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# Robust Positioning Control of Pneumatic Servo System with Pressure Control Loop

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## Abstract

*The goal of this paper is to attain a robust positioning control of pneumatic driving system. A positioning control system positively focusing on the pressure control is investigated, from the view that the pressure control is indispensable for improvement of control performances. A disturbance observer is employed to improve the pressure response and compensate the influence of friction force and parameter change. Consequently the improvements of robustness against payload and of positioning accuracy have been attained.*

## 1 Introduction

Pneumatic servo system has become one of interests lately in industrial fields. However, air compressibility makes the control characteristics easily affected by a friction force and parameter change of plant.

The characteristic of a pneumatic actuator depends on the pressure. So if the pressure follows the reference value which can compensate the influence of friction force and parameter change of plant, the system becomes robust against the friction force and parameter changes of plant.

Miyata et al. have carried out positioning control of piston of pneumatic cylinder by constructing pressure control loop[1]. The effectiveness of pressure control on the improvement of positioning performances has been confirmed, however the problem of the robustness for the payload or the friction force is still remaining.

A meter out speed control method, widely used in a pneumatic system, is employed. This method has an advantage of superior energy efficiency but gives the non-linear pressure response.

In this study, two disturbance observers are employed in proposed positioning control system. One is applied in the pressure control loop to improve robustness against the influence of piston speed and to compensate non-linearity owing to meter out method, the other is in the kinetic part, from pressure to control variable, in order to generate the compensating signal fed back to the pressure loop to counterbalance the friction force and parameter change.

The validity of this control method is confirmed through some experiments.

## 2 Construction of the control system

### 2.1 Modeling of the plant

A simplified model of pneumatic driving system is shown in Fig.1, where  $p_l, p_r, p_s, p_A$  are pressures of left chamber, right one, supply pressure and atmospheric one, respectively.  $m, b, A, V, T, x, f_c, S$  and  $G_p$  are mass of table, damping coefficient, effective sectional area of cylinder, volume of chamber, absolute temperature, control variable (piston position), Coulomb friction force, effective sectional area of valve and mass flow rate of air, respectively.

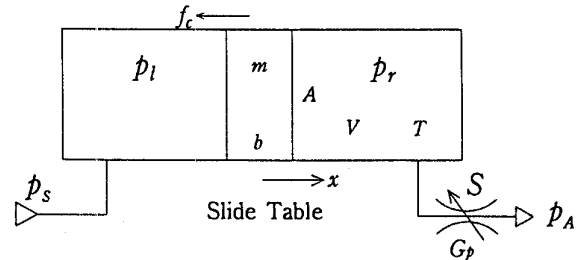


Figure 1: Simplified model of the plant

The both chambers are pressurized by supply pressure  $p_s$  at initial state. A characteristic equation of right chamber of cylinder is derived as

$$\frac{dp_r}{dt} = -\frac{\kappa RT}{V} G_p + \frac{\kappa p_r A}{V} \frac{dx}{dt} \quad (1)$$

In the case the air flow through the valve is considered as a sonic one, mass flow rate  $G_p$  is given as

$$G_p = S p_r \sqrt{\frac{2}{RT} \frac{\kappa}{\kappa - 1} \left( \frac{2}{\kappa + 1} \right)^{2/(\kappa - 1)}} \quad (2)$$

Here,  $p_d$  is introduced as a differential pressure between left and right chamber as  $p_d = p_l - p_r$ .

Assuming that, left side pressure is constant at supply pressure  $p_s$  as the effective sectional area of the contraction between  $p_s$  and  $p_l$  is enough large and effective sectional area of valve  $S$  is proportional to control signal  $u$ ,  $S = k_s u$ , then eq.(1) is rewritten as

$$\frac{dp_d}{dt} = \frac{\kappa RT k_s}{V} \sqrt{\frac{2}{RT} \frac{\kappa}{\kappa - 1} \left( \frac{2}{\kappa + 1} \right)^{2/(\kappa - 1)}} \quad *$$

$$* u(p_s - p_d) - \frac{\kappa(p_s - p_d)A}{V} \frac{dx}{dt} \quad (3)$$

$p_d$  has a non-linear characteristic for  $u$  and  $dx/dt$  owing that  $p_d$  is multiplied with  $u$  and  $dx/dt$ . In the case that the air flow through the valve is a subsonic flow, the non-linearity become stronger.

While the equation of motion is given by

$$p_d = \frac{m}{A} \frac{d^2 x}{dt^2} + \frac{b}{A} \frac{dx}{dt} + \frac{f_c}{A} \text{sgn}\left(\frac{dx}{dt}\right) \quad (4)$$

## 2.2 Pressure control system

### 2.2.1 Pressure control system using disturbance observer

Fig.2 shows the pressure control system using a disturbance observer[2], where  $P_{ref}$ ,  $P_{1n}$  and  $Q_1$  is a reference pressure, nominal model of transfer part from  $U$  to  $P_d$  and a filter to secure the stability, respectively, where a large letter represents the Laplace transformed variable.

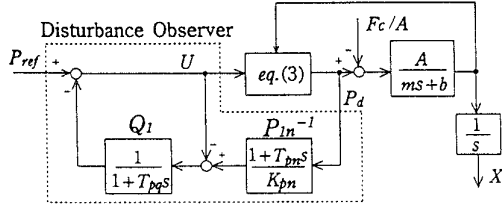


Figure 2: Pressure Control System

Assuming that,  $P_d$  is linearized as

$$P_d = \Phi U + \Psi sX \quad (5)$$

then the following relation is obtained.

$$P_d = \frac{(1 - Q_1)^{-1}}{\Phi^{-1} + P_{1n}^{-1} Q_1 (1 - Q_1)^{-1}} P_{ref} + \frac{\Phi^{-1} \Psi}{\Phi^{-1} + P_{1n}^{-1} Q_1 (1 - Q_1)^{-1}} sX \quad (6)$$

Complementary sensitivity function  $T$ , an indicator of robust stability in pressure control system, is given by

$$T = \frac{P_{1n}^{-1} Q_1 (1 - Q_1)^{-1}}{\Phi^{-1} + P_{1n}^{-1} Q_1 (1 - Q_1)^{-1}} \quad (7)$$

From eq.(6), the closer  $Q_1$  get to 1, the closer the transfer function between  $P_{ref}$  and  $P_d$  approaches nominal model  $P_{1n}$  and the smaller the influence of piston speed becomes. However, from eq.(7), to secure the robust stability  $Q_{1n}$  is desirable to be 0. Fortunately, a disturbance spectrum mainly exists in low frequency range and almost servo problems occur in band width of plant, while stability problem should be considered in high frequency one. So that the filter  $Q_1$  is designed to have a low pass filtering property and time constant  $T_{pq}$  is chosen by the trade off among these demands.

### 2.2.2 Comparison with PI control system

The basic control performances in pressure control loop are compared between the cases of using a disturbance observer and PI controller.

Fig.3 shows the pressure control loop using a PI controller. The pressure is fed back through a gain  $K_{pn}^{-1}$  to make the steady gain of closed loop transfer function equal in both control systems.

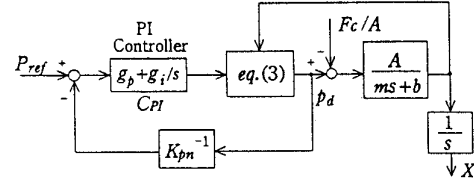


Figure 3: Pressure Control System Using PI controller

Similarly, the pressure  $P_d$  is represented in eq.(8), where  $C_{PI}$  shows a PI controller.

$$P_d = \frac{C_{PI}}{\Phi^{-1} + K_{pn}^{-1} C_{PI}} P_{ref} + \frac{\Phi^{-1} \Psi}{\Phi^{-1} + K_{pn}^{-1} C_{PI}} sX \quad (8)$$

Compared the second term in right side of eq.(6) with that of eq.(8), the sensitiveness becomes equivalent if

$$K_{pn}^{-1} C_{PI} = P_{1n}^{-1} Q_1 (1 - Q_1)^{-1} \quad (9)$$

Substituting each control parameter, following relations are obtained.

$$\left. \begin{aligned} g_p &= T_{pn} T_{pq}^{-1} \\ g_i &= T_{pq}^{-1} \end{aligned} \right\} \quad (10)$$

On the other hand, the gain of the closed loop transfer function of the control system using a disturbance observer, which is shown in the first term in right side of eq.(6) is  $|P_{1n} Q_1^{-1} K_{pn}^{-1}|$  times as much as that of PI control system shown in eq.(8).

Hence the relation of band width of both control systems are determined by that of time constant of  $P_{1n}$  and  $Q_1$ .

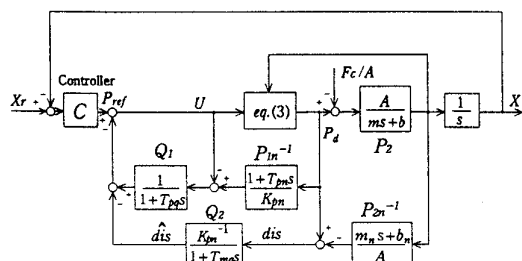
## 2.3 Positioning control system

First, the compensating signal which counterbalances the influence of friction force and parameter change is generated using an idea of disturbance observer. Each parameter of kinetic part is described using a nominal part and the fluctuation as

$$\left. \begin{aligned} \frac{m}{A} &= \frac{m_n}{A} + \Delta \frac{m}{A} \\ \frac{b}{A} &= \frac{b_n}{A} + \Delta \frac{b}{A} \end{aligned} \right\} \quad (11)$$

$$p_d = \frac{m_n}{A} \frac{d^2 x}{dt^2} + \frac{b_n}{A} \frac{dx}{dt} + dis \quad (12)$$
$$dis = \Delta \frac{m}{A} \frac{d^2 x}{dt^2} + \Delta \frac{b}{A} \frac{dx}{dt} + \frac{f_c}{A} \text{sgn}\left(\frac{dx}{dt}\right) \quad (13)$$

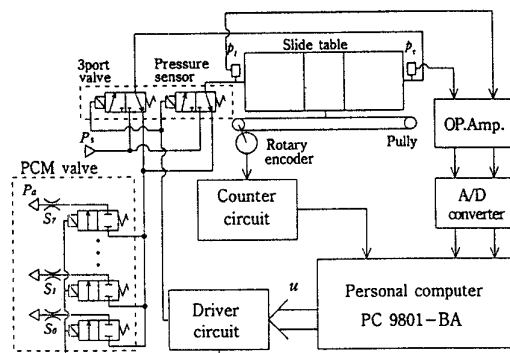
Fig.4 shows the proposed positioning control system.



The estimated disturbance  $\hat{dis}$  is fed back through a filter  $Q_2$  into pressure reference  $P_{ref}$  as a compensating signal. The controller  $C$  which prescribes the closed loop response is derived as follows.

$$sX = \frac{\frac{P_{1n}}{P_2^{-1}(1-Q_2P_{1n})+P_{2n}^{-1}Q_2P_{1n}}P_{ref}-\frac{F_c}{P_2^{-1}(1-Q_2P_{1n})+P_{2n}^{-1}Q_2P_{1n}}\frac{A}{A}}{1-Q_2P_{1n}} \quad (14)$$
$$\frac{sX}{P_{ref}} = P_{1n}P_{2n} = \frac{K_{pn}}{1 + T_{pn}s} \frac{A}{m_ns + b_n} \quad (15)$$
$$\frac{X}{X_r} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (16)$$
$$sX = \frac{\omega_n^2}{s + 2\zeta\omega_n}(X_r - X) \quad (17)$$
$$C = \frac{\omega_n^2 K_{pn}^{-1} A^{-1} (m_n s + b_n)}{s + 2\zeta\omega_n} \quad (18)$$
$$Q_1 P_{1n}^{-1} - Q_2 = \frac{s(T_{mq} + T_{pn} - T_{pq} + T_{mq}T_{pn}s)}{K_{pn}(1 + T_{pn}s)(1 + T_{mq}s)} \quad (19)$$
$$T_{mg} + T_{pn} - T_{pg} > 0 \quad (20)$$

Fig.5 shows the experimental equipment. A slide table comprising a pneumatic cylinder(800mm in stroke, 25mm in diameter) is set horizontally.



The position of piston is detected by rotary encoder (7.8  $\mu$ m in resolution) and pressure of each chamber is measured by pressure sensor (0.52 kPa in resolution). The control signal  $u$  calculated in computer drives the PCM digital control valve[3] and 3 port electromagnetic valve. The 3 port electromagnetic valves change the direction of piston motion and PCM valve regulates the speed of piston by meter out speed control scheme. The sign and the absolute value of  $u$  correspond to the driving signal of the 3 port valve and the PCM one, respectively. When  $u=0$ , both chamber of cylinder are pressurized by 3 port valves to raise a positioning stiffness.

## 4 Experiments and discussion

### 4.1 Effect of disturbance observer in pressure control system

Fig.6 shows the pressure response for various control signals  $u$  with fixing the table physically.

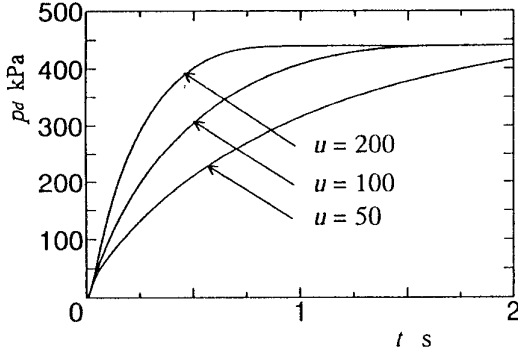


Figure 6: Pressure response

As seen from eq.(3), it is confirmed that pressure  $p_d$  converges to supply pressure  $p_s$  regardless of  $u$ , while transient response depends on  $u$ .

Before applying disturbance observer, each parameter of nominal model,  $T_{pn}$  and  $K_{pn}$ , have to be set. Assuming that closed loop transfer function of pressure control system is fixed to its nominal model  $P_{1n}$ , then in the frequency range where  $Q_2 P_{1n} = 1$ , closed loop transfer function is described as

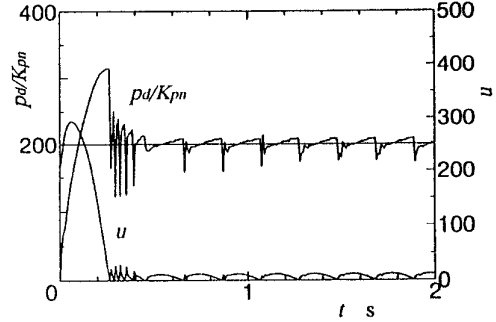
$$\frac{X}{X_r} = \frac{\omega_n^2}{s(s + 2\zeta\omega_n)(1 + T_{pn}s) + \omega_n^2} \quad (21)$$

Considering the maximum speed of piston,  $\omega_n$  and  $\zeta$  are set to 8 rad/s and 1.0, respectively. Therefore  $T_{pn}$  is set to 0.01s in order that an over shoot may not appear in step response.  $K_{pn}$  is set to 1 kPa by considering the maximum value of  $u$  and  $p_d$ .

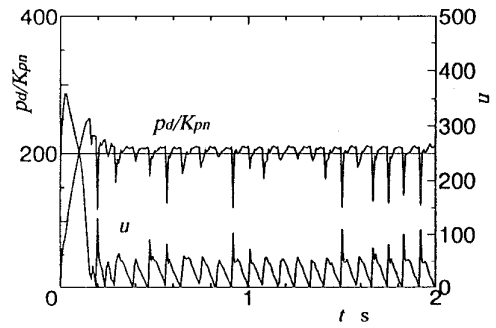
Fig.7 shows the step response of pressure using a disturbance observer shown in Fig.2. The cases of (a) and (b) correspond to the time constant of filter  $T_{pq}=0.05s$  and 0.01s, respectively.

Using the meter out method, a control signal  $u$  have to cycle around 0 in order to keep the pressure at a reference value. When both chamber of cylinder are pressurize at  $u=0$ ,  $p_d$  suddenly decreases because the pressures in both chambers approach to its supply pressure  $p_s$ .

As seeing from eq.(6), small  $T_{pq}$  makes the closed loop transfer function of the pressure control loop match to its nominal model. Therefore, in the case of (b), the pressure response is closer to the response of first order nominal model compared to (a). However, from Fig.4 the feedback loop of  $U$  has an integrator, and its gain corresponds to the inverse of  $T_{pq}$ , so that larger oscillation appears in the response of  $p_d$  and  $u$ .



(a)  $T_{pq}=0.05s$



(b)  $T_{pq}=0.01s$

Figure 7: Pressure response using disturbance observer

In steady state at positioning,  $u$  is desirable to be 0 to keep pressure at constant. An oscillating  $u$  may make the air leak which causes the fluctuation of pressure and consequently it lowers the positioning accuracy.

### 4.2 Effect of disturbance observer in kinetic part

First the nominal model  $P_{2n}$  is set as follows from an identification.

$$P_{2n} = \frac{Ab_n^{-1}}{1 + m_nb_n^{-1}s} = \frac{850}{1 + 0.08s} \quad (22)$$

The identification error can be considered as the parameter change from its nominal value, but from the robust stability, it is desirable to set to the nominal one as close as possible. Fig.8 shows the effect of disturbance observer in estimating the friction force and parameter change. A payload of 10 kg is added on the table and the control signal  $u$  is increased in step state at  $t=0.8s$ . Where  $\dot{d}is$  is the output of  $Q_2$  with  $T_{mq}=0.06s$  and  $K_{pn}=1.0$  in Fig.4.

From eq.(13), only the influence of friction force is appeared in  $\dot{d}is$  while the table has a constant speed ( $0.5 < t < 0.8$ ), when the table is accelerated the influ-

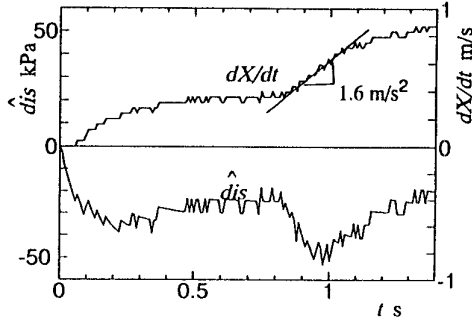


Figure 8: Effect of kinetic disturbance observer

ence of incremented mass is added to  $\hat{dis}$ . The adequateness of the  $\hat{dis}$  has been confirmed quantitatively. Therefore the simultaneous compensation of influence of a friction force and parameter change can be expected by feeding back  $\hat{dis}$ .

### 4.3 Positioning control

#### 4.3.1 Influence of filter in pressure control system

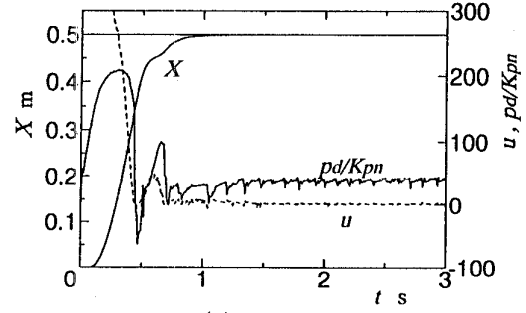
Fig.9 shows the step response of  $x_r=0.5m$ . The cases of (a) and (b) correspond to the time constant of filter in pressure control loop  $T_{pq}=0.05s$  and  $0.01s$ , respectively.

In the case of (a), a fluctuation can be seen in the positioning transient response because of the oscillation of pressure, but  $u$  converges to 0 in the steady state. Consequently  $p_d$  is kept to a certain value which balances  $f_c/A$  and piston never moves. On the contrary, the case of (b) shows the excellent transient response but  $u$  oscillates in the steady state. It leads to the worsening of positioning accuracy caused by a change of  $p_d$  due to the above mentioned air leak. Therefore in this study  $T_{pq}$  is set to  $0.05s$ .

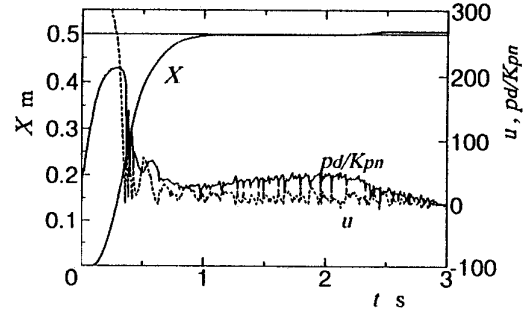
#### 4.3.2 Comparison with pressure control system using PI one

Fig.10 shows the step response, solid line and dashed one correspond to the case of pressure control loop using a disturbance observer and a PI controller, respectively. The control parameters of PI controller are set  $K_p=0.2$ ,  $K_i=20s^{-1}$  from eq.(10).

Time constant of pressure filter  $T_{pq}(=0.05s)$  is set larger than  $T_{pn}(=0.01s)$  in order to avoid the oscillation in  $u$ , then, as mentioned in section 2.2.2, the band width of closed pressure control loop becomes larger than that of PI one. Consequently, in the case of using a PI controller, the influence of pressure response lag can not be ignored and it becomes difficult to match the closed loop characteristic to the second order one. The over shoot is seemed to be caused by the pressure response lag. This influence of band width in pressure control loop appears seriously against payload shown



(a)  $T_{pq}=0.05s$



(b)  $T_{pq}=0.01s$

Figure 9: Influence of  $T_{pq}$  on positioning performances

in below. Fig.11 is the result of the same experiment with a payload. In the case of disturbance observer, the pressure  $p_d$  decreases rapidly due to the large band width and it helps to avoid an occurrence of an over shoot.

However, for the case  $x_r=0.2m$ , the over shoot appears as shown in Fig.12 (dotted line). Therefore we try to improve the robustness by setting more severe parameters. In Fig.12 the dashed line corresponds to the case where  $T_{mq}$  is decreased to  $0.04s$  which is a critical point in eq.(20). So the decrease of stability is confirmed from the response. Solid line is the case of

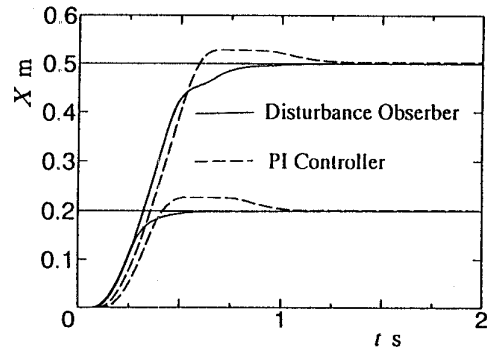


Figure 10: Step response without payload

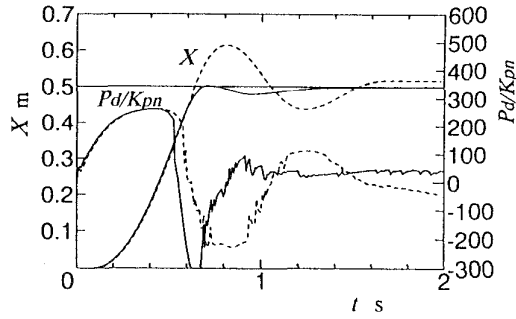


Figure 11: Comparison with PI control system in robustness

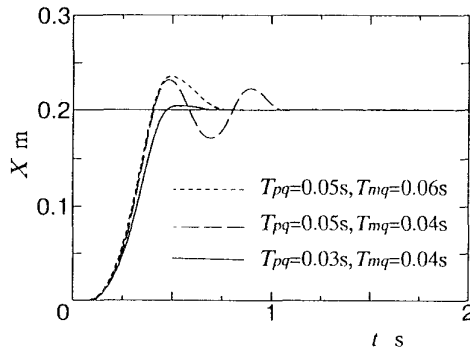


Figure 12: Improvement in robustness against the payload

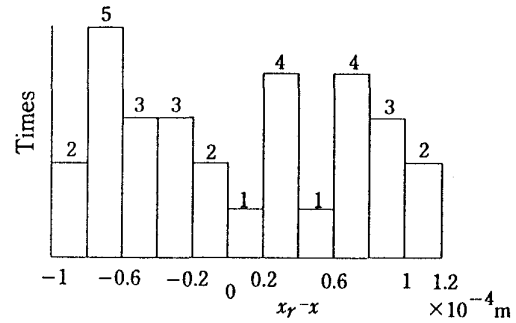
$T_{pq}$  is set to 0.03s to satisfy the eq.(20). The strong robustness can be confirmed. This robustness is not lowered for the case  $x_r = 0.5m$ .

However, as mentioned in 4.3.1, a minute oscillation appears on control signal  $u$  in the case  $T_{pq} = 0.03s$   $T_{mq} = 0.04s$ . Consequently the positioning accuracy is lowered slightly.

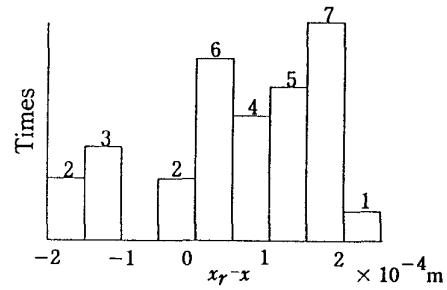
Finally the positioning accuracy is investigated. A step response has carried out 30 times for  $x_r = 0.5m$ . Fig.13 shows the positioning error at  $t=2.0s$ . (a) is the case of without payload (b) is that of adding payload of 10kg. Almost all positioning errors are included in  $\pm 0.1mm$  around the reference in the case (a) and  $\pm 0.2mm$  in that of (b).

## 5 Conclusion

A positioning control system including a pressure control loop is proposed. Two disturbance observers are employed. One is for a pressure control loop to compensate the non-linearity caused by meter out driving scheme and influence of piston speed which acts as disturbance, the other is for kinetic part to generate the compensating signal for pressure loop which counterbalance the influence of a friction force and parameter change.



(a) without payload



(b) with payload

Figure 13: Positioning accuracy

Comparing with a control system using a PI controller in pressure control loop, a strong robustness for payload has been attained.

By setting severe parameter of filter, more improvement of robustness can be expected, but a notice is necessary in order that control signal may not to oscillate.

Steady state error converges to almost  $\pm 0.1mm$  around the reference without a payload and  $\pm 0.2mm$  with a payload, which are enough accurate for pneumatic servo.

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