A new approach to the transient analysis of magnetic field with variable transformation based on eigenvalues

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A NEW APPROACH TO THE TRANSIENT ANALYSIS OF MAGNETIC FIELD WITH VARIABLE TRANSFORMATION BASED ON EIGENVALUES

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Abstract: A new approach to the transient analysis of magnetic field is presented using the formulation of a boundary element model in two-dimensional problems. In order to solve the transient problems, state equations are formed using the boundary element model and are solved by using the variable transformation based on eigenvalues. The computational accuracy of the proposed method is evaluated by a infinite plate model and an infinite cylinder model in which the impressed magnetic flux densities are given by a unit step function and a sinusoidal function, respectively.

INTRODUCTION

Various useful numerical methods for the analysis of electric and magnetic fields have been developed and applied to static, quasi-static and transient problems. In finite element methods[1], time differentiation in the transient problems is approximated by the time difference method. On the other hand, in boundary element methods, the time differentiation is approximated by the time difference method, the Laplace transformation and the exponential kernel function[2,3]. As another approach, a network method was solved by using eigenvalues and eigen vectors of the computation model[4].

In this paper, a new approach to the transient analysis is presented using the formulation of the boundary element method. State equations for unknown variables in the region to be analyzed are formed by a boundary element model. The state equations are solved by the variable transformation based on eigenvalues and simple matrix operations. In the proposed method, a larger step-width can be chosen as compared with time difference method because of analytical treatment for the time differentiation. Using a step by step method, the proposed method can be applied to the model with the arbitrary external condition which does not give Laplace transform. The computational accuracy of the proposed method is evaluated by two models: a diffusion model in magnetic field varying a unit step function and a infinite cylindrical model in alternating magnetic field. Step response and sinusoidal solution are evaluated by the diffusion model and the infinite cylindrical model, respectively.

FORMULATION

The differential equation to be solved in two-dimensional problems is obtained from Maxwell's equations as follows:

$$\nabla^2 B - \mu\sigma \frac{\partial B}{\partial t} = 0 \tag{1}$$

where B is the z-component of the magnetic flux density, μ is the permeability and σ is the conductivity. Furthermore, replacing B by vector potential A, we can expand Eq.(1) to the equation for the analysis using vector potential.

When $\partial B/\partial t$ is regarded as the impressed term, the fundamental solution of Eq. (1) becomes that of static problems and it is given by

$$\phi = \frac{1}{2\pi} \ln \frac{1}{r} \tag{2}$$

where r is the distance between the source point and the computation point.

Using the Green's theorem, the integral equations at computation point i is obtained from Eqs. (1) and (2) as follows:

$$\frac{C_i}{2\pi} B_i + \int_\Gamma B \frac{\partial \phi}{\partial n} d\Gamma = \int_\Gamma \phi \frac{\partial B}{\partial n} d\Gamma - \mu\sigma \int_\Omega \frac{\partial B}{\partial t} \phi \, d\Omega \tag{3}$$

where Γ is the boundary surface, Ω is the region to be analyzed, C_i is the angle subtended by Ω at i, and $\mathbf{n}$ is the normal unit vector at source point. This equation reduces on the smooth boundary element to

$$\frac{B_i}{2} + \int_\Gamma B \frac{\partial \phi}{\partial n} d\Gamma = \int_\Gamma \phi \frac{\partial B}{\partial n} d\Gamma - \mu\sigma \int_\Omega \frac{\partial B}{\partial t} \phi \, d\Omega \tag{4}$$

and in the region to be analyzed to

$$B_i + \int_\Gamma B \frac{\partial \phi}{\partial n} d\Gamma = \int_\Gamma \phi \frac{\partial B}{\partial n} d\Gamma - \mu\sigma \int_\Omega \frac{\partial B}{\partial t} \phi \, d\Omega \tag{5}$$

After discretizing, the following equations are obtained from Eqs. (4) and (5).

$$[H_\Gamma]\{B_\Gamma\} = [G_\Gamma]\{\frac{\partial B_\Gamma}{\partial n}\} - \mu\sigma[S_\Gamma]\{\frac{\partial B_\Omega}{\partial t}\} \tag{6}$$

$$\{B_\Omega\} + [H_\Omega]\{B_\Gamma\} = [G_\Omega]\{\frac{\partial B_\Gamma}{\partial n}\} - \mu\sigma[S_\Omega]\{\frac{\partial B_\Omega}{\partial t}\} \tag{7}$$

where subscripts Ω and Γ express the values concerning the region Ω and boundary surface Γ, respectively. Combining Eq. (6) with Eq. (7) to eliminate unknown variables on the boundary surface, the following state equation is obtained.

$$\{\frac{\partial B_\Omega}{\partial t}\} = [T]\{B_\Omega\} + \{\Phi\} \tag{8}$$

where $\{\Phi\}$ is the vector which is given by the boundary conditions and impressed field.

Using the variable transformation based on eigen vectors, $\{B\}$ can be defined as following equations[5].

$$\{B\} = [U]\{x\} \tag{9}$$

where

$$[U] = [\{\eta_1\}, \{\eta_2\}, \cdots, \{\eta_n\}]$$

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$\{\eta_1\}$ is the eigen vector. The following equation is obtained from Eqs. (8) and (9).

$$\{\frac{\partial x}{\partial t}\}=[U]^{-1}[T][U]\{x\}+[U]^{-1}\{\Phi\} \tag{10}$$

By using eigen values, the matrix $[U]^{-1}[T][U]$ can be written[4] as

$$[U]^{-1}[T][U]=[\Lambda] \tag{11}$$

where

$$[\Lambda]=\begin{bmatrix}\lambda_1 & & & 0\\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n\end{bmatrix} \tag{12}$$

λ_i is an eigen value. Therefore, Eq. (10) can be rewritten as follows:

$$\{\frac{\partial x}{\partial t}\}=[\Lambda]\{x\}+[U]^{-1}\{\Phi\} \tag{13}$$

When the impressed field is given by a step function, the particular solution of Eq. (13) is given by

$$\{x\}=[E]\{k\} \tag{14}$$

$$[E]=\begin{bmatrix}e^{\lambda_1 t} & & & 0\\ & e^{\lambda_2 t} & & \\ & & \ddots & \\ 0 & & & e^{\lambda_n t}\end{bmatrix} \tag{15}$$

where $\{k\}$ is the constant vector which is determined by the initial condition.

Using Eq. (9), we can obtain the following equations.

$$\{B_\Omega\}=[U][E]\{k\} \tag{16}$$

In static problems, the solution of Eq. (8) is given by

$$\{B_\Omega\}=-[T]^{-1}\{\Phi\} \tag{17}$$

Therefore, using Eq. (16) and Eq. (17), the solution of Eq. (8) is given by[5]

$$\{B_\Omega\}=[U][E]\{k\}-[T]^{-1}\{\Phi\} \tag{18}$$

Determining $\{k\}$ in Eq. (18) to satisfy the initial condition, we can obtain the final equation as follows:

$$\{B_\Omega\}=[U][E][U]^{-1}(\{B_0\}+[T]^{-1}\{\Phi\})-[T]^{-1}\{\Phi\} \tag{19}$$

where $\{B_0\}$ contains the initial values of the variables.

When the initial values $\{B_0\}$, the boundary conditions and impressed field are given, the magnetic flux density at any given time is obtained from Eq. (19).

COMPUTATION RESULTS

Infinite plate model

Figure 1 shows an infinite plate model[7]. The theoretical solution for this model is given by

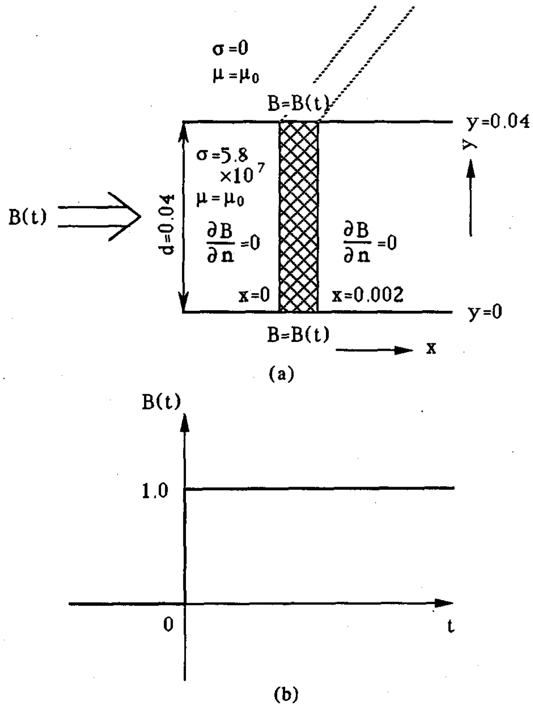


Fig. 1 Infinite plate model, (a)computation model, (b)impressed magnetic flux density.

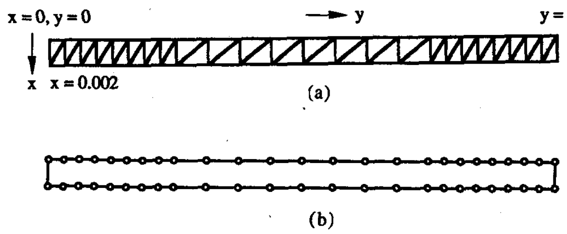


Fig. 2 Triangular mesh and boundary elements for the infinite plate model, (a)triangular mesh, (b)boundary elements.

$$B(y,t)=1-\sum_{n\ \mathrm{odd}}\frac{4}{n\pi}\sin\frac{n\pi y}{d}\,e^{-\lambda_n t} \tag{20}$$

where

$$\lambda_n=\frac{n^2\pi^2}{\mu\sigma d^2} \tag{21}$$

The triangular mesh and the boundary elements for the infinite plate model are shown in Fig. 2. The number of triangular elements and the number of boundary elements are 48 and 50, respectively. The unknown variables are defined to be constant on each element. Figure 3 shows the computation results at $t=1.2\mathrm{x}10^{-3}$(sec), $t=3.6\mathrm{x}10^{-3}$(sec) and $t=1.2\mathrm{x}10^{-2}$(sec). The computation results agree with theoretical values.

Figure 4 shows the eigenvalues and the eigenmodes from the first mode to the forth mode. The distribution of the eigen vectors agrees with the sinusoidal function in Eq.

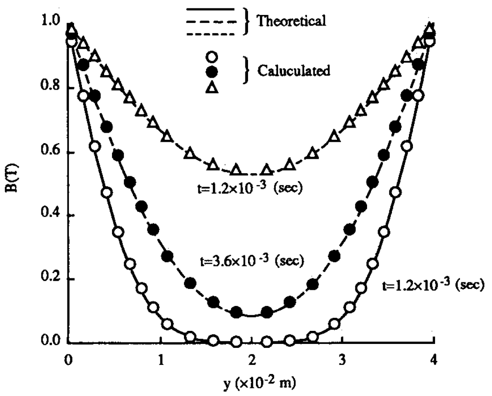


Fig. 3 Distributions of the magnetic flux density along y axis.

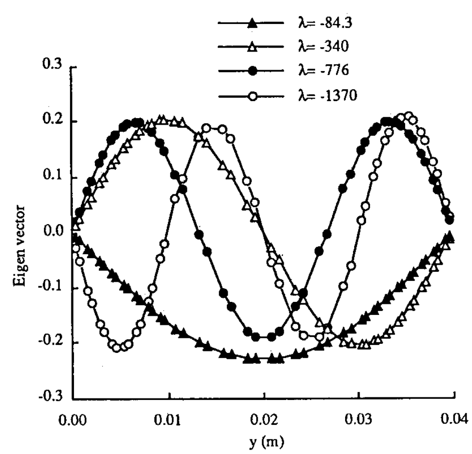


Fig. 4 Eigen modes of the infinite plate model from the first mode to the fourth mode.

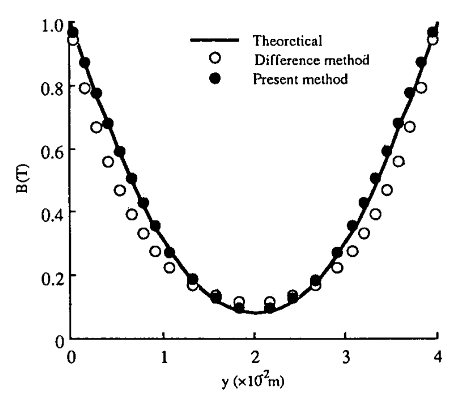


Fig. 5 Comparison between the proposed method and the difference method.

(20). The computation results of eigen values agree with theoretical values: λ_1=-82.97, λ_2=-331.6, λ_3=-746.3 and λ_4= -1326.

Figure 5 shows the computation results by the proposed method and the computation results by the time difference method[8] which use 3.6×10^{-3}(sec) as time step. From Fig. 5, it is clear that lager step-width of time can be chosen in the proposed method as compared with the time difference method.

Infinite cylinder model

Figure 6 shows an infinite cylinder model in which the impressed magnetic flux density is a sinusoidal function of time. For this model, Eq. (19) is modified as follows:

$$\{B_\Omega(t_2)\} = [U]\,[E]\,[U]^{-1}(\{B_\Omega(t_1)\} + [T]^{-1}\{\Phi(t_1)\}) - [T]^{-1}\{\Phi(t_2)\} \tag{22}$$

$$[E] = \begin{bmatrix} e^{\lambda_1 \Delta t} & & & 0 \\ & e^{\lambda_2 \Delta t} & & \\ & & \ddots & \\ 0 & & & e^{\lambda_n \Delta t} \end{bmatrix} \tag{23}$$

were $\Delta t = t_2 - t_1$. When $\{B_\Omega(t_1)\}$ is given, we can obtain the initial values for the next step, $\{B_\Omega(t_2)\}$, from Eq. (22). Figure 7 shows the computation results of the magnetic flux density after ten cycles of impressed field. The computation results almost agree with theoretical values[9].

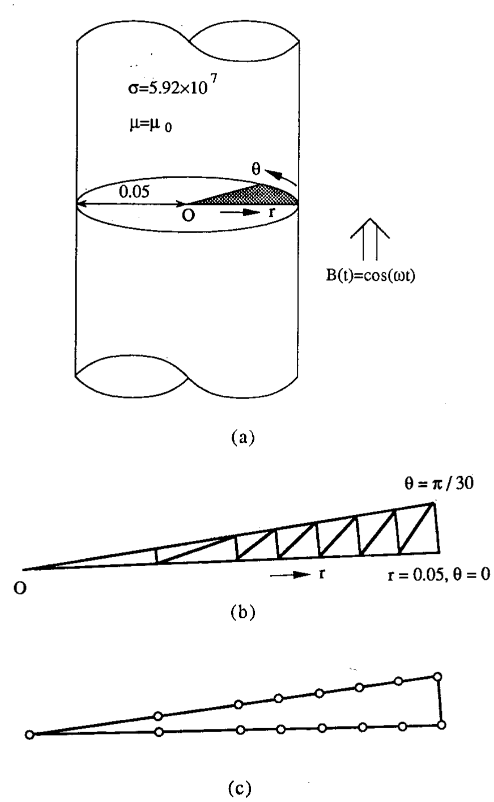
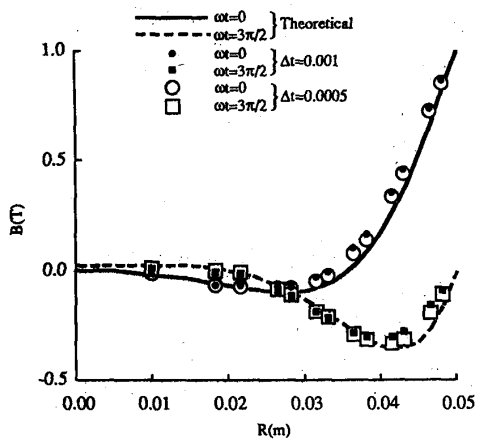
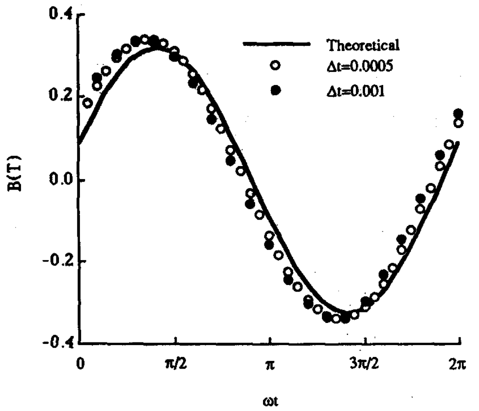


Fig. 6 Infinite cylinder model, (a)computation model, (b)triangular mesh, (c)boundary elements.



(a)



(b)

Fig. 7 Computed results of the infinite cylinder model, (a)distributions of the magnetic flux density along radial line, (b)distributions of the magnetic flux density at r=0.04(m) during eleventh cycle of the impressed field.

CONCLUSION

A new approach to the transient analysis of magnetic field was presented using the formulation of the boundary element model in two-dimensional problems. The state equations for the unknown variables in the region to be analyzed are introduced because the domain integrals are unavoidable in the transient problem. In order to obtain the state equations, the unknown variables on the boundaries are eliminated in the state equations. The state equation is solved by the variable transformations based on eigenvalues. The larger step-width of time can be chosen in the proposed method compared with the time difference method because the proposed method treats the time differentiation analytically. Furthermore, the applicability of the proposed method for time-changing impressed field has been shown through the computed results.

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