Recent progress in numerical analysis for electromagnetic devices

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RECENT PROGRESS IN NUMERICAL ANALYSIS FOR ELECTROMAGNETIC DEVICES (Invited)

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Abstract - Over the past 10 years, the ability of computers has progressed rapidly, and the techniques of numerical analysis are also improved vastly. The 3-D analysis of electrostatic and magnetostatic fields can be easily carried out on even a workstation, and results obtained contribute to the design and development of new products.

The paper reviews the main factors of progress in numerical analysis using the finite element method.

1. INTRODUCTION

The technical progress in computer and numerical computation enable us to analyze accurately the three-dimensional distributions of magnetic fields in electromagnetic devices. Thus, smaller, shorter, thinner and more efficient devices which save materials and energy can be developed within a very short period without any repetitions of trial productions.

In this paper, firstly the recent progress of analysis methods is discussed. Secondly, some difficult problems to be investigated are discussed. Lastly, the progress of computer technology and its environment to be expected are described.

2. PROGRESS IN NUMERICAL ANALYSIS

2.1 Classification of magnetic fields in electromagnetic devices

The phenomena occurring in electrical machines and electronic equipments can be classified as shown in Table 1. The figures show the degree of difficulty in the 3-D numerical analysis. The analysis of No.1 to No.4 is sufficiently practical even if using a workstation and No.5 is only practical using a supercomputer. The "example" in the Table shows the corresponding number of the FELIX[1] and TEAM[2] workshop models and the models of IEE of Japan[3,4] which are proposed to verify software.

Table 1 Classification of electromagnetic fields

field		linear		non-linear	
		difficulty	example	difficulty	example
static		1	IEEJ magnetostatic model	2	Problem 13
dynamic	transient	4	Problems 1,4,11,12	5	Problem 10
dynamic	steady state	3	Problems 2,3,5~9 IEEJ eddy current model	6	unpractical

1 ⟵⟶ 6
easy difficult

2.2 Necessary techniques for analysis of practical devices

2.2.1 Analysis of devices excited from voltage source

The windings of a capacitor motor shown in Fig.1 are excited from the external power source. The waveform of the applied voltage is usually sinusoidal. Although the effective value of the voltage Vo is given, the currents I_M and I_A of the main and auxiliary windings are unknown and the waveforms of these currents are distorted due to non-linearity of

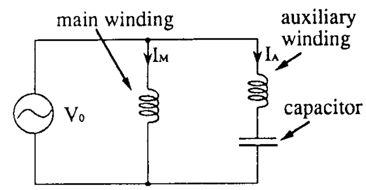


Fig.1 Connection diagram of capacitor motor.

the magnetic materials. In such a case, which is usual in electrical machines, Poisson's equation should be solved simultaneously combined with the circuit equation obtained from Kirchhoff's second law as shown in the following equations[5-7].

$$\mathrm{rot}\,(\nu\,\mathrm{rot}A) = J_0 - \sigma\left(\frac{\partial A}{\partial t} + \mathrm{grad}\phi\right) \tag{1}$$

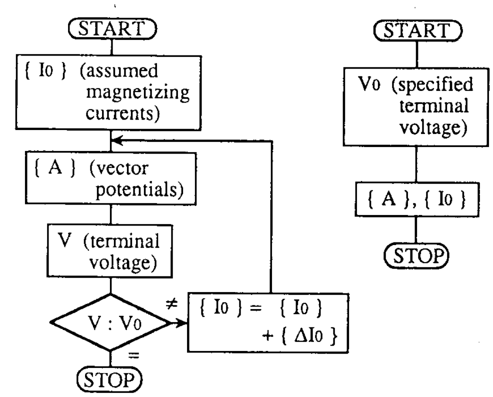
$$\eta = V_0 - \frac{\partial \Psi}{\partial t} - (R_c + R_0)\, I_0 - L_0 \frac{dI_0}{dt} - \frac{1}{C}\int I_0\, dt = 0 \tag{2}$$

where A and ϕ are the magnetic vector and electric scalar potentials respectively. J_0 is the current density. ν and σ are the reluctivity and conductivity respectively. η corresponds to the residual. V_0 and I_0 are the terminal voltage and current respectively. Ψ is the interlinkage flux with the winding and is a function of A. R_c is the dc resistance of the winding. R_0, L_0 and C are the resistance, inductance and capacitance which cannot be included in the finite element region. The flux distributions in non-linear models can be analyzed by solving the following Newton-Raphson equation repeatedly:

$$\begin{bmatrix} \left[\frac{\partial G_i}{\partial u_j}\right] & \left[\frac{\partial G_i}{\partial I_{0\ell}}\right] \\ \left[\frac{\partial \eta_k}{\partial u_j}\right] & \left[\frac{\partial \eta_k}{\partial I_{0\ell}}\right] \end{bmatrix} \begin{Bmatrix} \{\delta u_j\} \\ \{I_{0\ell}\} \end{Bmatrix} = \begin{Bmatrix} -\{G_i\} \\ -\{\eta_k\} \end{Bmatrix} \tag{3}$$

where G_i is the residual related to the unknown variable u_i.

Fig.2 shows the flow charts for the conventional and new methods solving above simultaneous equations. The conventional iterative method can not be expanded into three phase machines.



(a) conventional method (b) new method

Fig.2 Comparison of calculation process.

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Fig.3 shows an example of application to a fuel injector[8]. The optimal magnetic characteristics (permeability, conductivity, etc.) of materials in the magnetic circuit are examined by numerical analysis.

In the analysis of single sheet testers[9,10], the total interlinkage flux Ψ_0 with the test specimen is specified. Therefore, the following equation (4) should be used instead of Eq.(2).

$$\eta = \Psi_0 - \Psi = 0 \tag{4}$$

where Ψ is the interlinkage flux obtained from the calculation.

Although the combination of Eqs.(1) and (4) makes the coefficient matrix asymmetric, it can be solved efficiently[11].

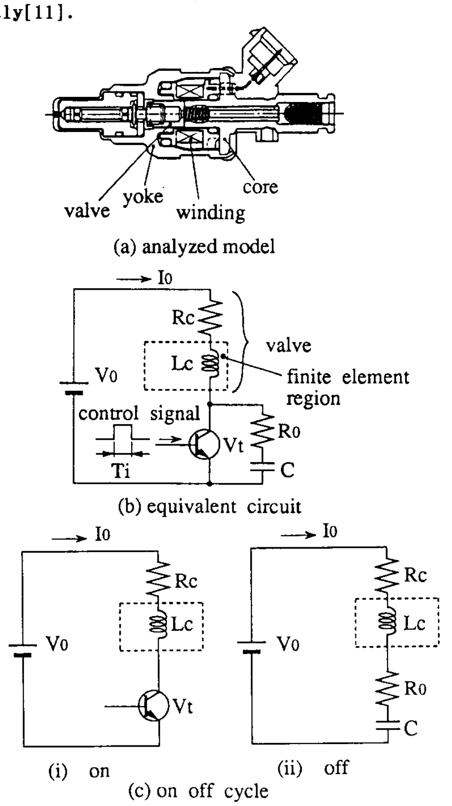


Fig.3 Fuel injector.

2.2.2 Eddy current analysis of moving conductors

Two coordinate systems can be conceived to analyze a moving conductor with eddy current as shown in Fig.4. The following Eq.(5) or (6) should be adopted when the coordinate system is moving one (X, Y, Z) or fixed one (x, y, z) respectively.

$$\mathrm{rot}\ \{\ \nu \mathrm{rot}\mathbf{A}\ (X,Y,Z)\ \} = -\ \sigma\ \frac{\partial \mathbf{A}\ (X,Y,Z)}{\partial t} \tag{5}$$

$$\mathrm{rot}\ \{\ \nu \mathrm{rot}\mathbf{A}\ (x,y,z)\ \} = -\ \sigma\ \left\{ \frac{\partial \mathbf{A}\ (x,y,z)}{\partial t} - \mathbf{v} \times \mathbf{B}\ (x,y,z) \right\} \tag{6}$$

where $\mathbf{v}$ and $\mathbf{B}$ are the velocity and flux density vectors respectively.

The method using Eq.(5) is superior for transient analysis from the viewpoints of accuracy, CPU time and computer storage. It will be expanded into steady state analysis[12].

Figs.5-7 show an application to a linear induction motor. The model is simplified from three phases to single one as shown in Fig.5. Figs.6 and 7 show the distributions of flux and eddy current densities respectively.

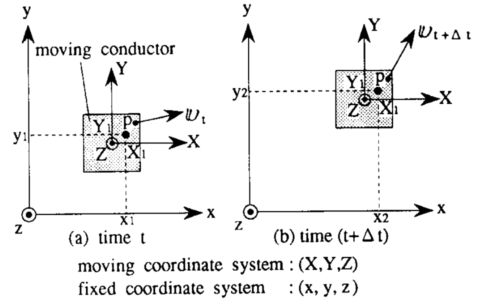


Fig.4 Coordinate systems for moving conductor.

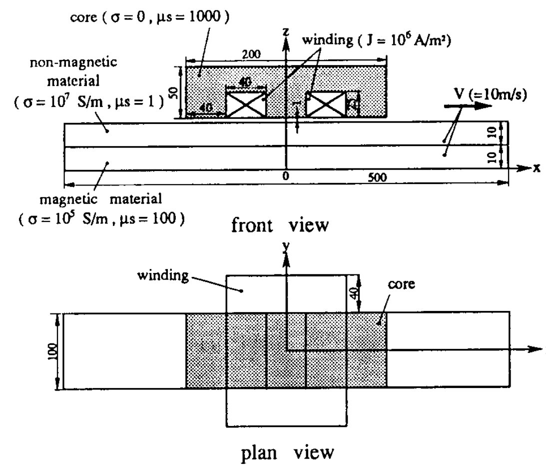


Fig.5 Linear induction motor.

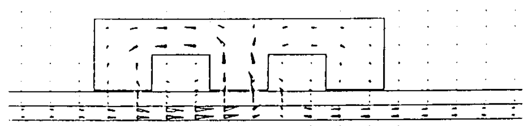
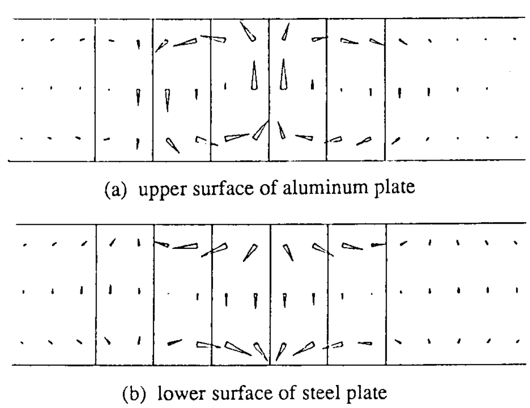


Fig.6 Distribution of flux densities (t = 0.01s, y = 0).



(a) upper surface of aluminum plate

(b) lower surface of steel plate

Fig.7 Distrubution of eddy current density (t = 0.01s).

2.3 Techniques to reduce CPU time and computer storage

The following various efficient techniques have been developed.

2.3.1 ICCG method

Instead of the Gaussian elimination method, the

ICCG (Incomplete Cholesky Conjugate Gradient) method[13] to solve linear equations has been developed to reduce the memory size and the CPU time. Although the ICBCG (Incomplete Cholesky Bi-Conjugate Gradient) method should be used for the complex matrix[14], it is not necessary to calculate the orthogonal direction vector and residual vector when the coefficient matrix is symmetric. As the algorithm of the complex ICBCG method is the same as that of the real ICCG method, it is better to call it the ICCG method. If the bandwidth of the coefficient matrix is minimized and the computer storage is enough for the Gaussian elimination method, the Gaussian method is favorable, because the CPU time using the Gaussian method is shorter than that using the ICCG method.

2.3.2 Edge element

The CPU time of numerical analysis is decreased considerably by introducing the edge element[15-19]. In the case of the IEEJ eddy current model[4], the CPU time using the nodal element for the A method is about 6 times longer than that using the edge element as shown in Table 2, although these elements show almost the same accuracy[4]. The analyzed results of the magnetostatic model (Problem 13 in TEAM workshop) show that the edge element is more favorable than the nodal element from the viewpoints of the CPU time and the accuracy[20].

Fig.8 shows a magnetic sensor which has a thin shielding case and is excited at a high frequency of 140kHz. In such a case, the ICCG method is not converged when the nodal element is used.

Table 2 Discretization data and CPU time

item	without hole				with hole			
	$A-\phi$		$T-\Omega$		$A-\phi$		$T-\Omega$	
	nodal	edge	nodal	edge	nodal	edge	nodal	edge
number of elements	14400							
number of nodes	16275							
number of unknowns	43417	41060	22844	22412	42885	41060	22844	22412
number of non-zero entries	1781644	653718	632859	423056	1734684	653718	632859	423056
computer storage (MB)	72.2	28.4	30.7	19.4	70.5	28.4	30.7	19.4
number of iterations of ICCG method	1306	513	172	192	1264	582	1141	327
CPU time (s)	6242	947	533	290	5870	1069	2001	442

Computer used : NEC supercomputer SX-1E (maximum speed : 285 MFLOPS)
convergence criterion of ICCG method : 10^{-7}

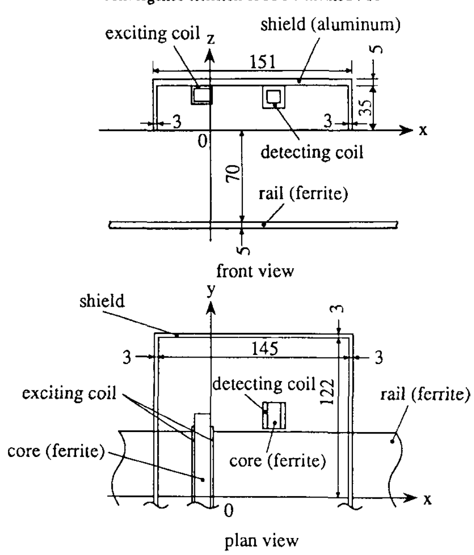


Fig.8 Magnetic sensor.

2.3.3 Time-periodic finite element method

The so-called "Time-Periodic Finite Element Method" to solve the non-linear periodic phenomena has been developed to reduce the CPU time[21,22]. If the conventional method is used, the solution for the steady state is obtained after a few cycles of transient phenomena as shown in Fig.9. The dots on the waveform show the points to be analyzed. The new method utilizes the relationship between the vector potentials at instants t and t+T/2 (T:period). When the waveform of a potential is symmetric and periodic with time as shown in Fig.10, the following relationship is hold.

$$A^{t+T/2} = - A^{t} \tag{7}$$

Using the above-mentioned relationship, all points within the interval of a half period are analyzed at the same time. The increase of the computer storage is negligible, and the CPU time can be reduced to less than 1/3 times by using the new method.

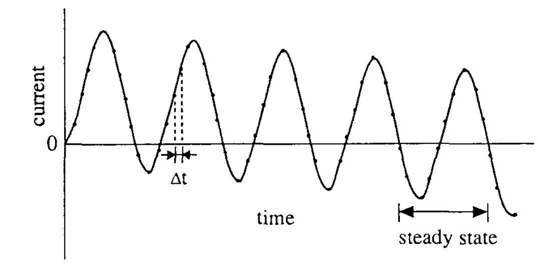


Fig.9 Waveform of current.

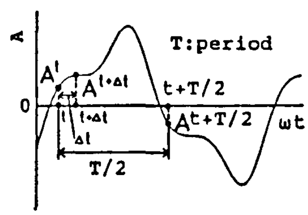


Fig.10 Periodic waveform.

2.3.4 Periodic boundary condition

The periodic boundary condition for 3-D magnetic fields has been introduced[23-25]. There are two kinds of periodic conditions as shown in Figs.11(a) and (b). Fig.11(a) shows the corner joint of laminated transformer core. The geometry and flux distribution are symmetrical with the line o-o'. Fig.11(b) shows a twisted multifilamentary superconducting cable. The geometry and the magnetic characteristics are repeated periodically along the axis. Figs.12(a) and (b) are corresponding meshes. Table 3 compares the computer storage and the CPU time with and without periodic conditions for Fig.11(a). The CPU time is decreased to less than 1/2.

2.3.5 Quasi 3-D analysis method

Various approximation methods for calculating the 3-D magnetic fields have been developed.

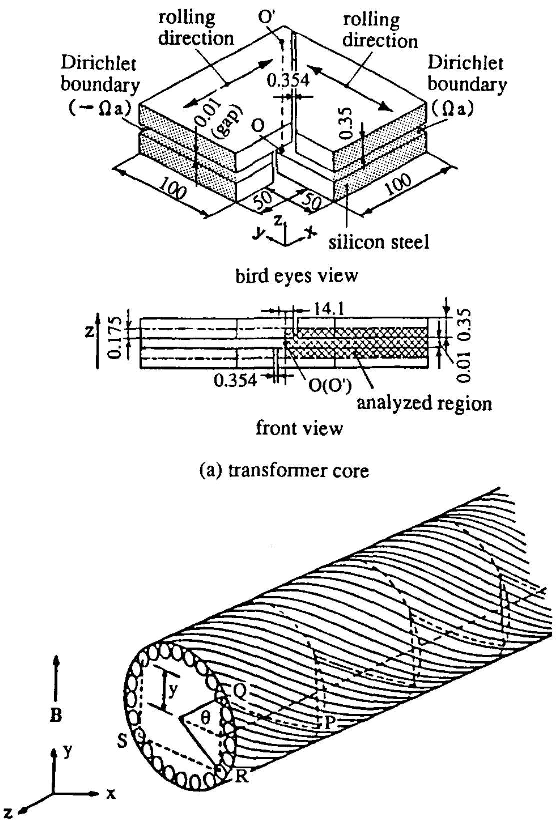
(i) Approximate analysis of laminated cores

Fig.13 shows the so-called scrap-less type three-phase transformer core. Laminations are alternately turned over. In the hatched parts, the angle between the rolling directions of the adjacent sheets is 90°. The flux density vectors at an instant in each layer of the hatched region are shown in Fig.14. The distribution ratio of the fluxes into two layers is determined from the minimum energy principle[26].

(ii) Approximate analysis of magnetic circuits composed of axisymmetric and rectangular regions

Fig.15 shows an electromagnet with rectangular yoke. The winding, pole piece and plunger are

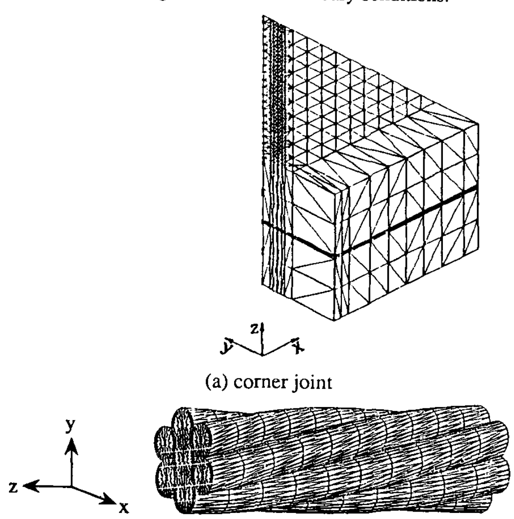
axisymmetric. A new vector potential is introduced in the axisymmetric region so that the continuity condition on the boundary between the two-dimensional region and the axisymmetric region is satisfied[27-29].
(iii) A brushless dc motor with permanent magnets has been analyzed approximately by representing the magnetization M by Fourier series[30].



(a) transformer core

(b) twisted multifilamentary superconducting ac cable

Fig.11 Periodic boundary conditions.



(a) corner joint

(b) twisted multifilamentary superconducting ac cable

Fig.12 Subdivisions.

Table 3 Comparison with and without periodic conditions

item	without periodic boundary condition	with periodic boundary condition
number of elements	24300	12150
number of nodes	4991	2576
number of unknowns	4851	2437
computer storage (MB)	11.0	5.5
CPU time (s)	389	26

computer used : NEC supercomputer SX-1E (maximum speed : 285MFLOPS)

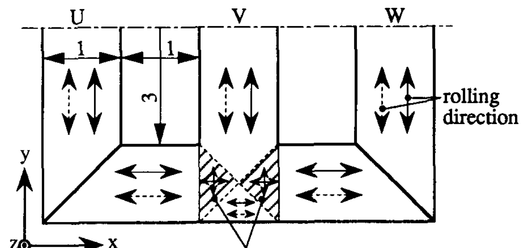
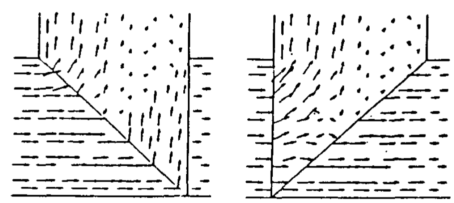


Fig. 13 Three - phase transformer core.



(a) first layer (b) second layer

Fig.14 Flux density vectors in each layer (M-5, 0.35mm, Bleg=1.7T, ωt=60°).

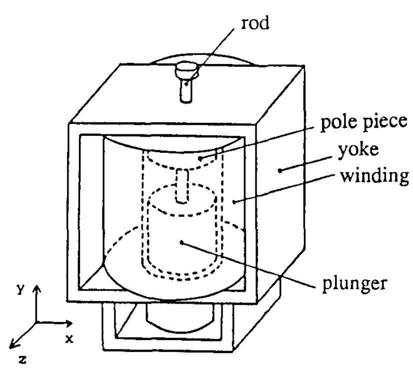


Fig.15 Electromagnet with rectangular yoke.

2.3.6 Special elements

Special elements such as gap element, expanding element, shielding element[31] have been developed.

(i) Gap element

The magnetic circuits contain often small gaps. Fig.16 shows a reactor with small gap. It can be assumed that the flux in the gap flows perpendicular to the magnetic material with high permeability. A special element can be inserted into the gap. It has no volume, but has nearly the same energy as the gap.

Table 4 shows an example of analysis with and without gap element.

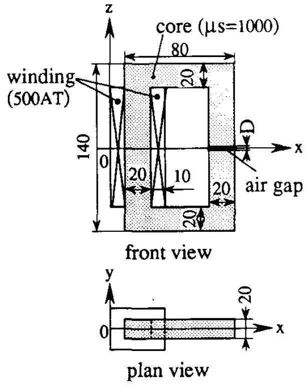


Fig. 16 Analyzed model with gap element.

Table 4 Discretization data and CPU time

item		using gap element	using ordinary element
number of elements	except gap region	4928	5150
	gap region	150	450
number of nodes		6072	6864
number of unknowns		12772	14539
number of non-zero entries		437908	500626
number of iterations of ICCG method		336	2080
CPU time (s)	ICCG	299	1445
	total	332	1482

computer used : NEC supercomputer SX-1E (maximum speed: 285MFLOPS)

gap D=0.3mm

(ii) Shielding element

The electromagnetic devices are often shielded by using thin conductors. Fig.17 shows a model of transformer tank shield. As the shielding plate is very thin, the electric scalar potential ϕ can be assumed to be constant in the perpendicular direction. The so-called shielding element can be put at the shielding plate. Table 5 shows the comparison of accuracy between shielding element and conventional element. The CPU time can be reduced to about 2/3 as shown in Table 6.

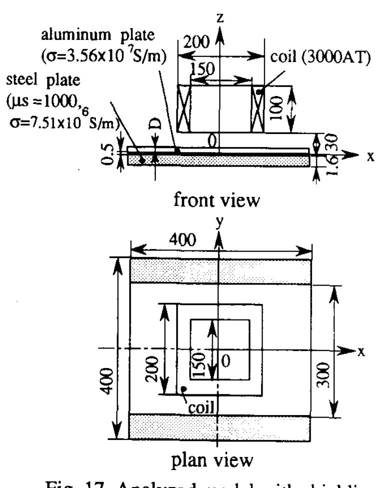


Fig. 17 Analyzed model with shielding element.

The concept of the "expanding element" is similar to above-mentioned special elements[31].

The special elements have the following advantages:

(i) It is easy to add or remove gaps, legs or conducting plates in desired positions in the mesh.

(ii) The modification of the length D in Figs.16 and 17 is also easy.

(iii) The CPU time using the special element can be reduced compared with that using the flat conventional element, because the number of iterations of the ICCG method is decreased.

Table 5 Comparison of eddy current densities (y=110, z=0mm)

coordinate x (mm)	eddy current density Jex ($\times 10^6$ A/m^2) with shielding element	eddy current density Jex ($\times 10^6$ A/m^2) without shielding element	error ε_J (%)
12.5	9.16	9.29	−1.40
37.5	8.55	8.67	−1.38
62.5	7.26	7.37	−1.49

Table 6 Discretization data and CPU time

item		using shielding element	using ordinary element
number of elements	except conducting region	2057	2115
	conducting region	63	63
number of nodes		2592	2736
number of nuknowns		5488	5971
number of non-zero entries		206884	231859
number of iterations of ICCG method		1531	1971
CPU time (s)	ICCG	805	1142
	total	821	1161

computer used : NEC supercomputer SX-1E (maximum speed : 285 MFLOPS)

plate D=1mm, 200Hz

3. SUBJECTS FOR FURTHER INVESTIGATION

3.1 Difficult problems

3.1.1 Anisotropy

The magnetic characteristics of cores laminated with grain oriented silicon steel are anisotropic. The permeabilities μx, μy and μz in respective directions are a function of the magnetic field strengths Hx, Hy and Hz in x-, y- and z-directions. It is very difficult to input such complicated functions in the computer. We need further experimental investigation.

3.1.2 Hysteresis

In the analysis of recording heads, the hysteresis should be taken into account. The mathematical modeling for the 3-D analysis is too complicated.

3.2 Techniques to be expected

We want a supercomputer with 1000 times faster CPU and larger memory in order to simulate the phenomena with practically sufficient accuracy. The optimal design of non-linear magnetic circuits with eddy currents would be done by using such a large computer.

The cooperation with the applied mathematicians is welcomed. We should willingly use the software developed by mathematicians, for examples Mathematica and Matlab.

The AI(Artificial Intelligence), fuzzy and neurocomputing techniques should be introduced to the pre- and post-processing. The scientific visualization using 3-D color graphic display is also important for the better understanding of the phenomena.

4. CONCLUSIONS

The establishment of verification systems of accuracy such as the TEAM workshop has contributed to improve the accuracy of software for dynamic fields. The 3-D analysis is absolutely necessary when the eddy current flows in electrical machines such as induction motors. We hope that the 3-D non-linear dynamic analysis of the steady state phenomena will become practical near future.

REFERENCES

[1] International Electromagnetic Workshop : "Test Problem" (1986).

[2] TEAM Workshops : "Test Problem" (1988).

[3] T.Nakata, N.Takahashi and K.Fujiwara : "3-D Finite Element Analysis of Magnetic Fields of IEEJ Model", Electromagnetic Fields in Electrical Engineering, 285 (1989) International Academic Publishers.

[4] T.Nakata, N.Takahashi and K.Fujiwara : "Verification of Softwares for 3-D Eddy Current Analysis Using IEEJ Model", Advances in Electrical Engineering Software (1990) Springer-Verlag.

[5] T.Nakata and N.Takahashi : "Direct Finite Element Analysis of Flux and Current Distributions under Specified Conditions", IEEE Trans. Magnetics, MAG-18, 2, 325 (1982).

[6] T.Nakata, N.Takahashi, K.Fujiwara and A.Ahagon : "3-D Finite Element Method for Analyzing Magnetic Fields in Electrical Machines Excited from Voltage Sources", ibid., MAG-24, 6, 2582 (1988).

[7] T.Nakata and N.Takahashi : "Numerical Analysis of Transient Magnetic Field in a Capacitor-Discharge Impulse Magnetizer", ibid., MAG-22, 5, 526 (1986).

[8] A.Kuromiya, K.Takeuchi, T.Nakata, N.Takahashi and K.Fujiwara : "Analysis of Electronic Fuel Injector Taking into Account External Electric Circuit and Valve Motor", Proceedings of the 11th Symposium on Computational Electrical and Electronic Engineering, 89 (1990).

[9] T.Nakata, N.Takahashi, Y.Kawase, M.Nakano, M.Miura and J.D.Sievert : "Numerical Analysis and Experimental Study of the Error of Magnetic Field Strength Measurements with Single Sheet Testers", IEEE Trans. Magnetics, MAG-22, 5, 400 (1986).

[10] T.Nakata, N.Takahashi, K.Fujiwara, M.Nakano and T.Kayada : "Effects of Eddy Currents in the Specimen in a Single Sheet Tester on Measurement Errors", ibid., MAG-26, 5, 1641 (1990).

[11] T.Nakata, N.Takahashi and K.Fujiwara : "Efficient Solving Techniques of Matrix Equations for Finite Element Analysis of Eddy Current", ibid., MAG-24, 1, 170 (1988).

[12] T.Nakata, N.Takahashi, K.Fujiwara and K.Muramatsu: "Investigation of Analysis Method for Moving Conductor", Compumag Conference, Sorrento (1991) (to be published).

[13] J.A.Meijerink and H.A. van der Vorst : "An Iterative Solution Method for Linear Systems of Which the Coefficient Matrix is a Symmetric M Matrix", Math. Comp., 31, 137, 148(1977).

[14] D.A.H.Jacobs : "Generalizations of the Conjugate Gradient Method for Solving Nonsymmetric and Complex Systems of Algebraic Equations", CERL Report RD/L/N70/80 (1980).

[15] A.Bossavit and J.C.Verite : "The Trifou Code : Solving the 3-D Eddy-Currents Problem by Using H as State Variable", IEEE Trans. Magnetics, MAG-19, 6, 2465 (1983).

[16] M.L.Barton and Z.J.Cendes : "New Vector Finite Elements for Three-Dimensional Magnetic Field Computation", Journal of Applied Physics, 61, 8, 3919 (1987).

[17] R.Albanese and G.Rubinacci : "Solution of Three Dimensional Eddy Current Problems by Integral and Differential Methods", IEEE Trans. Magnetics, MAG-24, 1, 98 (1988).

[18] A.Kameari : "Three Dimensional Eddy Current Calculation Using Edge Elements for Magnetic Vector Potential", Applied Electromagnetics in Materials, 225 (1989) Pergamon Press.

[19] K.Fujiwara, T.Nakata, N.Takahashi and T.Imai : "Effects of Gauge Condition on Accuracy and CPU time for 3-D Finite Element Method Using Edge Element", IEEE Trans. Magnetics, MAG-27 (1990) (to be published).

[20] T.Nakata and K.Fujiwara : "Summary of Results for Benchmark Problem 13 (3-D Non-Linear Magnetostatic Model", Proceedings of the European TEAM Workshop and International Seminar in Electromagnetic Field Analysis, 155 (1990).

[21] T.Nakata, N.Takahashi and Y.Kawase : "Non-Linear Analysis of Eddy Current Problems Using the Time-Periodic Finite Element Method", Proceedings of the International Symposium on Electromagnetic Fields in Electrical Engineering, 89 (1985).

[22] T.Nakata, N.Takahashi, K.Fujiwara and A.Ahagon : "3-D Non-Linear Eddy Current Analysis Using the Time-Periodic Finite Element Method", IEEE Trans. Magnetics, MAG-25, 5, 4150 (1989).

[23] T.Nakata, N.Takahashi, K.Fujiwara and A.Ahagon : "Periodic Boundary Condition for 3-D Magnetic Field Analysis and Its Applications to Electrical Machines", ibid., MAG-26, 6, 2694 (1988).

[24] T.Nakata, N.Takahashi, K.Fujiwara, P.Olszewski and K.Muramatsu : "Analysis of Magnetic Fields of 3-D Non-Linear Magnetostatic Model (Problem 13)", Proceedings of the European TEAM Workshop and International Seminar on Electromagnetic Field Analysis, 107 (1990).

[25] T.Nakata, N.Takahashi, Y.Fujii, M.Kitagawa, J.Takehara and K.Muramatsu : "3-D Finite Element Analysis of Coupling Currents in a Multifilamentary AC Superconducting Cable", IEEE Trans. Magnetics, MAG-27 (1990) (to be published).

[26] T.Nakata, N.Takahashi and Y.Kawase : "New Approximate Method for Calculating Three-Dimensional Magnetic Fields in Laminated Cores", ibid., MAG-21, 6, 2374 (1985).

[27] T.Nakata, N.Takahashi, Y.Kawase, H.Funakoshi and S.Ito : "Finite Element Analysis of Magnetic Circuits Composed of Axisymmetric and Rectangular Regions", ibid., MAG-21, 6, 2199 (1985).

[28] J.Takehara, M.Kitagawa, T.Nakata and N.Takahashi : "Finite Element Analysis of Inrush Currents in Three-Phase Transformers", ibid., MAG-23, 5, 2647 (1987).

[29] J.Takehara, M.Kitagawa, T.Nakata and N.Takahashi : "Numerical Analysis of Inrush Currents in Transformers", Electromagnetic Fields in Electrical Engineering, 129 (1988) Plenum Press.

[30] T.Nakata, N.Takahashi and K.Uehara : "Analysis of Magnetic Characteristics of a Brushless DC Motor Taking into Account the Distribution of Magnetization", IEEE Trans. Magnetics, MAG-22, 5, 1084 (1986).

[31] T.Nakata, N.Takahashi, K.Fujiwara and Y.Shiraki : "3-D Magnetic Field Analysis Using Special Elements", ibid., MAG-26, 5, 2379 (1991).

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