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Continuous-Time Anti-Windup Generalized Predictive Control of Uncertain Processes with Input Constraints and Time Delays

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Abstract—In this paper, a design problem of a continuous-time anti-windup generalized predictive control (CAGPC) system using coprime factorization approach for uncertain processes with input constraints and time delays is considered. The uncertainty of the process is considered as an uncertain time delay. To reduce the effect of the input constraint and uncertain delay, controller for strong stability of the closed-loop system is designed. As a practical appeal, the effectiveness of the proposed design scheme is confirmed by a simulated application to an industrial process with input constraint and uncertain time delay.

I. INTRODUCTION

A great deal of effort has been made for developing generalized predictive control of processes. As a result, generalized predictive control becomes a popular technique for process control. In general, the controlled process was assumed as a linear process. However, real process control systems must deal with uncertain time delay and some constraints such as actuator output limitation. Furthermore, the uncertain process is sometimes of non-minimum phase. Then, we have to consider the case of this kind of uncertain process with input constraints. In particular, usually the control of a nominal non-minimum phase system needs an unstable controller to cancel unstable zeros of the process and satisfactory results were obtained by using conventional generalized predictive control [12]. That is, the closed-loop pole polynomial has a factor of non-minimum phase and would give unstable controller for obtaining the desired results. However, real process has uncertainties, the problem of the above-mentioned processes with unstable controller for non-minimum phase is a problem of considerable practical importance to guarantee bounded controller output. Meanwhile, the increasing complexity of the uncertainty of the non-minimum phase process has to be considered, where the uncertain part including the non-minimum phase factor is considered as an uncertain time delay. The main reason is that high order or non-minimum phase exhibit similar dynamic characteristics to a reduced order model with a time delay [9,13]. This paper considers the above-mentioned processes.

The purpose of this paper is to consider the design problem of a continuous-time generalized predictive control system for the uncertain time delay processes with input constraints.

In general, in continuous-time generalized predictive control approach, controller design for processes with input constraints can be taken into account in two main ways. In the first approach, model predictive controls of constrained continuous-time systems were considered by using quadratic programming [1], [2]. That implementation requires solving a linear program during the control period. In the second case, two-step design paradigm [3] was discussed by using coprime factorization representation and Youla-Kucera parametrization [4], [5], [6]. The system having the following two stabilities is called to be strongly stable [4]. Namely, the controller is stable and the closed-loop system is stable. The design idea is simply stated as follows: design a linear strongly stable generalized predictive controller ignoring input constraint and then add a compensator to minimize the adverse effects of any input constraints on the closed-loop performance. The added controller is called anti-windup controller. It is proved that if the controller is strongly stable, then under some conditions the closed-loop system with input constraints is stable. Despite the above developments, generalized predictive control is difficult to offer a robust stability to the uncertain delay processes with input constraints. Recently, an example in [10] shows that the controller output signal increases to a huge value if the strong stability of the closed-loop system is not guaranteed. However, concerning strong stability of uncertain delay processes with input constraints, so far the design problem of a continuous-time anti-windup generalized predictive control (CAGPC) system using coprime factorization approach was not considered.

In this paper, to obtain the robustness to uncertain input delay, the design scheme for strong stability of continuous-time generalized predictive control is given to the case of non-minimum phase processes with uncertain time delay and input constraints by extending the design idea in...
Namely, we apply the method in [3, 10] to the above processes by using a balanced realization [9] of all-pass Padé approximation for uncertain time delay and by using robust evaluating method for coprime factor process description.

The paper is organized as follows. Section II states the problem setup. Section III gives details of the design of strongly stable robust CAGPC system. An example on a process is illustrated in Section IV to show the proposed method. Conclusion is drawn in Section V.

Notations: When $A$ is a function of $s$, $A(s)$ means a polynomial function of $s$, whereas, $A(s)$ is a rational function of $s$.

## II. Problem Setup

Consider a single-input single-output time-invariant linear non-minimum phase uncertain process modeled by the following transfer function:

$$
y_0(s) = \frac{B[s]}{A[s]}e^{-Ts}u_0(s) + \frac{C[s]}{A[s]}v_0(s)$$

where $A[s], B[s]$ are stable known polynomials in differential operator $s^\frac{1}{n}$. $Y_0(s), U_0(s)$ and $V_0(s)$ are the system output, control input and disturbance input respectively. $Y_0(s)$ and $U_0(s)$ are the Laplace transforms of $y(t)$ and $u(t)$. $A[s], B[s]$ are coprime polynomials of degree $n$ and $m$ respectively. $T$ is unknown time delay but suppose that the parameter range is known, and $0 < T^- \leq T \leq T^+$. The non-minimum phase factor is included in the time delay. $C[s]$ is a designed Hurwitz polynomial of degree $n - 1$, since no assumption is placed on the disturbance $V_0$. $C[s]$ can also be designed by choosing $\text{deg}(C[s]) = n$ when we would not wish to use the disturbance on the process output directly [12]. The control input $u(t)$ is subject to the following constraints (Fig.1).

$$u(t) = \sigma(u_1(t))$$

$$\sigma(v) = \begin{cases} 
  u_{\text{max}} & \text{if } v > u_{\text{max}} \geq 0 \\
  v & \text{if } u_{\text{min}} \leq v \leq u_{\text{max}} \\
  u_{\text{min}} & \text{if } v < u_{\text{min}} \leq 0
\end{cases}$$

where, $u_1(t)$ is the input prior to input constraint part. That is, $u_1(t)$ is the controller output. The objective is to design a strongly stable robust CAGPC system provided that the controller and the closed-loop system are stable by using coprime factorization representation and Youla-Kucera parametrization.

### III. Design of Strongly Stable Robust CAGPC System

#### 3.1 Design for the process without input constraint

One of the common techniques to handle the time delay system is to approximate the time delay by Padé approximation, since the approximation is preferable for capturing frequency response characteristic, and give a small gain error [14]. In this section, for the process (1), the effect of uncertain time delay for strong stability is evaluated.

The delay term of the controlled process will be approximated by a balanced realization of all-pass Padé approximation such that

$$e^{-T_D s} = \frac{B_1[s]}{A_1[s]} \approx \frac{(-1)^{l}s^l + (-1)^{l-1}q_1s^{l-1} + \cdots + q_l}{s^l + q_1s^{l-1} + \cdots + q_l}$$

and

$$q_i = \frac{(l+i)!}{(l-i)!} \frac{s^{-i}}{(T_D)^{i}}, \quad i = 1, 2, \ldots, l$$

where $T_D (0 < T^- \leq T_D \leq T^+)$ is nominal time delay and can be decided using the known ranges $T^-$ and $T^+$. Then, we have

$$\tilde{G}(s) = \frac{B[s]}{A[s]}e^{-Ts} = \frac{B[s]B_1[s] + (A_1[s]B[s]e^{-Ts} - B[s]B_1[s])}{A[s]A_1[s]}$$

From (4), the nominal non-minimum phase process of $\tilde{G}(s)$ is given as follows.

$$\tilde{G}(s) = \frac{B[s], (-1)^{l}s^l + (-1)^{l-1}q_1s^{l-1} + \cdots + q_l}{s^l + q_1s^{l-1} + \cdots + q_l}$$

The proposed controller for the process with input constraints is given by the following Youla-Kucera parametrization [3] (Fig.1).

$$R(s) = (Y(s) - Q(s)N(s))^{-1}K(s)$$

$$U(s) = X(s) + Q(s)D(s)$$

$$V(s) = Y(s) - Q(s)N(s)$$

where $Q(s) \in RH_{\infty}$ is a design parameter for ensuring a strongly stable feedback controller, and for tracking performance $K(s)$ is designed by the continuous-time GPC method [12]. The design of $X, Y, N, D, K$ and $Q$ is given as follows. First, $X(s) \in RH_{\infty}$ and $Y(s) \in RH_{\infty}$ satisfy the following Bezout identity.

$$X(s)N(s) + Y(s)D(s) = 1$$

Next, defining the stable closed-loop polynomial $T_0[s]$ of the process without input constraint, we choose the above
coprime factorizations as

\[
N(s) = \frac{B[s]B_1[s]}{T_0[s]}, \quad N(s) \in RH_{\infty}
\]

\[
D(s) = \frac{A[s]A_1[s]}{T_0[s]}, \quad D(s) \in RH_{\infty}
\]

\[
\Delta N(s) = \frac{A_1[s]B[s]e^{-Ts} - B[s]B_1[s]}{T_0[s]}
\]

(12)

where \( \Delta N(s) \) is the mismatch between the uncertain time delay and nominal time delay. Then, we have

\[
P(s) = \frac{N(s) + \Delta N(s)}{D(s)}
\]

where \( \Delta N(s) \in RH_{\infty}, X(s) \in RH_{\infty} \) and \( Y(s) \in RH_{\infty} \) are designed as follows.

\[
X(s) = \frac{gC[s] + F_0[s]}{C[s]}
\]

\[
Y(s) = \frac{C[s] + G_0[s]}{C[s]}
\]

\[
K(s) = g
\]

where, \( F_0(s), G_0(s) \) and \( L_0(s) \) are designed as follows [3, 12].

\[
F_0[s] = \sum_{i=1}^{N_y} k_i F_i[s]
\]

\[
G_0[s] = \sum_{i=1}^{N_y} k_i G_i[s]
\]

\[
L_0[s] = \sum_{i=1}^{N_y} k_i L_i[s]
\]

and \( N_y \) is predictor order, and \( F_0[s], G_0[s] \) and \( L_0[s] \) are given by

\[
\frac{s^kC[s]}{A[s]A_1[s]} = \frac{F_k[s]}{A[s]A_1[s]} + E_k[s]
\]

\[
\frac{B[s]B_1[s]E_k[s]}{C[s]} = \frac{G_k[s]}{C[s]} + H_k[s]
\]

\[
\frac{s^kB[s]B_1[s]}{A[s]A_1[s]} = H_k[s] + \frac{L_k[s]}{A[s]A_1[s]}
\]

Finally, \( Q(s) \) is chosen so as to satisfy the following inequality.

\[
\| \begin{bmatrix} S(s) & 1 \end{bmatrix} (D + NS)^{-1} \|_{\infty} \leq \| \begin{bmatrix} \Delta N & 0 \end{bmatrix} \|_{\infty} < 1 \quad (13)
\]

Then it is well known from small gain theorem [10, 11] that the perturbed closed-loop system will remain strongly stable, where \( S(s) = V^{-1}U \), \( U \) and \( V \) are given by (9) and (10). \( S(s) \) is the stable controller for the process without input constraint. Here, the maximum of \( \Delta N \) can be summarized as follows [9].

\[
|\Delta N(j\omega)| \leq \frac{|A_1[j\omega]B[j\omega]|}{T_0[j\omega]} \| \bar{G}_o(j\omega) \| T_2(\omega)
\]

\[
+ |\bar{G}_o(j\omega)|T_1(\omega), \omega \in [0, \infty)
\]

(14)

where

\[
T_1(\omega) = \begin{cases} |e^{-\omega T^+} - 1|, & \text{if } 0 \leq \omega \leq \frac{\pi}{T} \\ 2, & \text{if } \omega > \frac{\pi}{T} \end{cases}
\]

\[
T_2(\omega) = \begin{cases} |e^{-\omega T^-} - 1|, & \text{if } 0 \leq \omega \leq \frac{2\pi}{T} \\ |e^{-\omega T^+} + 1|, & \text{if } \frac{2\pi}{T+1} < \omega \leq \frac{2\pi}{T} \\ 2, & \text{if } \omega > \frac{2\pi}{T} \end{cases}
\]

\[
\bar{G}_e(s) = \frac{q_l + q_{k-2}s^2 + \cdots}{s^4 + q_1s^3 + \cdots + q_l}
\]

\[
\bar{G}_o(s) = \frac{q_l - s + q_{l-3}s^3 + \cdots}{s^4 + q_1s^3 + \cdots + q_l}
\]

Instead of \(|e^{-\omega T} - 1| \leq 2 \) and \(|e^{-\omega T} + 1| \leq 2\), in this paper we evaluate these terms using the range values of \( T^+ \) and \( T^- \). For example, when \( 10 \leq T \leq 20 \), the upper bound \( T_1(\omega) \) and \( T_2(\omega) \) can be obtained as Fig.2 [9].

![Gain diagram of T1(\omega) and T2(\omega) (10 ≤ T ≤ 20)](image)

The controller \( S(s) = V^{-1}U \) can be designed by using (9) and (10) under \( u = u_1 \).

Concerning the approximation of time delay, it is noted that if the time delay term is small relative to the dominant time constant, then 1st order (\( l = 1 \)) approximation will be sufficient. On the contrary, if the time delay term is large relative to the dominant time constant then a 2nd or higher order approximation of the delay term (\( l \geq 2 \)) would be considered provided that (13) is satisfied.
3.2 Design for the process with input constraint

In this section, an anti-windup controller for control system with input constraints is proposed by adding a controller. It also shows that the controller gives the robust stability of the closed-loop system under the condition of strong stability.

In general, CGPC in [12] can work well for the case of non-minimum phase processes without input constraints, even if the controller is unstable and the non-minimum phase process has uncertainties. However, in the case of uncertain non-minimum phase processes with input constraints, when unstable controller causes an excess output signal over an input constraint, the controller output will be huge but process input and output are bounded. An example is given in [10], and it is reviewed as follows (Figs.3, 4, 5). This example also confirms the theoretical analysis in [7]. However, concerning strong stability of uncertain delay processes with input constraints, robust design condition of the closed-loop system was not considered.

For guaranteeing strong stability for the anti-windup control system described in Fig.1, the object in the following is to obtain a control system in the presence of input constraint. In this paper, \( \phi(\cdot) \) satisfies the following relation [7,8].

\[
\phi(z) = z - 2\sigma(z), \phi \in \text{Cone}(0, 1) \tag{15}
\]

The input constraint \( \sigma(z) \) in (3) can be rewritten as

\[
\sigma(z) = \frac{1}{2}(z - \phi(z)), \sigma \in \text{Cone}(1/2, 1/2) \tag{16}
\]

From the definition of \( \sigma \), we have

\[
|\phi(z)| \leq |z| \tag{17}
\]

\[
u_2 = \phi(u_1) \tag{18}
\]

\[
u_1 = 2(I + V + UP)^{-1}VRw
+ (I + V + UP)^{-1}(V + UP - I)u_2 \tag{19}
\]

\[
y = (I + V + UP)^{-1}PVRw
- (I + V + UP)^{-1}Pu_2 \tag{20}
\]

\[
V + UP = Y - QN + (X + QD)(N(s) + \Delta N(s)) \over D(s)
= D^{-1}(1 + U\Delta N) \tag{21}
\]

The input-output relations from \((w, u_2)\) to \((u_1, y)\) are given by

\[
\begin{bmatrix}
  u_1 \\
  y
\end{bmatrix} =
\begin{bmatrix}
  G_{11}(s) & G_{12}(s) \\
  G_{21}(s) & G_{22}(s)
\end{bmatrix}
\begin{bmatrix}
  u_2 \\
  w
\end{bmatrix}
\]

where

\[
G_{11}(s) = -(I + V + UP)^{-1}(V + UP - I)
\]

\[
G_{12}(s) = 2(I + V + UP)^{-1}VR
\]

\[
G_{21}(s) = -(I + V + UP)^{-1}P
\]

\[
G_{22}(s) = (I + V + UP)^{-1}PVR
\]

**Theorem:** The closed-loop system described in Fig.1 is stable in the presence of \( \phi \), if

1. \( S(s) \) is stable
2. \( G_{11}(s) \in RH_{\infty} \)
3. \( MD(s)M^{-1} \) is strictly positive real
where $M$ is positive definite diagonal matrix. A stable predictive controller is guaranteed by Condition 1 $\sim$ 3, and the robustness of the control system is ensured by Condition 4.

Proof: From (14), $\Delta N$ is bounded. Based on the result in [3], from (21), we have that the closed-loop system is stable provided that $\| \frac{\Delta N(s)}{D} \|_\infty < 1$. The desired result is obtained under Condition 4.

Condition 1 of the theorem shows that the strong stability should be ensured for the robustness of the control system.

IV. EVALUATION OF THE PROPOSED METHOD ON A PROCESS

The purpose of the simulation is to demonstrate the benefit of the proposed method when input constraint and uncertain time delay exist. Simulation studies are conducted using the following uncertain process.

$$A[s] = 1500s^3 + 3500s^2 + 1000s + 6$$
$$B[s] = 56, T^- = 0.2$$
$$T^+ = 0.8, T_D = 0.5$$

(22)

The input constraints are set as $u_{max} = 0.2$ and $u_{min} = -0.2$. In the simulation, we select $l = 1$, then $G$ is of non-minimum phase.

The design parameters and some properties are shown in Table 1. By selecting $Q(s)$ based on [3], we can obtain a strongly stable system by the proposed method under consideration of uncertainty. However, when the above input constraint and uncertainty are present, the condition 4 of the theorem and (13) are violated for some values of $Q(s)$.

| CGPC | Reference input: $w = 1$
Predictor order: $N_y = 6$
Control horizon: $N_u = 1$
Control weighting: $\lambda = 0.1$
Min. prediction horizon: $T_1 = 0$
Max. prediction horizon: $T_2 = 29$
Design parameters: $u_n = -1; ud = 1$
Design parameter: $C[s] = 0.5s^2 + s + 0.1$

| Str. stable | Closed-loop characteristic poly.: $T_0(s) = 1500s^3 + 492.2s^2 + 36.6s + 1.4$
The unstable pole of the CGPC [12]: $0.0026 + 0.4756i; 0.0026 - 0.4756i$
The poles of the former method [3] $-0.0021 + 0.4815i; -0.0021 + 0.4815i$
$-0.2599; -0.0501; -0.0088$

Fig. 6. Process output

The control law proposed in this paper is given by

$$U(s) = \frac{0.5s^2 - 0.0026s + 0.1131}{0.5s^2 + s + 0.1} + \frac{Q * 1500s^3 + 492.2s^2 + 36.6s + 1.4}{56} - \frac{-5.4752 - 1.9286s + 0.0025}{0.5s^2 + s + 0.1} - \frac{Q * 1500s^3 + 3500s^2 + 1000s + 1}{1500s^3 + 492.2s^2 + 36.6s + 1.4}$$

The detailed example for selecting $Q(s)$ was given in [10]. In this paper, we omit it. In the case of input constraint and uncertainty being present, Fig.6 shows the process output (dashed line) under $T = 0.8$ and the process output (solid line) for the same conditions under $T = 0.5$ by using the proposed design scheme, where $Q = 0.025$. Meanwhile, Fig.7 shows the process input for $T = 0.8$ (dashed line) and the process input for $T = 0.5$ (solid line). Comparing the preceding simulation results in Fig.6, desired tracking performances have been obtained under the existence of the uncertainty of time delay.

Fig. 7. Process input

Table 1 Design parameters
V. Conclusion

In this paper, a design problem of a continuous-time anti-windup generalized predictive control system using coprime factorization approach for uncertain delay processes with input constraints was considered. Under the existence of input constraints and uncertainty of time delays, strong stable controller design scheme for the uncertain process is given. The effectiveness of the proposed method is confirmed by a simulation of an uncertain process.

References