Effective optimal design of 3-D magnetic device having complicated coil using edge element and Biot-Savart method

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Effective Optimal Design of 3-D Magnetic Device Having Complicated Coil Using Edge Element and Biot–Savart Method

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***Abstract*—It is troublesome and time consuming to treat a coil of complicated shape in the optimization method using edge element, because the current vector potential in the coil of changed shape (changed mesh) should be calculated at each iteration of optimization. Then, an effective optimal design method using the Biot–Savart method ($\boldsymbol{A}_{\rm s}$ method) and the evolution strategy is investigated. As it is not necessary to generate a mesh for the coil by using the $\boldsymbol{A}_{\rm s}$ method, the mesh becomes simple and, moreover, it is not necessary to calculate the current vector potential in the changed coil at each iteration in order to give the current vector potential in the coil. The usefulness of the proposed method is illustrated by applying it to the optimal design of a deflection coil for a cathode-ray tube.**

***Index Terms*—Deflection coil, finite element method, optimal design method.**

I. Introduction

ALTHOUGH there are many reports of the optimal design method using the finite element method, the optimal design using three dimensional (3-D) finite element method (FEM) is few due to the troublesome mesh generation, long central processing unit (CPU) time, etc. If the edge element, which is widely used for 3-D analysis, is introduced to the optimal design, the current vector potential in the coil of changed shape should be calculated at each iteration of optimization. This causes the difficulty of generating mesh for a coil of complicated shape and requires huge CPU time.

In this paper, a technique for optimal design of 3-D magnetic device having complicated coil is investigated. The analyzed region except coils is subdivided using edge elements. The vector potential produced by the source current in a coil is calculated using the Biot–Savart law (this is called as $\boldsymbol{A}_{\rm s}$ method). As it is not necessary to subdivide a coil to finite elements, the change of coil at each iteration is very easy. The $\boldsymbol{A}_{\rm s}$ method and the $(1+1)$-evolution strategy (ES) [1] are introduced for practical design. The usefulness of the optimal design method is illustrated by applying it to the design of a deflection coil [2] of complicated shape.

II. Optimization Using $\boldsymbol{A}_{\rm s}$ Method

A. $\boldsymbol{A}_{\rm s}$ Method

In the $\boldsymbol{A}_{\rm s}$ method, the vector potential $\boldsymbol{A}$ is treated as the sum of the vector potential $\boldsymbol{A}_{\rm s}$ produced by the source current and the reduced vector potential $\boldsymbol{A}_{\rm r}$ [3] as follows:

$$\boldsymbol{A} = \boldsymbol{A}_{\rm s} + \boldsymbol{A}_{\rm r}. \tag{1}$$

$\boldsymbol{A}_{\rm s}$ is calculated using the Biot–Savart's law as follows:

$$\boldsymbol{A}_{\rm s} = \int \frac{\mu_0 I}{4\pi r}\, d\boldsymbol{s} \tag{2}$$

where I is the source current and r is the distance from the line segment $d\boldsymbol{s}$ to the field point. The basic equation of the $\boldsymbol{A}_{\rm s}$ method is given by

$$\mathrm{rot}(\nu\, \mathrm{rot}\boldsymbol{A}_{\rm r}) = 0 \tag{3}$$

where ν is the reluctivity.

From (1) and (3), the following equation is obtained:

$$\mathrm{rot}(\nu\, \mathrm{rot}\boldsymbol{A}) = \mathrm{rot}(\nu_0\, \mathrm{rot}\boldsymbol{A}_{\rm s}) \tag{4}$$

where ν_0 is the reluctivity of vacuum.

As it is not necessary to generate a mesh for the coil by using the $\boldsymbol{A}_{\rm s}$ method, the mesh becomes simple and, moreover, it is not necessary to calculate the current vector potential in the changed coil at each iteration in order to give the current vector potential in the coil.

B. $(1+1)$ *ES*

As the generation of mesh of coil is not necessary in the $\boldsymbol{A}_{\rm s}$ method, it is easy to change the shape of coil. Therefore, the $\boldsymbol{A}_{\rm s}$ method is suitable for the optimal design method. $(1+1)$-ES is used as an optimization method, which is useful to obtain a global minimum. ES is an optimization method which applies the law of nature, namely the mutation and the natural selection. The mutation is a generation of the new design values (child vector), which is modified from a previous design values (parent vector) by normal random numbers. At the selection, the parent vector in the next generation, is selected so that the objective function becomes smaller. The initial value of standard deviation [4] is chosen as three.

Fig. 1 shows the flow chart of optimization by utilizing $\boldsymbol{A}_{\rm s}$ method and $(1+1)$-ES.

III. Analyzed Model and Objective Function

A. *Model of Deflection Coil*

Fig. 2 shows a model of deflection coil. The deflection coil is composed of three kinds coils A, B and C. The ampere turns of the upper (A), middle (B), and lower (C) coils are 16, 8, and 4 AT, respectively. The shape of coil is defined by the points

Manuscript received July 5, 2001.

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Publisher Item Identifier S 0018-9464(02)02058-7.

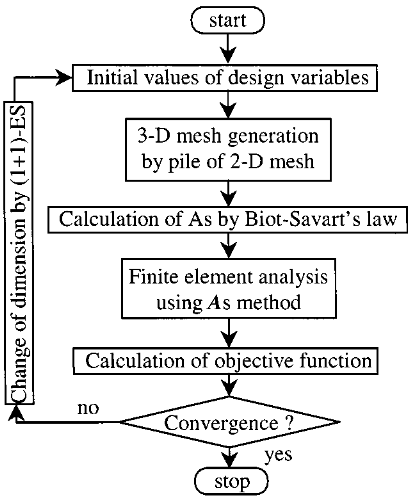


Fig. 1. Flow chart.

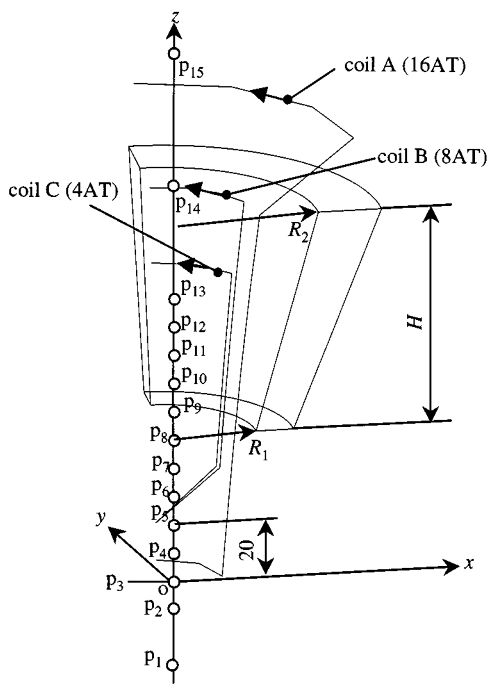


Fig. 2. Model of a deflection coil (analyzed region: 1/4).

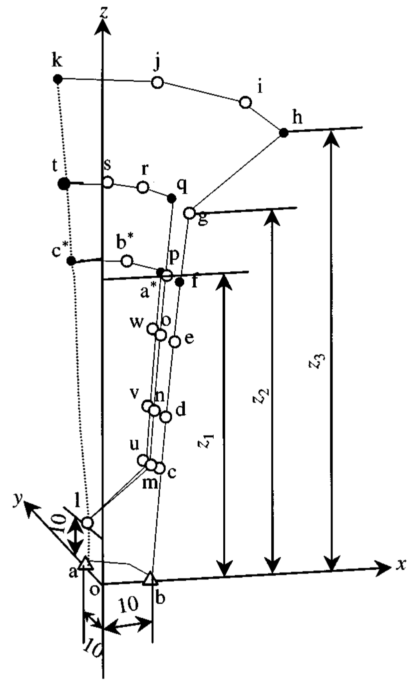
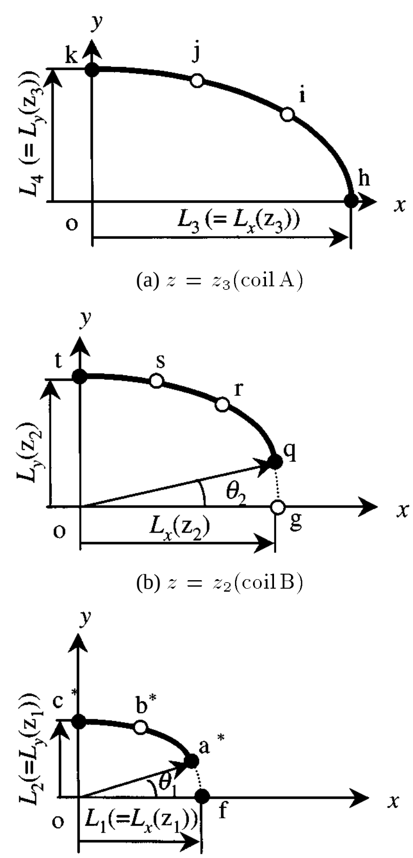


Fig. 3. Definition of shape of coils.



(a) $z = z_3$(coil A)

(b) $z = z_2$(coil B)

(c) $z = z_1$(coil C)

Fig. 4. Coils in xy planes.

(△, •, ∘) shown in Fig. 3. The curve between two points is approximated by a straight line.

The coordinates of points a and b of triangles (△) are given as a (10,0,0) and b (0,10,0).

The coordinates of points of black circles (•) are given by the design variables as follows: Fig. 4 shows the shape of coils in the xy planes which are cut at $z = z_1$, z_2, and z_3. These z coordinates z_1, z_2, and z_3 are chosen as design variables. The long and short axes L_1, L_2, L_3, and L_4 of the ellipse and the angles θ_1 and θ_2 in Fig. 4 are also chosen as design variables. The equation of the ellipse shown in Fig. 4 is given by

$$\left(\frac{x}{L_x(z)}\right)^2 + \left(\frac{y}{L_y(z)}\right)^2 = 1 \tag{5}$$

where $L_x(z)$ and $L_y(z)$ are the long and short axes in the x and y directions.

The coordinates of points of white circle (∘) are given as follows: The positions of points b*, r, s, i, and j are automatically determined by considering that they are on the ellipse. The points c, d, e, g, m, n, o, p, u, v, and w are determined

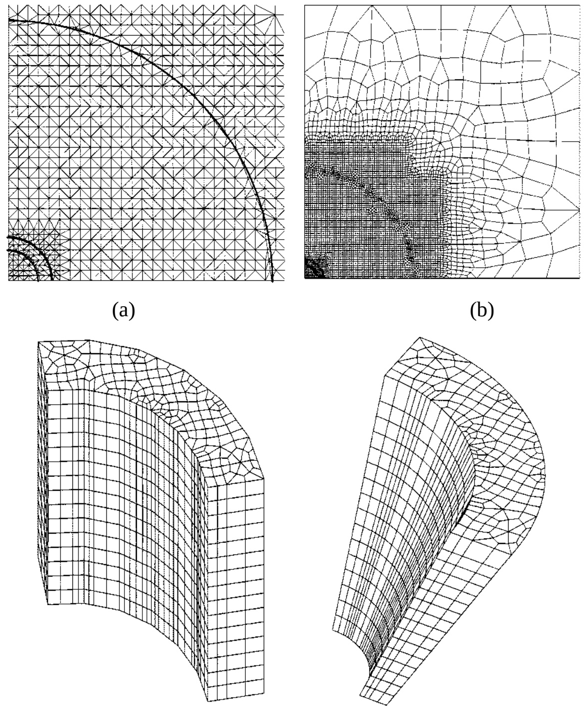


Fig. 5. 3-D mesh generation. (a) Outline of region. (b) 2–D mesh. (c) Pile of yoke. (d) Modified mesh.

by approximating the curves b–c–d–e–f–g–h, a–l–c*–t–k, m–n–o–p–q, and u–v–w–a* by quadratic functions.

Outside of the coil, there is an inclined cylindrical core of which the thickness is 10 mm. The radius R_1 and R_2, which are inside diameters at the bottom and the top of core, and the height H shown in Fig. 2 are chosen as design variables in the optimization.

A quarter of the region ($x > 0$, $y > 0$) is analyzed. 3-D mesh of hexahedral edge elements is easily generated by the pile of two-dimensional (2-D) mesh [Fig. 5(a)–(c)] and by the transformation [Fig. 5(d)], because it is not necessary to generate a mesh of coil.

B. Constraints and Objective Functions

The constraints of design variables are chosen as follows:

$$\begin{array}{ll} 0 < L_1 < 50 \text{ [mm]} & 0 < L_2 < 50 \text{ [mm]} \\ 0 < L_3 < 30 \text{ [mm]} & 0 < L_4 < 30 \text{ [mm]} \\ 60 < z_1 < 150 \text{ [mm]} & 90 < z_2 < 160 \text{ [mm]} \\ 100 < z_3 < 160 \text{ [mm]} & 0 < \theta_1 < 50 \text{ [deg]} \\ 0 < \theta_2 < 60 \text{ [deg]} & 45 < R_1 < 100 \text{ [mm]} \\ 50 < R_2 < 100 \text{ [mm]} & 45 < H < 100 \text{ [mm]}. \end{array} \quad (6)$$

Moreover, the coils must not penetrate into the core region in the optimization process. This constraint is considered by specifying that the distance between the coil and the inside of the core is larger than 10 mm.

These design variables that satisfy the above-mentioned constraints and the specified values of y component of flux density are optimized using $(1 + 1)$-ES. The objective function W is defined by

$$W = \sum_{i-1}^{ns} (B_{yi0} - B_{yi})^2 \quad (7)$$

where B_{yi} is the y component of flux density at a point $\mathrm{p}_i (i = 1, \ldots, ns, ns = 15)$ along the z axis shown in Fig. 2. B_{yi0} is the specified value.

TABLE I
COMPARISON OF RESULTS

	initial	optimal
L_1 [mm]	25	36.4
L_2 [mm]	25	38.2
L_3 [mm]	50	91.7
L_4 [mm]	50	76.1
R_1 [mm]	75	78.6
R_2 [mm]	57.5	34.7
z_1 [mm]	90	60.2
z_2 [mm]	110	133.6
z_3 [mm]	130	154.4
θ_1 [deg]	25	18.6
θ_2 [deg]	30	55
H [mm]	50	77.6
W	5.36×10^{-7}	6.18×10^{-11}
number of iterations	2250	
CPU time [h]	22.7	

computer used: VT-Alpha600 (SPECfp95 :27)

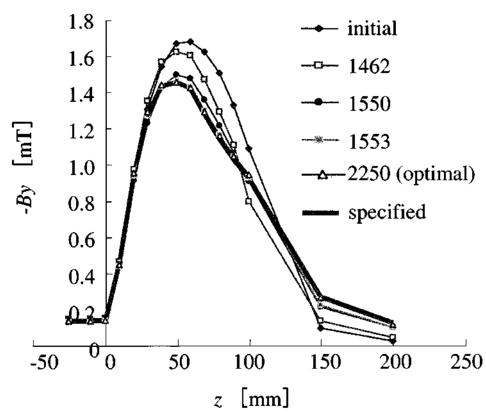


Fig. 6. y component of flux density along z axis.

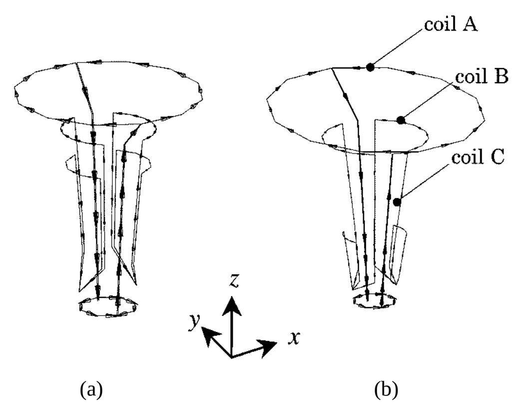


Fig. 7. Initial and optimal shape of coil. (a) Initial shape. (b) Optimal shape.

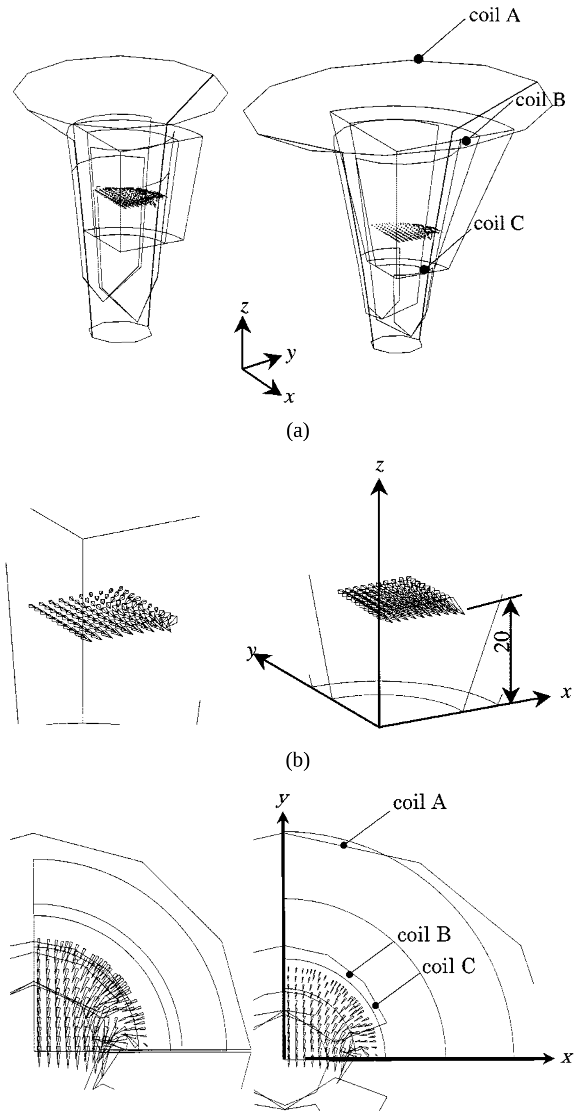


Fig. 8. Flux distributions. (a) Whole view. (b) Enlarged. (c) xy plane (z =60 mm).

IV. Optimization and Discussion

Table I shows the results of the initial and optimal shape. The value of objective function of optimal shape is reduced than that of initial one within a reasonable CPU time. Fig. 6 shows the y component of flux density along the z axis at some iterations. During all iterations from first to 1462th iteration and some of the iterations after 1462th iteration, the obtained parents of $(1+1)$-ES were outside the constraints, then FEM analysis is skipped during these iterations. Therefore, only about 190 FEM calculations are carried out in the iterations of optimization. In the optimal shape, the flux density is exactly the same with the specified value B_{yi0}. Figs. 7 and 8 show the shape of coils and flux distributions.

V. Conclusion

The results obtained are summarized as follows:

1) The optimization of 3-D magnetic device having complicated shape of coil can be easily carried out by using the $\boldsymbol{A}_{\mathrm{s}}$ method;
2) It is shown that the shape of deflection coil that produces the desired flux distribution can be obtained using the proposed optimization method within a reasonable CPU time.

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