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Improvements of convergence characteristics of Newton-Raphson method for nonlinear magnetic field analysis

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IMPROVEMENTS OF CONVERGENCE CHARACTERISTICS OF NEWTON-RAPHSON METHOD FOR NONLINEAR MAGNETIC FIELD ANALYSIS

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Abstract—**In order to overcome the divergence of the Newton-Raphson iteration in the nonlinear magnetic field analysis, a relaxation factor is introduced, and its optimum value is examined. It is shown that the modified Newton-Raphson method proposed in this paper shows quick and successful convergence even in the case when the conventional Newton-Raphson method fails in convergence.**

I. INTRODUCTION

When the magnetic scalar potential method was applied to the analysis of 3-D nonlinear magnetic fields (for example, the TEAM Workshop Problem 13), very often the nonlinear iteration using the conventional Newton-Raphson method failed to converge[1]. If the relaxation factor was introduced, the convergence characteristics were fairly improved[1]. Therefore, most codes, which employed the magnetic scalar potential method, introduced such a factor[2,3] when Problem 13 was analyzed[4].

In this paper, a method for determining the optimum relaxation factor, which utilizes the residual of Galerkin method, is developed. Moreover, some techniques to find the optimum value briefly are discussed. The effectiveness of the proposed method is illustrated quantitatively by applying the method to practical problems.

II. MODIFIED NEWTON-RAPHSON METHOD

A. Method for Determining Optimum Relaxation Factor

In the modified Newton-Raphson method, the obtained magnetic scalar potential $\Omega_i^{(k+1)}$ of a node i at the (k+1)-th iteration can be represented by the following equation :

$$\Omega_i^{(k+1)} = \Omega_i^{(k)} + \alpha^{(k)} \cdot \delta\Omega_i^{(k)} \tag{1}$$

where $\delta\Omega$ is the increment of Ω. α is the relaxation factor introduced. The case of α=1 corresponds to the conventional Newton-Raphson method. The optimum value α_{opt} of the relaxation factor can be determined using the linear search method[5].

The residual G_i can be written as follows[6]:

$$G_i^{(k+1)} = -\iiint \text{grad } N_i \cdot \{\mu^{(k+1)} (T_o - \text{grad } \Omega^{(k+1)})\} dV \tag{2}$$

where N_i is the interpolation function and μ is the permeability. T_o is the current vector potential corresponding to the magnetizing current density. If Ω converges, G_i will approach zero. Therefore, α_{opt} should be determined so that the following objective function W becomes a minimum:

$$W^{(k+1)} = \sum_{i=1}^{nu} \{G_i^{(k+1)}\}^2 \tag{3}$$

where nu is the total number of unknown variables.

B. Techniques for Reducing CPU Time

Although our new method is superior from the standpoint of the convergence, the computing time for finding the optimum relaxation factor is a problem. In this section, some techniques for reducing the CPU time for finding the optimum value are discussed.

1) Calculation of objective function: The objective function W defined by (3) can be separated into two parts, which are W_s and W_a in the nonlinear and linear regions, as follows :

$$W^{(k+1)} = W_s^{(k+1)} + W_a^{(k+1)} \tag{4}$$

$$W_s^{(k+1)} = \sum_{i=1}^{ns} \{G_i^{(k+1)}\}^2 \tag{5}$$

$$W_a^{(k+1)} = \sum_{j=1}^{na} \{G_j^{(k+1)}\}^2 \tag{6}$$

where ns and na are the number of unknown variables in the nonlinear region and that in the linear region respectively. Since W_a is negligibly small compared with W_s (this will be

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shown in the next Section), the following approximation can be done:

$$W^{(k+1)} \doteqdot W_s^{(k+1)} \tag{7}$$

When ns is much less than na, the CPU time can be considerably saved using (7).

2) Golden section method: There are many kinds of methods to determine α_{opt} which gives the minimum W_s. The golden section method[5], which is a kind of the linear search method, is superior from the standpoint of the CPU time. Since the CPU time for the golden section method is dominated by the number of iterations in it, the number is discussed in the next Section.

III. EXAMPLES OF APPLICATION

A. Analyzed Model

As mentioned above, the TEAM Workshop Problem 13[1] which is shown in Fig.1 is chosen as the analyzed model which fails to converge by the conventional Newton-Raphson method. Since the convergence characteristics depend on the subdivision, two kinds of meshes which are shown in Fig.2 and Table I are compared. The 1st-order brick nodal element and the $\dot{T}$-Ω method[6] are applied. The periodic boundary condition[1] which is applied on the y-z plane enables us to analyze only 1/4 of the whole region.

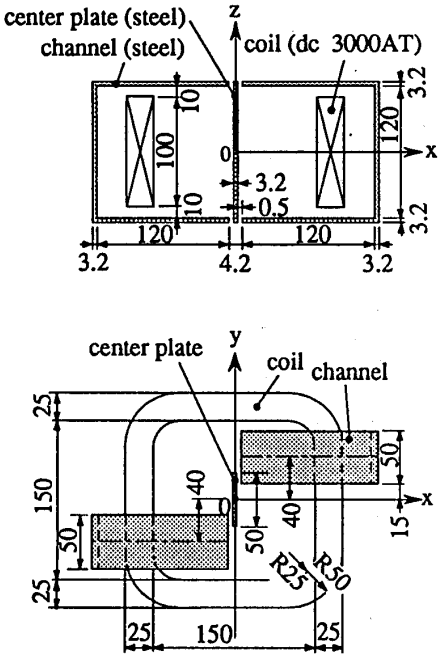


Fig. 1 3-D nonlinear magnetostatic model (TEAM Workshop Problem 13).

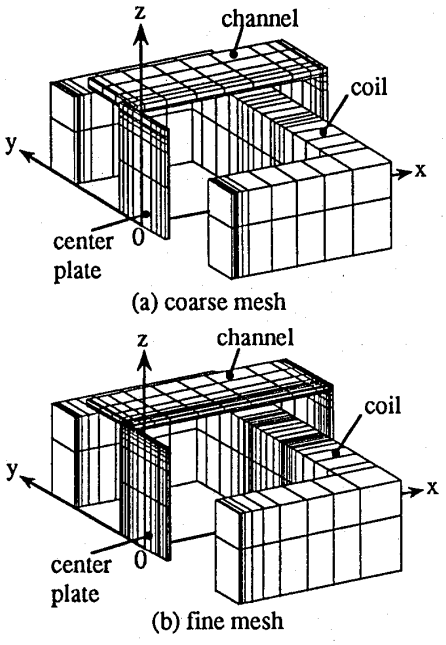


Fig. 2 Meshes.

Table I Discretization data

mesh	coarse	fine
number of elements	3,564	6,930
number of nodes	4,370	8,184
number of unknowns	2,944	5,950
number of non-zero entries	36,012	74,949

B. Results and Discussion

In order to examine the method for finding the optimum relaxation factor α_{opt}, the relationship between the relaxation factor $\alpha^{(1)}$ and the objective function $W^{(2)}$ at the second step of iteration for the modified Newton-Raphson method shown in Fig.3 is investigated. The curve for the coarse mesh has two local minima. The optimum values for the coarse and fine meshes are 0.3 and 0.2 respectively.

Figure 4 shows α_{opt} at each iteration for the modified Newton-Raphson method. Since α_{opt} changes sharply with iteration, it should be determined for every iteration.

Figures 5(a) and (b) show the objective functions W_s and W_a in the nonlinear and linear regions defined by (5) and (6) at two different steps for the modified Newton-Raphson method. W_a is negligibly small compared with W_s at any step and for any α. Therefore, the approximation using (7) is permissible.

Table II shows the numbers ns and na of unknown variables in the nonlinear and linear regions. Since ns is usually very small compared with na as shown in this table, we can use the speedup method mentioned in Sections II *B 1)*.

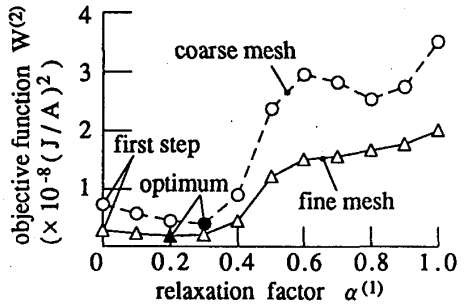


Fig. 3 Relationship between relaxation factor α and objective function W (second step of Newton-Raphson iteration).

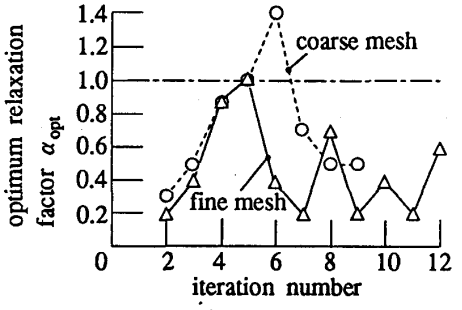


Fig. 4 Optimum relaxation factor α_{opt} at each iteration.

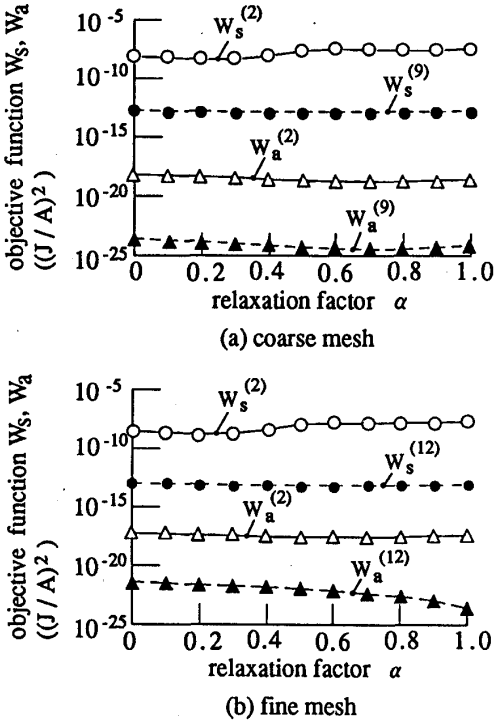
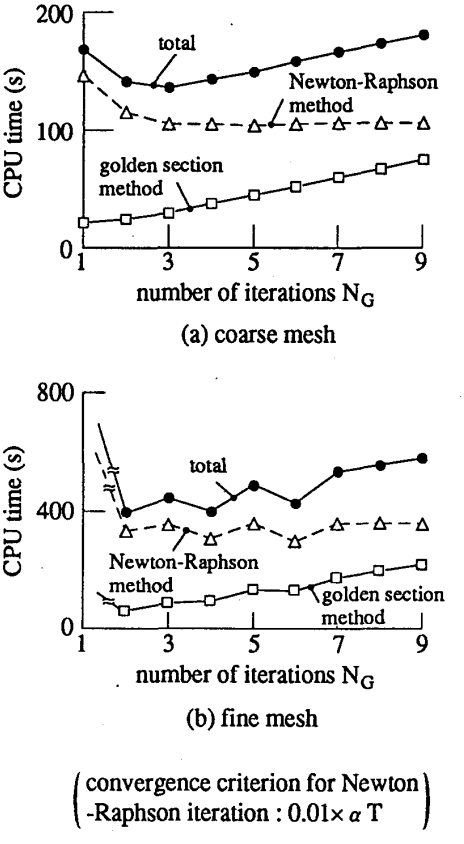


Fig. 5 Objective functions W_S and W_a in nonlinear and linear regions.

Table II Number of unknown variables

region		mesh	
		coarse	fine
total	(nu)	2,944(1)	5,950(1)
nonlinear	(ns)	408(0.14)	988(0.13)
linear	(na)	2,536(0.86)	4,962(0.87)

Figures 6(a) and (b) show the CPU time for the coarse and fine mesh respectively. The convergence criterion for the Newton-Raphson iteration is chosen as $0.01\times\alpha T$. Although α_{opt} may become larger than 1 as shown in Fig.4, the range of search of α for finding α_{opt} should be limited to a range between 0 and 1. This is, because the nonlinear iteration should be decelerated by an underrelaxation factor ($\alpha<1$). The CPU time for Newton-Raphson method does not include that for the golden section method. The CPU time for the golden section method increases considerably with the number of iterations N_G for the golden section method. The total CPU time for the coarse mesh has a minimum at $N_G=3$. Although the suitable number of iterations for the fine mesh is not obvious due to the oscillation, $N_G=3$ can be acceptable. Therefore, N_G is fixed at 3.

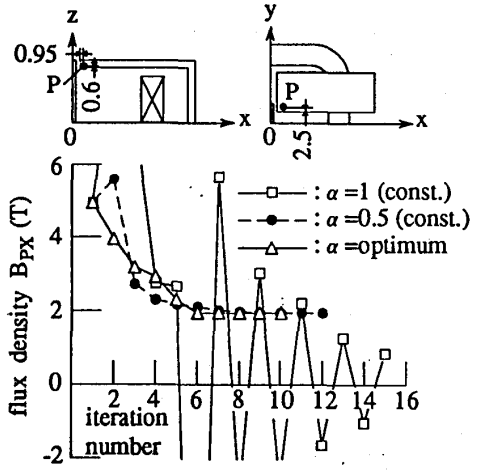


(convergence criterion for Newton-Raphson iteration : $0.01\times\alpha T$)

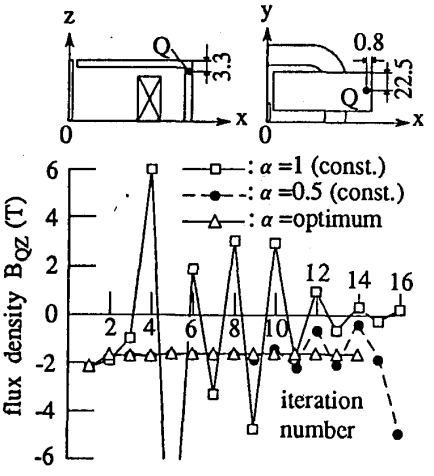
Fig. 6 CPU time.

Figure 7(a) shows the x-component B_{PX} of flux density at the point P for the coarse mesh. The point P (near the corner of the channel) is chosen from the standpoint that the flux density at the point P contains remarkable error because the flux density changes steeply near here. Although the flux density oscillates when the conventional Newton-Raphson method (α=1) is used, it converges for the cases of α=0.5 and α_{opt}. Figure 7(b) shows the z-component B_{QZ} of flux density at the point Q for the fine mesh. As B_{PX} does not oscillate violently with the number of iterations, another flux density B_{QZ} at the point Q is chosen. The convergence characteristics are fairly stabilized when α_{opt} is used for every iteration.

Table III shows the number of iterations for Newton-Raphson method and the CPU time. Since many iterations for the golden section method are required for the determination of α_{opt}, the CPU time for each Newton-Raphson iteration is increased. However, the number of iterations for Newton-Raphson method is reduced.



(a) coarse mesh



(b) fine mesh

Fig. 7 Convergence characteristics of Newton-Raphson method.

Table III Number of iterations for Newton-Raphosn method and CPU time

relaxation factor α	number of iterations N_R		CPU time (s)	
	coarse	fine	coarse	fine
1 (const.)	osc.*	osc.*	∞*	∞*
0.5 (const.)	12	osc.*	130	∞*
optimum**	10	14	136	447

computer used : NEC supercomputer SX-1E
(maximum speed : 285MFLOPS)
convergence criterion for
Newton-Raphson method : $0.01\times\alpha$ T
convergence criterion for ICCG method : 10^{-5}
* number of iterations for
Newton-Raphson method : 30
** number of iterations for golden section method : 3

IV. CONCLUSIONS

The results obtained can be summarized as follows:

(1) A modified Newton-Raphson method is introduced.
(2) The optimum relaxation factor can be obtained by minimizing the total square residual of Galerkin method.
(3) Methods for reducing the CPU time for finding the optimum relaxation factor are proposed.
(4) The method is applied to a practical problem and compared the convergence characteristics and the CPU time with the conventional methods.

The possibility of expansion of this method to the analysis using the magnetic vector potential will be reported in another paper.

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