\partial A_k}{\partial x} \right) \]

\[ \frac{\partial x}{\partial Fy_1} = \sum_{n=1}^{3} \left( \frac{\partial h_k}{\partial y} \right) \delta A_k \frac{\partial A_k}{\partial y} \right) \]

\[ \delta A_k \text{ are given by} \]

\[ \delta A_k = \int \left( \frac{\partial A_k}{\partial x} \right) \delta Fx_1 \]

\[ \delta A_k = \int \left( \frac{\partial A_k}{\partial y} \right) \delta Fy_1 \]

\[ \frac{\partial x}{\partial Fx_1} \text{ and } \frac{\partial x}{\partial Fy_1} \text{ are given by} \]

\[ \frac{\partial x}{\partial Fx_1} = \sum_{n=1}^{3} \left( \frac{\partial h_k}{\partial x} \right) \delta A_k \frac{\partial A_k}{\partial x} \right) \]

\[ \frac{\partial x}{\partial Fy_1} = \sum_{n=1}^{3} \left( \frac{\partial h_k}{\partial y} \right) \delta A_k \frac{\partial A_k}{\partial y} \right) \]

\[ \delta A_k \text{ are given by} \]

\[ \delta A_k = \int \left( \frac{\partial A_k}{\partial x} \right) \delta Fx_1 \]

\[ \delta A_k = \int \left( \frac{\partial A_k}{\partial y} \right) \delta Fy_1 \]

\[ \delta A_k \text{ are given by} \]

\[ \delta A_k = \int \left( \frac{\partial A_k}{\partial x} \right) \delta Fx_1 \]

\[ \delta A_k = \int \left( \frac{\partial A_k}{\partial y} \right) \delta Fy_1 \]

2.5 Calculation

Figure 3 shows the steps of calculation.

1. The initial values of vector potentials \( \{A\} \) and flux distribution ratios \( \{Fx_1\} \) and \( \{Fy_1\} \) are set.
2. The increments of the vector potentials \( \delta A \) can be calculated from Eq. (26).
3. The increments of flux distribution ratios \( \delta Fx_1 \) and \( \delta Fy_1 \) are calculated from Eqs. (34) and (35). \( \{\varepsilon_1\} \) and \( \{\varepsilon_2\} \) are small numbers for the decision of convergence.
4. The above steps are repeated alternately until the final solution is obtained. \( \{\varepsilon_0\} \) is a small number for determining convergence.

(a) first layer

(b) second layer

Fig. 4 Flow chart.
are a function of the fluxes in each layer denoted in Figs. 4, 6 and 8.

The results presented here were calculated within about 70(%) increase of computing time than a two-dimensional analysis.

4. CONCLUSIONS

By modifying the two-dimensional finite element method, it has become possible to analyze easily the flux distribution in each layer of laminated cores made of grain-oriented silicon steel. In our new method, the distribution of flux in each layer is determined by the minimum energy principle. The method has the following advantages:

(a) The approximate three-dimensional analysis is possible by adding several lines to the usual two-dimensional program.
(b) The computing time is within twice as much as two-dimensional one.

The results obtained provide information for designing the most suitable laminated core.

The method will be improved so that the energy due to the z-component of the flux and eddy currents can be taken into account.

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