Optimal design of tank shield model of transformer

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Optimal Design of Tank Shield Model of Transformer

Norio Takahashi, Fellow, IEEE, Tetsuro Kitamura, Makoto Horii, and Jun Takehara

Abstract—A tank shield model of transformer which is proposed by the Investigation Committee of IEE of Japan is analyzed. This is the model having the constraint that the maximum eddy current density should be less than a specified value. The automatic 3-D mesh generation technique for hexahedral element is investigated for the optimal design of such a model. It is shown that the reasonable results that satisfy the specified constraints can be obtained using the Rosenbrock’s method within the acceptable CPU time. The experimental verification is also carried out.

Index Terms—Optimization, Rosenbrock’s method, tank shield, transformers.

I. INTRODUCTION

It is important to examine the problems in applying the optimal design method to the practical design of magnetic devices. The “Investigation Committee on Highly Advanced Optimization Problems”, IEE of Japan has proposed five kinds of benchmark models for comparing optimal design methods. The “tank shield model of transformer” is a benchmark model for reducing the volume of shielding plate and for constraining the eddy current density in the tank within a specified value.

In this paper, tank shield model is optimized using the Rosenbrock’s method [1] and the finite element method (FEM). The effectiveness of the technique of generating 3-D mesh of hexahedral elements for the optimal design of practical model is discussed. The comparison with measurement is also shown.

II. TANK SHIELD MODEL OF TRANSFORMER

Fig. 1 shows the tank shield model of transformer. The tank plate is made of steel and the eddy current is taken into account (conductivity: 0.75 × 10⁻⁷ S/m). The shielding plate is made of grain-oriented silicon steel (grade(JIS): 30G140) in which no eddy current flows. The rolling direction of steel is the y-direction. The ampere-turns of the coil are 5484 AT(max) (12 A(max) × 475 turns, 60 Hz). As the steel is not saturated, the phasor method (so-called \( j_0 \) method) is applied in 3-D eddy current analysis by assuming that the magnetic characteristics of magnetic material is linear. The relative permeability of tank plate is assumed as 1000, and those in the rolling and transverse directions of silicon steel are assumed as 3000 and 30, respectively.

The dimension of analyzed region is 1000 mm × 1000 mm × 1000 mm. The dimensions \( L_1-L_4 \) of the shielding plate should be determined so that the following objective function \( W \) becomes minimum, and the maximum value of the eddy current density in the tank plate should be less than the specified value \( J_{\text{em}} \) in order to avoid the local heating:

\[
W = V + P
\]

where \( V \) is the volume of shielding plate and \( P \) is the penalty function which are defined by

\[
V = 10 \times (L_1 + L_2 + L_3 + L_4) \times 10^{-3} \text{ [m}^3]\]

\[
P = \begin{cases} 0 & (J_{\text{em}} < J_{\text{ema}}) \\ J_{\text{em}} & (J_{\text{em}} \geq J_{\text{ema}}) \end{cases}
\]

\( J_{\text{em}} \) is given by

\[
J_{\text{em}} = \sqrt{|j_{ex}|^2 + |j_{ey}|^2 + |j_{ez}|^2}
\]
TABLE I
ORTHOGONAL ARRAY

<table>
<thead>
<tr>
<th>model no.</th>
<th>design variable (mm)</th>
<th>$V$ ($10^3$ m$^3$)</th>
<th>$J_{em}$ ($10^4$ A/m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 1 1 1</td>
<td>1.0</td>
<td>0.392</td>
</tr>
<tr>
<td>2</td>
<td>1 2 2 2</td>
<td>1.75</td>
<td>0.400</td>
</tr>
<tr>
<td>3</td>
<td>1 3 3 3</td>
<td>2.5</td>
<td>0.406</td>
</tr>
<tr>
<td>4</td>
<td>2 1 3 2</td>
<td>2.0</td>
<td>0.203</td>
</tr>
<tr>
<td>5</td>
<td>2 2 1 3</td>
<td>2.0</td>
<td>0.203</td>
</tr>
<tr>
<td>6</td>
<td>2 3 2 1</td>
<td>2.0</td>
<td>0.203</td>
</tr>
<tr>
<td>7</td>
<td>3 1 2 3</td>
<td>2.25</td>
<td>0.131</td>
</tr>
<tr>
<td>8</td>
<td>3 2 3 1</td>
<td>2.25</td>
<td>0.131</td>
</tr>
<tr>
<td>9</td>
<td>3 3 1 2</td>
<td>2.25</td>
<td>0.130</td>
</tr>
<tr>
<td>10</td>
<td>— — — —</td>
<td>—</td>
<td>3.462</td>
</tr>
</tbody>
</table>

where $\cot(\cdot)$ means the complex number. As $J_{em}$ is of the order of $10^3$ and $V$ is of the order of $10^{-4}$, (1) and (3) can be approximated as follows:

$$W = \begin{cases} V [m^3] & (J_{em} < J^{\text{opt}}) \\ J_{em} [A/m^2] & (J_{em} \geq J^{\text{opt}}) \end{cases}$$

The constraint of $L_2 - L_4$ is given by

$$0 < L_1, L_2, L_3, L_4 < 0.01 \text{ [m]}$$

III. METHOD OF OPTIMIZATION

Although the evolution strategy, the simulated annealing method etc. are suitable to obtain the global minimum of the objective function, the number of iterations becomes huge and not useful for 3-D optimization. Therefore, the Rosenbrock’s method (RBM), which is the direct search method, is used for the optimization from the standpoint of the CPU time. The experimental design method (EDM, Taguchi’s method) [2] is used to determine the upper value of eddy current density which is suitable for the optimization method of tank shield model and to determine the appropriate initial values. In this case, constraints are divided into three levels (1: low (2.5 mm), 2: medium (5.0 mm), 3: high (7.5 mm)).

The volume $V$ of shielding plate and the maximum eddy current density $J_{em}$ in the tank plate, which has nine patterns, are calculated from the orthogonal array (Nos. 1–9) shown in Table I. $J_{em}$ calculated using 3-D FEM for the respective combinations of design variables $L_1, L_2, L_3$ and $L_4$ and the volume $V$ of shielding plate are also shown in Table I. $V$ and $J_{em}$ at the models Nos. 4, 5 and 6 are nearly the middle values and the minimum (optimal) value of $V$ may exist if the upper limit of eddy current density is determined as around these values. Then, the specified value $J^{\text{opt}}$ of the maximum eddy current density is assumed as $0.24 \times 10^6$ A/m$^2$.

Although $0.24 \times 10^6$ A/m$^2$ is not enough to heat the tank plate, it is assumed that the tank plate is overheated when $J_{em}$ is larger than $0.24 \times 10^6$ A/m$^2$ in order to become possible to carry out the experiment in laboratory.

![3-D mesh. (a) Whole view. (b) Enlarged view.](image)

**Fig. 2.** 3-D mesh. (a) Whole view. (b) Enlarged view.

**TABLE II**
RESULTS OF OPTIMIZATION

<table>
<thead>
<tr>
<th></th>
<th>case A</th>
<th>case B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>initial</td>
<td>optimal</td>
</tr>
<tr>
<td>design $L_1$</td>
<td>5.0</td>
<td>4.08</td>
</tr>
<tr>
<td>variable $L_2$</td>
<td>5.0</td>
<td>4.51</td>
</tr>
<tr>
<td>(mm) $L_3$</td>
<td>5.0</td>
<td>0.59</td>
</tr>
<tr>
<td>$J_{em}$ ($10^4$ A/m$^2$)</td>
<td>0.203</td>
<td>0.247</td>
</tr>
<tr>
<td>number of iterations</td>
<td>51</td>
<td>28</td>
</tr>
<tr>
<td>CPU time (h)</td>
<td>49.3</td>
<td>25.7</td>
</tr>
</tbody>
</table>

Computer used: VT-Alpha533 (SPECint95:22.5)

The result without shielding plate is denoted as model no. 0 in Table I.
Fig. 3. Initial shape (case B). (a) Shape of $x$-$y$ plane, (b) flux distribution ($\omega t = 0$ deg), (c) eddy current distribution on tank plate ($x = 110$ mm), (d) contour line of eddy current density ($x = 110$ mm).

IV. 3-D MESH GENERATION FOR OPTIMAL DESIGN

In the optimal design using the finite element method, the mesh must be changed according to the obtained design variables at each iteration.

As there is no universal automatic mesh generator for hexahedral elements [3], a technique for making the pile in the $z$-direction of 2-D mesh of quadrilateral element in the $x$-$y$ plane as shown in the following process is introduced:

a) The 3-D model is projected on the $x$-$y$ plane as shown in Fig. 1(b). Delaunay method is applied to divide the region having the obtained shape (in principle, arbitrary shape correspond to design variables) into 2-D triangular elements. Then, the mesh of quadrilateral elements is obtained from these triangular elements as shown in Fig. 2(b). The skin depth region (0.75 mm) of iron steel is subdivided into three-layers to obtain accurate results. In this model, the tank plate is subdivided into eight layers in the $x$-direction.

b) The 2-D mesh of quadrilateral elements is piled up in the $z$-direction, then the mesh of hexahedral elements is obtained. The number of quadrilateral elements is about 3200 in this case. The total number of hexahedral elements is about 30,000.

Although this mesh generator for hexahedral element is not universal, the optimization of the model having arbitrary shape in 2-D plane and having brick shape in the pile up direction ($z$-direction) is possible by utilizing this mesh generator.

V. RESULTS AND DISCUSSION

Table II shows results obtained using only Rosenbrock’s method (RBM, case A) and the combined method of EDM
and RBM (case B) [4]. In the combined method (case B), the initial values of design variables for Rosenbrock’s method are determined by using EDM. In this case, the variables of the model no. 5 in Table I is used as the initial value. If the change of design variables \( L_1 - L_4 \) becomes less than 0.1 mm in the process of direct search of RBM, it is judged that the final result is obtained.

The volume \( V \) in the case B of which the initial values are determined by EDM is smaller (about 9\%) than that in the case A having the same initial values (\( L_1 = L_2 = L_3 = L_4 = 5 \) mm). And the CPU time in the case B is shorter than that in the case A. Therefore, the combined method of EDM + RBM may be effective.

Figs. 3 and 4 show the shapes, flux and eddy current distributions and contour lines of eddy current density at the initial shape and the final shape in the case B. Figures (c) and (d) are the distributions on the surface of tank plate which is observed from the coil side. Figs. 3 and 4 and Table II denote that about 60\% of the volume \( V \) of shielding plate is reduced, and the eddy current density can be limited within the specified value by the shielding plate.

Fig. 5 shows the comparison of the \( y \)-component \( B_y \) of flux density calculated by 3-D FEM along the line c-d in Fig. 1(b) and measured value without shielding plate. As this is the open circuit model, the error of the case without tank plate is larger than that of the case with tank plate. The result of nonlinear analysis for the case with tank plate is also shown. The CPU time of nonlinear analysis is about 120 hours using VT-Alpha600 (SPECfp95:27.0). As the difference between the linear and nonlinear analysis is small, the optimization
VI. CONCLUSION

The obtained results can be summarized as follows:

a) It is shown that the technique for making the pile in the $z$-direction of 2-D mesh of quadrilateral element in the $x$-$y$ plane is effective in 3-D optimization.

b) The optimal dimension of shielding plate can be obtained by considering 3-D eddy current within the acceptable CPU time.

REFERENCES


