Transient performance improvement in dynamic certainty equivalent adaptive controllers by including a fixed compensator

Shiro Masuda
Okayama University

Akira Inoue
Okayama University

Proceedings of the 35th
Conference on Decision and Control
Kobe, Japan • December 1996

Transient Performance Improvement in Dynamic Certainty Equivalent Adaptive Controllers by Including a Fixed Compensator

Shiro Masuda, Akira Inoue
Faculty of Engineering, Okayama University
3-1-1, Tsushima-naka Okayama, JAPAN, 700
siro@suri.it.okayama-u.ac.jp

Abstract

In this paper we propose a modified Morse's dynamic certainty equivalent (DyCE) adaptive controllers in the continuous-time, single-input single-output linear time-invariant plant for the purpose of improving the transient performance. In the new scheme the additive feedback loop through a fixed compensator, which means non-adaptive one, is included. Furthermore a design method for the fixed compensator is also given, and performance analysis for the proposed DyCE adaptive controller is examined in terms of the mean square tracking error criterion and the $\mathcal{L}_\infty$ tracking error bound. According to the results of the paper the transient performance can be improved arbitrarily by the properly designed fixed compensator. Finally a numerical example is illustrated in order to show the effectiveness of the proposed method.

1 Introduction

Recently much concern has been given to the dynamic certainty equivalent (DyCE) adaptive controller proposed by Morse [1]. Using the DyCE schemes, we can assure the stability of the system without the error augmentation and tuning error normalization required in the traditional model reference adaptive control systems (MRACS) based on the certainty equivalence (CE) principle. Since the error augmentation and tuning error normalization may bring about undesirable transient performance, the DyCE adaptive controllers are considered to be ones of efficient schemes to improve the transient performance.

Indeed performance analysis in DyCE schemes has been studied, and the direct computable performance bounds in terms of both the $\mathcal{L}_2$ and $\mathcal{L}_\infty$ criteria were given [2], [3]. However controller structures studied so far were the same as the traditional MRACS. For the purpose of improving the transient performance of adaptive systems, the modification of the controller structure is also an efficient approach [5]-[7]. Hence by introducing the idea of the modified adaptive system to the DyCE adaptive systems, still more excellent transient performance are expected. However this method has been mainly applied to MRACS based on the CE principle and the transient performance improvement in DyCE adaptive controllers using the modification of the controller structure has not been studied yet. In [4] a fixed compensator was used for the DyCE adaptive controller. However the aim to adopt the fixed compensator was the disturbance attenuation and the robust stability in the presence of unmodeled dynamics, and the design method of the fixed compensator for transient performance improvement was not given.

Therefore in this paper we propose a modified Morse's dynamic certainty equivalent (DyCE) adaptive controllers in the continuous-time, single-input single-output linear time-invariant plant for the purpose of improving the transient performance. In the new scheme the additive feedback loop through a fixed compensator, which means non-adaptive one, is included. Furthermore a design method for the fixed compensator is also given, and the performance analysis for the proposed DyCE adaptive controller is examined in terms of the mean square tracking error criterion and the $\mathcal{L}_\infty$ tracking error bound. According to the results of the paper the transient performance can be improved arbitrarily by the properly designed fixed compensator.

The following notations are used. $(\cdot)^T$, $|\cdot|$, $\|\cdot\|_\infty$ $\|\cdot\|_2$ represent transpose, Euclid norm of vector, $\mathcal{L}_\infty$ norm and $\mathcal{L}_2$ norm. When $\|\cdot\|_\infty$ is used for transfer function, it represents $\mathcal{H}_\infty$ norm. $\|\cdot\|_{pg}$ represents peak gain, namely $\mathcal{L}_1$ norm of impulse responses. $\boldsymbol{RH}_\infty$ represents the ring of proper stable rational functions.

2 Problem Statements

Consider the following single-input, single-output, linear-time-invariant system

$$y_p(t) = P(s)u(t), \quad P(s) = g_p\frac{\alpha_p(s)}{\beta_p(s)} \tag{2.1}$$

0-7803-3590-2/96 $5.00 © 1996 IEEE

where $y_p(t)$ and $u(t)$ are the measured output and input respectively, and $\alpha_p(s)$ and $\beta_p(s)$ are coprime monic polynomials of degrees n and m respectively. The plant is assumed to be strictly proper. In this paper we only consider the plant for the case of $g_p = 1$.

Let the reference model is described as

$$y_m(t) = P_M(s)r(t) \tag{2.2}$$

where $r(t)$ and $y_m(t)$ are the reference model input and output respectively, and $r(t)$ is a piecewise continuous function of time.

The following assumptions are made for the plant and the reference model.

(A.1) $\alpha_p(s)$ is a stable monic polynomial.

(A.2) The relative degree $n^*$of $P(s)$ is known.

(A.3) The upper bound on the order n is known.

(A.4) The relative degree of $P_M(s)$ is greater than or equal to that of the plant.

The control objective is to determine a differentiator free controller so that all the signals in the closed-loop system remain bounded and the tracking error tends to zero asymptotically, namely

$$\lim_{t\to\infty} e(t) = \lim_{t\to\infty} (y_p(t) - y_m(t)) = 0. \tag{2.3}$$

3 A Construction of DyCE Adaptive Controller Including a Fixed Compensator

In this section we construct a proposed DyCE adaptive controller including a fixed compensator and the stability of the system is proved.

Let the following exact model matching (EMM) controller with the free parameter $Q(s) \in \boldsymbol{RH}_\infty$ be introduced [7].

$$\begin{aligned} u(t) &= -\theta^T\omega(t) + K(s)r(t) \\ &-Q(s)\left\{y_p(t) - \frac{1}{\beta_m(s)}\left(\theta^T\omega(t) + u(t)\right)\right\}, \\ &Q(s) \in \boldsymbol{RH}_\infty \end{aligned} \tag{3.1}$$

where $\omega(t)$ is the state variable filter defined as

$$\omega(t)=\frac{1}{\lambda(s)}\left[s^{n-2}u(t),\cdots,u(t),s^{n-1}y_p(t),\cdots,y_p(t)\right]^T$$

and θ is the parameter vector of order $2n-1$ so that the control law (3.1) allows us to obtain an EMM system [7]. $K(s)$ is the feedforward compensator which belong to $\boldsymbol{RH}_\infty$such that the next equation is satisfied.

$$P_M(s) = \frac{1}{\beta_m(s)}K(s), \quad K(s) \in \boldsymbol{RH}_\infty. \tag{3.2}$$

Such a $K(s)$ exists because the assumption (A.2) are made for the reference model $P_M(s)$. The EMM controller (3.1) has an optional compensator $Q(s)$, which can be selected freely among the proper stable rational transfer functions ($\boldsymbol{RH}_\infty$). In [7] replacing unknown parameters of the EMM controller (3.1) with their estimates based on the CE principle and regarding the free parameter $Q(s)$ as a fixed compensator, MRACS including a fixed compensator was constructed.

Based on the DyCE principle, in this paper, we construct MRACS including a fixed compensator using the EMM control law (3.1). In DyCE schemes high order times derivatives of unknown parameters of the EMM system are replaced with their estimates generated by an appropriate high order estimator. In order to adopt the DyCE principle we rewrite the EMM control law (3.1) as follows.

$$\begin{aligned} u(t)=&-\xi(s)\left(\theta^T\zeta(t)\right) + K(s)r(t) \\ &-Q(s)\left\{y_p(t) - \frac{1}{s+\kappa}\left(\theta^T\zeta(t) + \nu(t)\right)\right\} \end{aligned} \tag{3.3}$$

where $\xi(s)$ and $(s+\kappa)$ are the factors of $\beta_m(s)$, namely

$$\beta_m(s) = (s+\kappa)\xi(s), \qquad \kappa > 0 \tag{3.4}$$

and $\zeta(t)$ and $\nu(t)$ are defined as

$$\zeta(t) = \frac{1}{\xi(s)}\omega(t), \quad \nu(t) = \frac{1}{\xi(s)}u(t).$$

Since $\xi(s)$ is a dynamic operator in (3.3), we can obtain a DyCE controller by replacing the parameter θ with their estimates $\hat{\theta}(t)$. Thus the following control law can be obtained.

$$\begin{aligned} u(t)=&-\xi(s)\left(\hat{\theta}(t)^T\zeta(t)\right) + K(s)r(t) \\ &-Q(s)\left\{y_p(t) - \frac{1}{s+\kappa}\left(\hat{\theta}(t)^T\zeta(t) + \nu(t)\right)\right\}. \end{aligned} \tag{3.5}$$

The control law (3.5) is the DyCE adaptive controller including a fixed compensator proposed in this paper. Here $Q(s)$ is a fixed compensator, and has the ability to improve the transient performance. The detail of how to design $Q(s)$ will be given in the subsequent section. The additive feedback loop through $Q(s)$ is the main difference between the proposed control law and conventional ones [2], [3]. Indeed when $Q(s)$ equals to zero, the control law (3.5) turns to be equivalent to the conventional ones.

The role of the dynamic operator $\xi(s)$ is similar to the conventional ones. Namely $\xi(s)$ reduces the relative degree of the transfer function involving the error equation. In fact the tracking error $e(t)$ can be calculated in the following way when the control law (3.5) is utilized.

$$\begin{aligned} e(t) \quad = \quad & -\frac{1}{s+\kappa}\left(1 - Q(s)\frac{1}{\beta_m(s)}\right)\left[\tilde{\theta}(t)^T\zeta(t)\right] \\ & +\epsilon(t) \end{aligned} \tag{3.6}$$

where $\tilde{\theta}(t)$ is the parameter error vector defined as

$$\tilde{\theta}(t) = \hat{\theta}(t) - \theta. \qquad (3.7)$$

and $\epsilon(t)$ denotes an exponentially decaying term due to the initial conditions in the plant and the controller filters. As mentioned above, the relative degree of the transfer function $-\frac{1}{s+\kappa}\left(1 - Q(s)\frac{1}{\beta_m(s)}\right)$ in (3.6) is one due to the dynamic operator $\xi(s)$. However since the transfer function may not be strictly positive real, we introduce the filtered error $\hat{e}(t)$ defined as

$$\hat{e}(t) = -\left(1 - Q(s)\frac{1}{\beta_m(s)}\right)^{-1} e(t). \qquad (3.8)$$

Then the error equation (3.6) becomes

$$\hat{e}(t) = \frac{1}{s+\kappa}\left(\tilde{\theta}(t)^T\zeta(t)\right) + \epsilon(t). \qquad (3.9)$$

When the $\hat{e}(t)$ is regarded as the error signal, we can see that the error equation (3.9) is strictly positive real, and the obtained error equation (3.9) is similar to the conventional one. Hence the similar high order estimator to the conventional method [1]-[3] can be utilized.

$$\begin{aligned} \dot{\eta}_i(t) &= -\gamma\zeta_i(t)\hat{e}(t), \qquad \eta_i(0) = \xi[0]\hat{\theta}_i(0) \\ \dot{x}_i(t) &= \rho^2(t)\left[Ax_i(t) + b\eta_i(t)\right], \\ x_i(0) &= -\xi[0]A^{-1}b\hat{\theta}_i(0), \\ \hat{\theta}_i(t) &= c^T x_i(t), \quad \rho^2(t) = 1 + \mu\left|\zeta(t)\right|^2 \end{aligned} \qquad (3.10)$$

where (A, b, c^T) is a minimal realization of $\frac{1}{\xi(s)}$, and $\zeta_i(t)$, $\hat{\theta}_i(t)$ is the i-th component of $\zeta(t)$ and $\hat{\theta}(t)$ respectively. μ, γ are positive constants. In fact the next theorem on the stability of the system is satisfied.

Theorem 3.1 *Consider the MRACS with the control law (3.5) and the high order estimator (3.10) for the unknown plant (2.1) except assumptions (A.1)-(A.3). It is assumed that*

$$\mu > \frac{q\gamma}{2\kappa}\left|P_0A^{-1}b\right|^2 \qquad (3.11)$$

where P_0 is defined as follows.

$$P_0 \triangleq P_1 + \xi[0]cc^T \qquad (3.12)$$

where P_1 is the symmetric positive definite solution of $A^TP_1 + P_1A = -I$, and $q = 2n - 1$ is the order of parameter vector θ. Furthermore $Q(s) \in \boldsymbol{RH}_\infty$ is assumed to be satisfied with such a condition.

$$\left(1 - Q(s)\frac{1}{\beta_m(s)}\right)^{-1} \in \boldsymbol{RH}_\infty. \qquad (3.13)$$

Under these conditions, all the signals of the closed loop system are uniformly bounded, and the control objective (2.3) is achieved.

Proof: At first in order to give the error model of the high order estimator (3.10), we define

$$\begin{aligned} \tilde{x}_i(t) &\triangleq x_i(t) + \xi[0]A^{-1}b\theta_i, \quad i = 1, \cdots, q, \quad (3.14) \\ \tilde{\eta}_i(t) &\triangleq \eta_i(t) - \xi[0]\theta_i, \quad i = 1, \cdots, q. \quad (3.15) \end{aligned}$$

Using (3.15), we can rewrite (3.10) as

$$\begin{aligned} \dot{\tilde{\eta}}_i(t) &= -\gamma\zeta_i(t)\hat{e}(t), \qquad \tilde{\eta}_i(0) = \xi[0]\tilde{\theta}_i(0) \\ \dot{\tilde{x}}_i(t) &= \rho^2(t)\left[A\tilde{x}_i(t) + b\tilde{\eta}_i(t)\right], \\ \tilde{x}_i(0) &= -\xi[0]A^{-1}b\tilde{\theta}_i(0), \\ \tilde{\theta}_i(t) &= c^T\tilde{x}_i(t), \quad \rho^2(t) = 1 + \mu\left|\zeta(t)\right|^2. \end{aligned} \qquad (3.16)$$

The next in order to show that the error model of the estimator (3.16) has a positivity property, we consider a quadratic function

$$\begin{aligned} V[\chi(t)] &\triangleq \chi(t)^T\Pi\chi(t) \\ \chi(t) &\triangleq \left[\tilde{x}_1^T(t), \cdots, \tilde{x}_q^T(t), \tilde{\eta}_1(t), \cdots, \tilde{\eta}_q(t)\right]^T \\ \Pi &\triangleq \begin{bmatrix} \Lambda_{P_1} & \vdots & \Lambda_{P_3} \\ \cdots & \cdots & \cdots \\ \Lambda_{P_3}^T & \vdots & \Lambda_{P_2} \end{bmatrix} \end{aligned} \qquad (3.17)$$

where $\Lambda_{P_1} \in \mathbf{R}^{q(n^*-1)\times q(n^*-1)}$, $\Lambda_{P_2} \in \mathbf{R}^{q\times q}$, $\Lambda_{P_3} \in \mathbf{R}^{q(n^*-1)\times q}$ are block diagonal matrices with $P_1 \in \mathbf{R}^{q\times q}$, $P_2 \in \mathbf{R}$ and $P_3 \in \mathbf{R}^q$ respectively. And P_2, P_3 are defined as

$$P_2 \triangleq \frac{1}{\xi[0]} + b^TA^{-T}P_1A^{-1}b \qquad (3.18)$$

$$P_3 \triangleq P_1A^{-1}b \qquad (3.19)$$

Calculating the derivative of $V[\chi(t)]$ evaluated along the trajectories of (3.16), we obtain

$$\begin{aligned} \dot{V}[\chi(t)] &= -\rho^2(t)\sum_{i=1}^{q}|z_i(t)|^2 - 2\gamma\psi(t)\hat{e}(t) \\ &\quad -2\gamma\sum_{i=1}^{q} z_i(t)^TP_0A^{-1}b\zeta_i(t)\hat{e}(t) \\ &\leq -\sum_{i=1}^{q}|z_i(t)|^2 + 2\gamma k|\hat{e}(t)|^2 - 2\gamma\psi(t)\hat{e}(t) \end{aligned} \qquad (3.20)$$

where k, $\psi(t)$ and $z_i(t)$ are defined as

$$k \triangleq \frac{q}{2\mu}\gamma\left|P_0A^{-1}b\right|^2 \qquad (3.21)$$

$$\psi(t) \triangleq \tilde{\theta}(t)^T\zeta(t) \qquad (3.22)$$

$$z_i(t) \triangleq \tilde{x}_i(t) + A^{-1}b\tilde{\eta}_i(t). \qquad (3.23)$$

Integrating (3.20) from 0 to t and rearranging, we can derive

$$\int_0^t \psi(\tau)\hat{e}(\tau)d\tau \leq k\int_0^t \hat{e}(\tau)^2\, d\tau$$

$$- \frac{1}{2\gamma}V[\chi(t)] + \frac{1}{2\gamma}V[\chi(0)]$$
$$- \frac{1}{2\gamma}\int_0^t \sum_{i=1}^{q} |z_i(\tau)|^2 d\tau. \quad (3.24)$$

On the other hand, from the error equation (3.9), we get

$$\int_0^t v(\tau)\hat{e}(\tau)\, d\tau = \frac{1}{2}\hat{e}(t)^2 - \frac{1}{2}\hat{e}(0)^2 + \kappa\int_0^t \hat{e}(\tau)^2\, d\tau - \int_0^t \hat{e}(\tau)\epsilon_1(\tau)\, d\tau \quad (3.25)$$

where $\epsilon_1(t) = (s+\kappa)\epsilon(t)$. Therefore from (3.24) and (3.25) it follows that

$$\left(\kappa - k - \frac{1}{2h}\right)\int_0^t \hat{e}(\tau)^2\, d\tau \le -\frac{1}{2}\hat{e}(t)^2 + \frac{1}{2}\hat{e}(0)^2 + \frac{h}{2}\int_0^t \epsilon_1(\tau)^2\, d\tau - \frac{1}{2\gamma}V[\chi(t)] + \frac{1}{2\gamma}\xi[0]|\tilde{\theta}(0)|^2 - \frac{1}{2\gamma}\int_0^t \sum_{i=1}^{q}|z_i(\tau)|^2 d\tau \quad (3.26)$$

for all positive constants h. From (3.11) it follows that $\kappa - k > 0$. Hence there exists h sufficiently large so that $\kappa - k - \frac{1}{2h} > 0$. Therefore we conclude that $\int_0^t \hat{e}(\tau)^2\, d\tau$ and $V[\chi(t)]$ is bounded, and we can obtain that $\tilde{\theta}(t)$ and $n^* - 1$ times derivative of the estimated parameter $\hat{\theta}(t)$ is bounded. This means that we can extend the solutions of the error equation on $[0, \infty)$. Hence it follows that $\hat{e}(t) \in \mathcal{L}_2$. Furthermore since (3.26) is satisfied for all t, it follows that $\hat{e}(t) \in \mathcal{L}_\infty$. Therefore from (3.8) and (3.13) we conclude that $e(t) \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ and $y_p(t) \in \mathcal{L}_\infty$. On the other hand, from the definition of $\zeta(t)$ we obtain

$$\zeta(t) = W(s)y_p(t) \quad (3.27)$$

where $W(s)$ is defined as

$$W(s) \triangleq \frac{1}{\xi(s)\lambda(s)}\left[P(s)^{-1}s^{n-2}, \cdots, P(s)^{-1}, s^{n-1}, \cdots, 1\right]^T. \quad (3.28)$$

Since $W(s) \in \boldsymbol{RH}_\infty$, $\zeta(t) \in \mathcal{L}_\infty$. From (3.6) and $\tilde{\theta}(t), \zeta(t) \in \mathcal{L}_\infty$, it follows that $\dot{e}(t) \in \mathcal{L}_\infty$. Hence from $e(t) \in \mathcal{L}_2$, it is concluded that $e(t)$ converges to zero.

Finally we show the boundedness of control input u. Noting that the boundedness of $\dot{y}_m(t)$, it follows from the boundedness of $\dot{e}(t)$ that $\dot{y}_p(t)$ is bounded. Hence $\dot{\zeta}(t)$ is bounded from (3.27). Furthermore from (3.6) and the boundedness of $\dot{\tilde{\theta}}(t)$ and $\dot{\zeta}(t)$, it follows that $\ddot{e}(t)$ is bounded. When the relative degree of the plant is greater than or equal two, one of the reference model becomes greater than or equal two. Hence $\ddot{y}_m(t)$ is bounded, and so $\ddot{y}_p(t)$ is bounded. Since we can repeat the similar discussion until the high order derivative of $y_m(t)$ does not exist, the relative degree times derivative of $y_p(t)$ is bounded. Therefore noting that $P(s)$ is minimal phase, the inverse system from the relative degree times derivative of $y_p(t)$ to input $u(t)$ can be described as a proper stable rational function. Hence from the boundedness of high order derivative of $y_p(t)$ it follows that $u(t)$ is bounded. Therefore we can show the boundedness of all signal of the closed loop system. ■

4 The design method of the fixed compensator and performance analysis

In this section we give the design method of the fixed compensator for the performance improvement. Furthermore we show that the transient performance can be improved arbitrarily by using the proposed fixed compensator in terms of the mean square tracking error criterion and the $\mathcal{L}_\infty$ tracking error bound.

Using (3.6) the mean square tracking error can be evaluated by

$$\frac{1}{t}\int_0^t e(\tau)^2 d\tau \le \left\|\frac{1}{s+\kappa}\left(1 - Q(s)\frac{1}{\beta_m(s)}\right)\right\|_\infty \frac{1}{t}\int_0^t \left(\tilde{\theta}(\tau)^T\zeta(\tau)\right)^2 d\tau + \|\epsilon(t)\|_2 \quad (4.1)$$

and the $\mathcal{L}_\infty$ norm of the tracking error is evaluated by

$$\|e(t)\|_\infty \le \left\|\frac{1}{s+\kappa}\left(1 - Q(s)\frac{1}{\beta_m(s)}\right)\right\|_{pg} \left\|\tilde{\theta}(t)^T\zeta(t)\right\|_\infty + |\epsilon(t)|. \quad (4.2)$$

Using the relation between $\|\cdot\|_{pg}$ and $\mathcal{H}_\infty$ norm [10], we get

$$\left\|\frac{1}{s+\kappa}\left(1 - Q(s)\frac{1}{\beta_m(s)}\right)\right\|_{pg} \le (2m_1 + 1)\left\|\frac{1}{s+\kappa}\left(1 - Q(s)\frac{1}{\beta_m(s)}\right)\right\|_\infty \quad (4.3)$$

where m_1 is the McMillan degree of

$$\frac{1}{s+\kappa}\left(1 - Q(s)\frac{1}{\beta_m(s)}\right). \quad (4.4)$$

Therefore in order to improve the transient performance in terms of the mean square tracking error criterion and the $\mathcal{L}_\infty$ tracking error bound, we have only to reduce the $\mathcal{H}_\infty$ norm of (4.4), while (3.13) is satisfied. As such a fixed compensator, in this paper, we propose the next fixed compensator $Q(s)$.

$$Q(s) = \frac{\beta_m(s)}{(\tau s + \tau + 1)^{n^*}}, \quad \tau > 0 \tag{4.5}$$

where τ is a positive constant.

Lemma 4.1 *For given $Q(s)$ in (4.5), $1 - Q(s)\frac{1}{\beta_m(s)}$ is unimodular over $\boldsymbol{RH}_\infty$.*

Proof: From $Q(s)$ defined in (4.5) it follows that

$$\left\| Q(s)\frac{1}{\beta_m(s)} \right\|_\infty = \left\| \frac{1}{(\tau s + \tau + 1)^{n^*}} \right\|_\infty \leq \frac{1}{(\tau + 1)^{n^*}} < 1. \tag{4.6}$$

Hence from Lemma 2.8 in [8] it follows that $1 - Q(s)\frac{1}{\beta_m(s)}$ is unimodular over $\boldsymbol{RH}_\infty$. ■

From Lemma 4.1 $Q(s)$ defined in (4.5) is satisfied with (3.13). Therefore the condition of Theorem 3.1 is satisfied and we can see that the control objective is achieved using the fixed compensator (4.5). Furthermore the $\mathcal{H}_\infty$ norm of (4.4) can be evaluated as given in the following Lemma by using the fixed compensator (4.5).

Lemma 4.2 *For given $Q(s)$ in (4.5) there exists a positive constant α which is independent of τ such that*

$$\left\| \frac{1}{s+\kappa}\left(1 - Q(s)\frac{1}{\beta_m(s)}\right) \right\|_\infty < \tau\alpha. \tag{4.7}$$

Proof: Substituting $Q(s)$ in (4.5) into (4.4) we get

$$\frac{1}{s+\kappa}\left(1 - Q(s)\frac{1}{\beta_m(s)}\right) = \frac{\tau(s+1)}{s+\kappa}\sum_{i=1}^{n^*}\left(\frac{1}{(\tau s + \tau + 1)^i}\right). \tag{4.8}$$

Noting that $\left\| \frac{1}{(\tau s + \tau + 1)^i} \right\|_\infty < 1$, $i = 1, \cdots, n^*$, we can derive

$$\left\| \frac{1}{s+\kappa}\left(1 - Q(s)\frac{1}{\beta_m(s)}\right) \right\|_\infty < \tau n^* \left\| \frac{s+1}{s+\kappa} \right\|_\infty \leq \tau n^* \max\left[1, \frac{1}{\kappa}\right] \tag{4.9}$$

Define α as $n^* \max\left[1, \frac{1}{\kappa}\right]$, then α is independent of τ and (4.7) is satisfied. ■

The next we show that $\frac{1}{t}\int_0^t \left(\tilde{\theta}(\tau)^T\zeta(\tau)\right)^2 d\tau$ and $\|\tilde{\theta}(t)^T\zeta(t)\|_\infty$ are bounded irrespective the value of τ. In order to show them, we prepare the following Lemma.

Lemma 4.3 *There exist positive constants C_1, C_2, C_3, C_4 which are independent of τ such that*

$$\begin{array}{ll} \|\zeta(t)\|_\infty \leq C_1, & \|\dot{\tilde{\theta}}(t)\|_2 \leq C_2 \\ \|\tilde{\theta}(t)\|_\infty \leq C_3, & \|\dot{\zeta}(t)\|_\infty \leq C_4. \end{array} \tag{4.10}$$

Proof: We begin with the evaluation of $\|\zeta(t)\|_\infty$. Using (3.8), (3.27) and (3.28) we can obtain

$$\begin{aligned}\zeta(t) &= W(s)e(t) + W(s)y_m(t) \\ &= -W(s)\left(1 - Q(s)\frac{1}{\beta_m(s)}\right)\hat{e}(t) + W(s)y_m(t). \end{aligned} \tag{4.11}$$

Hence $\|\zeta(t)\|_\infty$ can be evaluated as

$$\begin{aligned}\|\zeta(t)\|_\infty \leq & \left\| W(s)\left(1 - Q(s)\frac{1}{\beta_m(s)}\right) \right\|_{pg} \|\hat{e}(t)\|_\infty \\ & + \|W(s)\|_{pg}\|y_m(t)\|_\infty. \end{aligned} \tag{4.12}$$

Then using the relation between $\|\cdot\|_{pg}$ and $\mathcal{H}_\infty$ norm [10], we get

$$\begin{aligned} & \left\| W(s)\left(1 - Q(s)\frac{1}{\beta_m(s)}\right) \right\|_{pg} \\ \leq & \ (2m_2 + 1)\left\| W(s)\left(1 - Q(s)\frac{1}{\beta_m(s)}\right) \right\|_\infty \\ \leq & \ (2m_2 + 1)n^*\|W(s)\|_\infty \end{aligned} \tag{4.13}$$

where m_2 is the McMillan degree of

$$W(s)\left(1 - Q(s)\frac{1}{\beta_m(s)}\right).$$

Using (3.26) we get

$$\hat{e}(t) \leq \hat{e}(0) + \sqrt{h}\|\epsilon(t)\|_2 + \sqrt{\frac{\xi[0]}{\gamma}}|\tilde{\theta}(0)| \tag{4.14}$$

Hence from (4.12), (4.13) and (4.14), we get

$$\|\zeta(t)\|_\infty \leq c_1|\tilde{\theta}(0)| + d_1 \tag{4.15}$$

where

$$c_1 = (2m_2 + 1)n^*\|W(s)\|_\infty\sqrt{\frac{\xi[0]}{\gamma}} \tag{4.16}$$

$$\begin{aligned} d_1 = & \ (2m_2 + 1)n^*\|W(s)\|_\infty\left(\hat{e}(0) + \sqrt{h}\|\epsilon(t)\|_2\right) \\ & + \|W(s)\|_{pg}\|y_m(t)\|_\infty \end{aligned} \tag{4.17}$$

Define C_1 as $c_1|\tilde{\theta}(0)| + d_1$, then C_1 is independent of τ, and (4.10) is satisfied.

The next we show that $\dot{\tilde{\theta}}$ belong to $\mathcal{L}_2$. From (3.16) we can derive

$$\begin{aligned}|\dot{\tilde{\theta}}(t)|^2 &= \sum_{i=1}^{q} |c^T \dot{\tilde{x}}_i(t)|^2 \\ &= \sum_{i=1}^{q} |c^T \rho^2(t) A(\tilde{x}_i(t) + A^{-1} b \tilde{\eta}_i(t))|^2 \\ &\le \rho^4(t) |c^T A|^2 \sum_{i=1}^{q} |z_i(t)|^2. \qquad (4.18)\end{aligned}$$

Hence it follows that

$$\begin{aligned}\left\|\dot{\tilde{\theta}}(t)\right\|_2^2 &\le \sqrt{\int_0^\infty \rho^2(t)|c^T A| \sum_{i=1}^{q} |z_i(t)|^2 \, dt} \\ &\le \|\rho^2(t)\|_\infty |c^T A| \sqrt{\int_0^\infty \sum_{i=1}^{q} |z_i(t)|^2 \, dt}. \qquad (4.19)\end{aligned}$$

From (3.26) we get

$$\sqrt{\int_0^\infty \sum_{i=1}^{q} |z_i(t)|^2 \, dt} \le \sqrt{\xi[0]}|\tilde{\theta}(0)| + \sqrt{\gamma}\hat{e}(0) + \sqrt{\gamma h}\|\epsilon_1(t)\|_2. \qquad (4.20)$$

And from the definition of $\rho^2(t)$ and (4.15) and the evaluation of $\|\zeta(t)\|_\infty$, we get

$$\|\rho^2(t)\|_\infty \le 1 + \mu\left(c_1|\tilde{\theta}(0)| + d_1\right)^2. \qquad (4.21)$$

Therefore it follows from (4.19), (4.20) and (4.21) that $\dot{\tilde{\theta}} \in \mathcal{L}_2$ and we can get

$$\begin{aligned}\|\dot{\tilde{\theta}}\|_2 &\le \left\{1 + \mu\left(c_1|\tilde{\theta}(0)| + d_1\right)^2\right\} |c^T A| \\ &\times \left(\sqrt{\xi[0]}|\tilde{\theta}(0)| + \sqrt{\gamma}|\hat{e}(0)| + \sqrt{\gamma h}\|\epsilon_1(t)\|_2\right) \qquad (4.22)\end{aligned}$$

Define C_2 as the right-hand side of (4.22), then C_2 is independent of τ and (4.10) is satisfied.

The next we show the evaluation of $\|\tilde{\theta}(t)\|_\infty$. From (3.26) we get

$$V[\chi(t)] \le \gamma \hat{e}(0)^2 + \gamma h \int_0^t \epsilon_1(\tau)^2 \, d\tau + \xi[0]|\tilde{\theta}(0)|^2. \qquad (4.23)$$

On the other hand, from the definition of $V[\chi(t)]$ in (3.17), we get

$$\frac{\lambda_{\min}[\Pi]}{|c|^2}|\tilde{\theta}(t)|^2 \le \lambda_{\min}[\Pi]|\tilde{x}|^2 \le V[\chi(t)] \qquad (4.24)$$

where $\lambda_{\min}[\cdot]$ represets the minimal eigenvalue. Define C_3 as

$$C_3 = \frac{|c|\left(\sqrt{\gamma}\hat{e}(0) + \sqrt{\gamma h}\|\epsilon_1(\tau)\|_2 + \sqrt{\xi[0]}|\tilde{\theta}(0)|\right)}{\sqrt{\lambda_{\min}[\Pi]}}, \qquad (4.25)$$

then we get $\|\tilde{\theta}(t)\|_\infty \le C_3$. Since C_3 is independent of τ, (4.10) is satisfied.

Finally we evaluate $\|\dot{\zeta}(t)\|_\infty$. Using (3.9) and (4.11) we get

$$\begin{aligned}\dot{\zeta}(t) &= -W(s)\frac{s}{s+\kappa}\left(1 - Q(s)\frac{1}{\beta_m(s)}\right)\tilde{\theta}(t)^T\zeta(t) \\ &\quad + W(s)\dot{\epsilon}(t) + W(s)\dot{y}_m(t). \qquad (4.26)\end{aligned}$$

Hence using (4.13), $\|\zeta(t)\|_\infty \le C_1$ and $\|\tilde{\theta}(t)\|_\infty \le C_3$, we can evaluate $\|\dot{\zeta}(t)\|_\infty$ as

$$\begin{aligned}&\|\dot{\zeta}(t)\|_\infty \\ &\le \left\|W(s)\frac{s}{s+\kappa}\left(1 - Q(s)\frac{1}{\beta_m(s)}\right)\right\|_{pg} \|\tilde{\theta}(t)^T\zeta(t)\|_\infty \\ &\quad + \|W(s)\|_{pg}\|\dot{\epsilon}(t)\|_\infty + \|W(s)\|_{pg}\|\dot{y}_m(t)\|_\infty \\ &\le (2m_3+1)\|W(s)\|_\infty \frac{n^*}{\kappa} C_1 C_3 + \|W(s)\|_{pg}\|\dot{\epsilon}(t)\|_\infty \\ &\quad + \|W(s)\|_{pg}\|\dot{y}_m(t)\|_\infty \qquad (4.27)\end{aligned}$$

where m_3 is the McMillan degree of

$$W(s)\frac{s}{s+\kappa}\left(1 - Q(s)\frac{1}{\beta_m(s)}\right).$$

Define the right-hand side of (4.27) as C_4, then C_4 is independent of τ and (4.10) is satisfied. ■

Using Lemma 4.3 we can give the evaluation of $\frac{1}{t}\int_0^t \left(\tilde{\theta}(\tau)^T\zeta(\tau)\right)^2 d\tau$ and $\|\tilde{\theta}(t)^T\zeta(t)\|_\infty$.

Lemma 4.4 *There exist positive constants C_5, C_6, C_7 which are independent of τ such that*

$$\begin{aligned}\frac{1}{t}\int_0^t \left(\tilde{\theta}(\tau)^T\zeta(\tau)\right)^2 d\tau &\le \frac{1}{t}C_5 + C_6 \qquad (4.28) \\ \|\tilde{\theta}(t)^T\zeta(t)\|_\infty &\le C_7. \qquad (4.29)\end{aligned}$$

Proof: From (3.9) we get

$$\begin{aligned}\hat{e}(t) &= \int_0^t e^{-\kappa(t-\tau)}\tilde{\theta}(\tau)^T\zeta(\tau)d\tau + \epsilon(t) \\ &= e^{-\kappa t}\int_0^t e^{\kappa\tau}\tilde{\theta}(\tau)^T\zeta(\tau)d\tau + \epsilon(t). \qquad (4.30)\end{aligned}$$

Integrating $\int_0^t e^{\kappa\tau}\tilde{\theta}(\tau)^T\zeta(\tau)d\tau$ by part, we get

$$\begin{aligned}&\int_0^t e^{\kappa\tau}\tilde{\theta}(\tau)^T\zeta(\tau)d\tau \\ &= \left[\frac{1}{\kappa}e^{\kappa\tau}\tilde{\theta}(\tau)^T\zeta(\tau)\right]_0^t - \int_0^t \frac{1}{\kappa}e^{\kappa\tau}\frac{d}{d\tau}\left(\tilde{\theta}(\tau)^T\zeta(\tau)\right)d\tau \\ &= \frac{1}{\kappa}e^{\kappa t}\tilde{\theta}(t)^T\zeta(t) - \frac{1}{\kappa}\tilde{\theta}(0)^T\zeta(0) \\ &\quad - \frac{1}{\kappa}\int_0^t e^{\kappa\tau}\frac{d}{d\tau}\left(\tilde{\theta}(\tau)^T\zeta(\tau)\right)d\tau. \qquad (4.31)\end{aligned}$$

Hence it follows that

$$\hat{e}(t) = \frac{1}{\kappa}\tilde{\theta}(t)^T\zeta(t) - \frac{1}{\kappa}e^{-\kappa t}\tilde{\theta}(0)^T\zeta(0) - \frac{1}{\kappa}\int_0^t e^{-\kappa(t-\tau)}\frac{d}{d\tau}\left(\tilde{\theta}(\tau)^T\zeta(\tau)\right)d\tau. + \epsilon(t) \quad (4.32)$$

Rearranging (4.32), we get

$$\tilde{\theta}(t)^T\zeta(t) = \kappa\hat{e}(t) + e^{-\kappa t}\tilde{\theta}(0)^T\zeta(0) + \int_0^t e^{-\kappa(t-\tau)}\frac{d}{d\tau}\left(\tilde{\theta}(\tau)^T\zeta(\tau)\right)d\tau + \kappa\epsilon(t). \quad (4.33)$$

Therefore we can derive

$$\begin{aligned}\int_0^t (\tilde{\theta}(\tau)^T\zeta(\tau))^2 d\tau \\ \le\ & \kappa\|\hat{e}(t)\|_2^2 + |\tilde{\theta}(0)^T\zeta(0)|^2\|e^{-\kappa t}\|_2^2 \\ & +\|e^{-\kappa t}\|_1^2\int_0^t\left(\frac{d}{dt}\left(\tilde{\theta}(\tau)^T\zeta(\tau)\right)\right)^2 d\tau \\ & +\kappa\|\epsilon(t)\|_2. \qquad (4.34)\end{aligned}$$

Here we used Theorem 6.75 in [11]. Using Lemma 4.3, we get

$$\begin{aligned}&\int_0^t\left(\frac{d}{dt}\left(\tilde{\theta}(\tau)^T\zeta(\tau)\right)\right)^2 d\tau \\ &\le \int_0^t\left(\dot{\tilde{\theta}}(\tau)^T\zeta(\tau)\right)^2 d\tau + \int_0^t\left(\tilde{\theta}(\tau)^T\dot{\zeta}(\tau)\right)^2 d\tau \\ &\le \|\zeta(t)\|_\infty^2\,\|\dot{\tilde{\theta}}(t)\|_2^2 + t\left\|\tilde{\theta}(t)\right\|_\infty^2\left\|\dot{\zeta}(t)\right\|_\infty^2 \\ &\le C_1^2C_2^2 + tC_3^2C_4^2. \qquad (4.35)\end{aligned}$$

And from (3.26) we get

$$\|\hat{e}(t)\|_2 \le c_2|\tilde{\theta}(0)| + d_2 \quad (4.36)$$

where

$$c_2 = \left(\frac{\xi[0]}{2\gamma\left(\kappa - k - \frac{1}{2h}\right)}\right)^{\frac{1}{2}} \quad (4.37)$$

$$d_2 = \left(\frac{\frac{h}{2}\|\epsilon_1(t)\|_2^2 + \frac{1}{2}\hat{e}(0)^2}{\kappa - k - \frac{1}{2h}}\right)^{\frac{1}{2}}. \quad (4.38)$$

Hence from (4.34), (4.35) and (4.36) it follows that

$$\begin{aligned}&\int_0^t(\tilde{\theta}(\tau)^T\zeta(\tau))^2 d\tau \\ &\le \kappa(c_2|\tilde{\theta}(0)| + d_2)^2 + \frac{1}{2\kappa}|\tilde{\theta}(0)^T\zeta(0)|^2 \\ &\quad +\frac{1}{\kappa^2}(C_1^2C_2^2 + tC_3^2C_4^2) + \kappa\|\epsilon(t)\|_2. \qquad (4.39)\end{aligned}$$

Therefore defining C_5 and C_6 as

$$C_5 = \kappa(c_2|\tilde{\theta}(0)| + d_2)^2 + \frac{1}{2\kappa}|\tilde{\theta}(0)^T\zeta(0)|^2 + \frac{1}{\kappa^2}C_1^2C_2^2 + \kappa\|\epsilon(t)\|_2 \quad (4.40)$$

$$C_6 = \frac{1}{\kappa^2}C_3^2C_4^2 \quad (4.41)$$

we can show that (4.28) is satisfied. On the other hand from Lemma 4.3 defining $C_7 = C_1C_3$, we can show that (4.29) is satisfied. ∎

Using Lemmas obtained so far, we can evaluate the mean square tracking error criterion and the $\mathcal{L}_\infty$ tracking error bound for the proposed DyCE MRACS.

Theorem 4.1 *Given MRACS which consists of the control law (3.5), the high order estimator (3.10) and the fixed compensator (4.5), there exist positive constants C_5, C_6 and C_7 which is independent of τ such that*

$$\frac{1}{t}\int_0^t e(\tau)^2 d\tau < \tau\alpha\left(\frac{1}{t}C_5 + C_6\right) + \|\epsilon(t)\|_2 \quad (4.42)$$

$$\|e(t)\|_\infty < \tau\alpha(2m_1+1)C_7 + |\epsilon(t)| \quad (4.43)$$

Proof: It is clear from (4.1), (4.2), (4.3), Lemma 4.2 and Lemma 4.4. ∎

From Theorem 4.1 we can see that transient performance can be improved arbitrarily in terms of the mean square tracking error criterion and the $\mathcal{L}_\infty$ tracking error bound except influences of exponentially decaying term due to the initial condition of the system.

5 Numerical Example

In this section a numerical example of the proposed DyCE adaptive controller is shown in order to illustrate the effectiveness of the proposed design method of the fixed compensator. Single-input single-output plant with the transfer function

$$P(s) = \frac{1}{(s-1)^2} \quad (5.1)$$

is studied. The plant is assumed to be unknown except for the following *a priori* information.

1) The highest frequency gain g_p is known($g_p = 1$).
2) The maximal system order n is known($n = 2$).
3) The plant is minimum phase.

The control objective is to follow the reference model $P_M(s) = \frac{1}{(s+1)^2}$ and the boundedness of all the signals are assured. The reference signal is sinusoidal wave with period of 2π and amplitude of 1, that is, $r(t) = \sin t$. The characteristic polynomial of the regressor vector is $\lambda(s) = (s+2)$ and one of the second filter is $\xi(s) = s+1$. $\kappa = 1$ and $\gamma = 10$ and $\mu = 50$ is selected. $\mu = 50$ is satisfied with the condition (3.11). All the initial condition of the states and the initial value of the control parameter are assumed to be zero.

The simulation results of DyCE adaptive systems both without and with the fixed compensator are given in Figure 1 and Figure 2. In Figure 2 τ is selected as $\tau = 0.005$. The solid line and the dashed line represent the plant output and the reference model output respectively.

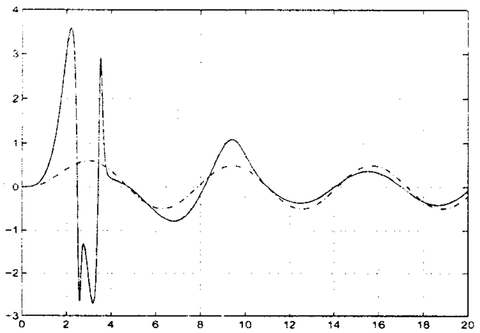

Figure 1: Plant output(solid line) and reference model output(dashed line) of DyCE adaptive control system without a fixed compensator

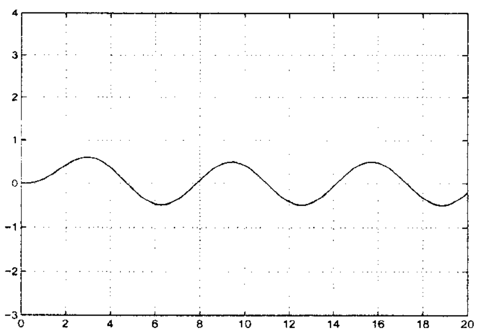

Figure 2: Plant output(solid line) and reference model output(dashed line) of DyCE adaptive control system with a fixed compensator ($\tau = 0.005$)

From Figure 1 and Figure 2, it follows that the transient performance can be improved according as the value of τ which is the design parameter of the fixed compensator tends to be small.

6 Concluding Remarks

In this paper we have proposed the modified Morse's DyCE adaptive controller in order to improve the transient performance. In the new scheme the additive feedback loop through a fixed compensator is included. The design method of the fixed compensator for the purpose of improving the transient performance is also given. Furthermore we show that the transient performance can be improved arbitrarily in terms of the mean square tracking error criterion and the $\mathcal{L}_\infty$ tracking error bound by the properly designed fixed compensator.

Essentially the transient performance improvement in the proposed method is achieved by high gain feedback. Hence the mechanism of the performance improvement is similar to ones in the modified MRACS based on the CE principle [5]-[7]. However due to the DyCE principle, the computable performance bounds of the tracking error can be obtained, which was not given based on the CE principle. As a future work, the effect of unmodeled dynamics on the proposed MRACS remains to be studied.

References

[1] A. S. Morse, "A Comparative Study of Normalized and Unnormalized Tuning Errors in Parameter-Adaptive Control", *Int. J. Adaptive Control and Signal Processing*, vol.6, pp.309–pp.318, 1992.

[2] R. Ortega, "On Morse's New Adaptive Controller: Parameter Convergence and Transient Performance', *IEEE Trans. Automat. Contr.*, vol.38, no.8, pp.1191–1202, 1993.

[3] A. Datta and P. A. Ioannou, "Directly Computable L_2 and L_∞ Performance Bounds for Morse's Dynamic Certainty Equivalence Adaptive Control", *Proc. 1995 American Contr. Conf.*, pp.589–591, 1995.

[4] H. Ohmori and Y. Shrivastava, "Robust Adaptive Control System with Fixed Compensator Attaining Robust Positive Realness", *Proc. 1995 American Contr. Conf.*, pp.592–596, 1995.

[5] J. Sun, "A Modified Model Reference Adaptive Control Scheme for Improved Transient Performance", *IEEE Trans. on Automat. Contr.*, vol. AC-38, no. 8, pp. 1255–1259, 1993.

[6] A. Datta and P. A. Ioannou, "Performance Analysis and Improvement in Model Reference Adaptive Control", *IEEE Trans. on Automat. Contr.*, vol. AC-39, no. 12, pp. 2370–2387, 1994.

[7] S. Masuda, A. Inoue and K. Sugimoto, "A Construction of Multivariable MRACS with Fixed Compensator Using Coprime Factorization Approach", *Proc. 33rd IEEE Conf. Dec. Contr.*, vol. 2, pp. 1688–1693, 1994.

[8] M. Vidyasagar, *Control System Synthesis. A Factorization Approach*, The MIT Press, 1985.

[9] S. Sastry, M. Bodson, *Adaptive Control - Stability, Convergence, and Robustness*, Prentice-Hall, 1989.

[10] S. P. Boyd, C. H. Barratt, *Linear Controller Design, Limits of Performance*, Prentice-Hall, 1991, pp. 118.

[11] M. Vidyasagar, *Nonlinear Systems Analysis*, Prentice-Hall, 1993, pp. 306.