Variable structure model reference adaptive control based on two-degree-of-freedom compensators scheme

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Variable Structure Model Reference Adaptive Control Based on Two-Degree-of-Freedom Compensators Scheme

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Abstract

In this paper, a variable structure model reference adaptive control system (MRACS) based on two-degree-of-freedom compensators scheme (2 d.o.f) is proposed. First, an MRACS with fixed compensator is constructed using the 2 d.o.f. proposed in [9]. Second, the fixed compensator is replaced by a switching compensator. The switching algorithm is determined in order that the estimated parameter can converge more rapidly. Since the proposed method is based on the 2 d.o.f structure, the switching compensator works only when the parameter uncertainties are large. The boundedness of all signals in the closed-loop system and the convergence of the output error are proved. Finally, simulation results are illustrated to show the effectiveness of the proposed method.

1. Introduction

An adaptive control scheme that uses switching function for controlling plant with unknown parameters is referred to “Variable Structure Model Reference Adaptive Control (VS-MRAC)”. Such a VS-MRAC was developed by adding the adaptive control to the variable structure control (VSC) [1] by several authors [2]-[4]. Especially a VS-MRAC developed by Hsu and Costa [2] is closely related with the conventional adaptive controllers and reveals the good transient behavior. However the VSC has the disadvantage of suffering from undesirable high-frequency signals due to the discontinuity in the control law in the switching compensator.

Therefore, in this paper, we propose a new VS-MRAC scheme which reveals a good transient performance and avoids the undesirable high-frequency signals. The switching compensator is especially effective when parameter uncertainties are large, that is, the deviation from true value is large. Therefore it is desirable to activate the switching compensator only when parameter uncertainties are large and suppress high-frequency oscillation caused by large parameter uncertainties. In order to realize the aim mentioned above, 2 d.o.f. compensators scheme proposed in [9] is used in this paper. The proposed 2 d.o.f. compensators scheme in [9] has a compensator freely designed independently of specification for the reference commands, and the compensator only works in the presence of parameter uncertainties. Therefore if we construct the switching compensator in VS-MRAC by using a freely designed compensator in the 2 d.o.f. compensators scheme, the switching compensator only works when the parameter uncertainties are large.

The design scheme of the proposed VS-MRAC is as follows. First, by using the model matching control law obtained from 2 d.o.f. compensators scheme, MRACS with a fixed compensator is constructed. This MRACS is constructed by rewriting the model matching law into linear form of the unknown control parameters and adding parameter adaptive law to the form. Second, the fixed compensator is replaced by a switching compensator. The switching law is determined so that the estimated parameter can converge more rapidly.

This paper is organized as follows. Section 2 gives problem statements and a priori information. In section 3 MRACS with a fixed compensator based on the 2 d.o.f. compensators scheme is given in order to design the VS-MRAC proposed in this paper. Section 4 shows that the new VS-MRAC is constructed using MRACS with a fixed compensator given in the previous section, and the stability of the system is proved. In section 5 simulation results of the proposed MRACS are illustrated to show the effectiveness of proposed VS-MRAC. Section 6 concludes the paper.

2. Problem Statements

Consider an unknown linear, time-invariant, finite dimensional, single-input single-output plant characterized by a transfer function \( P(s) \),

\[
y_p(t) = P(s)u(t), \quad P(s) = \frac{\alpha_p(s)}{\beta_p(s)}
\]  

(2.1)

where \( u(t), y_p(t) \) are the plant input and output, respectively. It is assumed that \( P(s) \) is strictly proper. Let the (right) coprime factorization of \( P(s) \) over \( RH_\infty \) be \((N_p(s), D_p(s))\), that is, \( N_p(s), D_p(s) \in RH_\infty \) are relatively right coprime over \( RH_\infty \), and

\[
P(s) = N_p(s)D_p(s)^{-1}
\]  

(2.2)

is satisfied. Throughout the paper, it is noted that \( s \) represents a differential operator. In constructing MRACS the factorization of the given plant over \( RH_\infty \) is not necessary to be calculated. But these notations are used to
parametrize reference model transfer functions and derive the controller structure of MRACS.

The transfer matrix of the reference model is denoted as \( P_M(s) \) which is strictly proper and asymptotically stable. And \( r(t) \), piecewise continuous and uniformly bounded, is the reference input and \( y_m(t) \) is the model output. \( P_M(s) \) is assumed to be satisfied with the necessary and sufficient condition for the model matching. This condition is given as follows.

\[
P_M(s) = N_p(s)K(s), \quad K(s) \in \mathbb{RH}_\infty.
\]  

(2.3)

The following assumptions are made for the plant [10].

(A.1) The order \( n \) (or an upper bound of \( n \)) of the transfer function \( P(s) \) is known.

(A.2) The relative degree \( n^* \) of \( P(s) \) is known and it is greater than or equal to that of \( P_M(s) \).

(A.3) The plant is minimum phase, that is, the zeros of \( P(s) \) lie in the open left-half plane.

(A.4) The high frequency gain \( g_p \) is known. In this paper, without loss of generality, it is assumed to be \( g_p = 1 \).

According to the assumptions (A.2) and (A.3), the rational function \( K \) satisfying the model matching condition (2.3) exists and belongs to \( \mathbb{RH}_\infty \).

Based on the description above, the control problem is stated as follows. Let the plant (2.1) be unknown except for the assumptions (A.1)-(A.4). Then determine a differentiator free controller which generates a bounded control input signal so that all the signals in the closed loop system remain bounded and the following equation is satisfied.

\[
\lim_{t \to \infty} |e(t)| = \lim_{t \to \infty} |y_p(t) - y_m(t)| = 0. 
\]  

(2.4)

3. MRACS with a Fixed Compensator Based on the 2 D.O.F Compensators Scheme

In this section we introduce MRACS with a fixed compensator based on the 2 d.o.f. compensators scheme in order to design the VS-MRAC proposed in this paper. The MRACS is derived from the model matching law based on the 2 d.o.f. compensators scheme proposed in [9]. The model matching control law is shown as follows: (see Figure 1)

\[
u = (1 - Y_p)u - X_p y_p + K_T \theta - Q_1 \{ y_p - N_p (P_{up} + X_p y_p) \}, \quad \forall Q_1 \in \mathbb{RH}_\infty \]  

(3.1)

where \( K \in \mathbb{RH}_\infty \) is chosen so that the model matching condition (2.3) is satisfied, and \( X_p \in \mathbb{RH}_\infty \) and \( Y_p \in \mathbb{RH}_\infty \) is the solution of the following Bezout identity over \( \mathbb{RH}_\infty \).

\[
X_p N_p + Y_p D_p = 1. 
\]  

(3.2)

Eq. (3.1) can be regarded as the model matching control law based on the 2 d.o.f. compensator scheme because \( Q_1 \) is a freely designed parameter and can be chosen independently of specifications for reference command.

![Figure 1: Parametrization of 2 d.o.f. compensators that stabilize a plant](image-url)

In constructing MRACS the given plant is unknown. Hence \( X_p \) and \( Y_p \) are unknown and need to be identified in order to achieve the control objective (2.4). The next lemmas are necessary to rewrite the control law to a linear form in terms of unknown parameters in \( X_p \) and \( Y_p \).

**Lemma 3.1** [6][7] Let \( \beta_{n[s]} \) be a monic Hurwitz polynomial of order \( n^* \), which is the relative degree of the plant \( P(s) \) and is known from the assumption (A.2). Then the right coprime factorization of the plant \( P(s) \) is given by

\[
N_p(s) = \beta_{n[s]}^{-1}, \quad D_p(s) = P(s)^{-1} \beta_{n[s]}^{-1}. 
\]  

(3.3)

**Lemma 3.2** [6][7] Let \( \lambda[s] \) be a monic Hurwitz polynomial of order \( n - 1 + p \) (\( p \geq 0 \)). Then there exist a unique polynomial \( z_x[s] \) of order \( n - 1 \) and a unique monic polynomial \( z_y[s] \) of order \( n - 1 + p \), such that

\[
X_p(s) = \frac{z_x[s]}{\lambda[s]}, \quad Y_p(s) = \frac{z_y[s]}{\lambda[s]} 
\]  

(3.4)

are solutions of eq. (3.2).

Using Lemma 3.1 and Lemma 3.2 the control law (3.1) can be rewritten into

\[
u(t) = -\theta^T \omega(t) + K_T r(t) - Q_1 \{ y_p(t) - \frac{1}{\beta_{n[s]}} (u(t) + \theta^T \omega(t)) \} 
\]  

(3.5)

where \( \omega(t) \) is a filter function and given by

\[
\omega(t) = [\omega_1(t)^T, \omega_2(t)^T]^T 
\]  

(3.6)

\[
\omega_1(t)^T = \frac{1}{\lambda[s]} [s^{n+p-2}, \ldots, 1]^T u(t) 
\]  

(3.7)

\[
\omega_2(t)^T = \frac{1}{\lambda[s]} [s^{n-1}, \ldots, 1]^T y_p(t) 
\]  

(3.8)

and parameter vector \( \theta \) is defined such that

\[
\theta^T = [\theta_1^T, \theta_2^T] 
\]  

(3.9)
are satisfied. Thus we have obtained a linear relation in terms of unknown parameters \( \theta \). Replacing the unknown parameter \( \theta \) by adjustable parameter \( \dot{\theta}(t) \) and giving an adaptive adjusting law, MRACS can be constructed.

In [9] we proposed MRACS with the fixed compensator \( Q_1 \in RH_\infty \) and proved that the stability of the system is ensured for arbitrarily chosen \( Q_1 \in RH_\infty \). On the other hand, in this paper we propose the replacement of the fixed compensator by a switching function in order to improve the parameter convergence. In the subsequent section we derive a new VS-MRAC using the control law (3.5) which is based on the 2 d.o.f. compensators scheme.

4. A New VS-MRAC Based on the 2 D.O.F Compensators Scheme

In this section we construct a new VS-MRAC based on the 2 d.o.f compensators scheme. In the proposed VS-MRAC the switching function \( F(\rho) \), which changes the value according to the sign of \( \rho \), is used instead of \( Q_1 \) in eq.(3.5), and unknown parameter \( \theta \) is identified simultaneously using an adaptive adjusting law. Hence the control law is given as

\[
u(t) = -\dot{\theta}(t)^T \omega(t) + \dot{\theta}(t)^T q(t) - F(\rho, X(t))
\]

where \( X(t) \) is defined as

\[
X(t) = y_p(t) - \frac{1}{\beta_m[s]}(u(t) - \dot{\theta}(t)^T \omega(t))
\]

First we will give an adaptive adjusting law and later switching functions \( F(\rho, X(t)) \) and \( \rho \) are given. In order to give the adaptive adjusting law, we derive an error equation. Let the parameter deviation \( \phi(t) \) from the true value of parameter vector \( \theta \) be defined as

\[
\phi(t) = \dot{\theta}(t) - \theta.
\]

Then the error equation can be calculated in the following way.

\[
e(t) = y_p(t) - y_m(t)
\]

\[
= \frac{1}{\beta_m[s]}(u(t) + \dot{\theta}(t)^T \omega(t)) - F(\rho, X(t)).
\]

Here an auxiliary error signal \( e_a(t) \) is generated as

\[
e_a(t) = -\dot{\theta}(t)^T \frac{1}{\beta_m[s]} \omega(t) + \frac{1}{\beta_m[s]} \dot{\theta}(t)^T \omega(t)
\]

\[
- F \left( \rho, \frac{1}{\beta_m[s]} X(t) \right) + \frac{1}{\beta_m[s]} F(\rho, X(t)).
\]

Let \( \epsilon(t) \) and \( \zeta(t) \) be defined as

\[
\epsilon(t) = e(t) + e_a(t)
\]

and

\[
\zeta(t) = \frac{1}{\beta_m[s]} \omega(t).
\]

The following adaptive adjusting law is used in this paper.

\[
\frac{d}{dt} \dot{\theta}(t) = \frac{\epsilon(t) \zeta(t)}{\kappa + \zeta(t)^T \zeta(t)}
\]

where \( \kappa \) is a positive constant and \( \Gamma \) is a positive definite matrix. And the switching function \( F(\rho) \) is given as follows:

\[
F(\rho, X(t)) = \begin{cases} 
0, & |X(t)| > M, \text{ or } |ho| \leq \delta \\
-fX(t), & |X(t)| < M, \text{ and } \rho > \delta \\
fX(t), & |X(t)| < M, \text{ and } \rho < -\delta 
\end{cases}
\]

where \( f, M \) and \( \delta \) are positive constants and \( \rho \) is defined to be

\[
\rho = \epsilon(t) \frac{1}{\beta_m[s]} X(t).
\]

Thus VS-MRAC proposed in this paper is established (See Figure 2).

![Figure 2: The block diagram of the proposed VS-MRAC](image)

The next theorem shows that the proposed VS-MRAC assures the global stability of the system.

**Theorem 4.1** All the internal signals in the system which consists of the plant (2.1), the reference model (2.3), the control law (4.11), the estimated error (4.16), the adaptive adjusting law (4.18) and the switching function (4.19) are uniformly bounded, and the control objective (2.4) is achieved.
Since the proof of Theorem 4.1 can be done in a similar way as [10], it will be omitted.

When a Lyapunov function candidate \( V(t) \) is chosen as

\[ V(t) = \frac{1}{2} \phi(t)^T \Gamma^{-1} \phi(t), \quad (4.21) \]

the time derivative of \( V(t) \) along the trajectories of eq. (4.18) is given by

\[ \dot{V}(t) = \frac{1}{\kappa + \zeta(t)^2 \zeta(t)} \left\{ -e(t)^2 - e(t) F \left( \frac{1}{\beta_m[s]} X(t) \right) \right\}. \quad (4.22) \]

From the definition of the switching function (4.19) and (4.20) it follows that

\[ e(t) F \left( \frac{1}{\beta_m[s]} X(t) \right) \geq 0. \quad (4.23) \]

Hence the estimated parameter in the proposed method converges to a constant value more rapidly than one in the standard MRACS, because the absolute value of the \( \dot{V} \) becomes large by the switching function \( F(\rho) \) as is shown in eq. (4.22).

Further the switching function works only when the estimated error is large and it doesn’t work when the estimated error is small enough. Therefore we can see that the undesirable high-frequency signals and excessive control activity due to the switching function can be avoided. This advantage results from the controller structure based on 2 d.o.f compensators scheme.

**5. Simulation Results**

In this section simulation results of the proposed VS-MRAC are illustrated to show the effectiveness of the proposed method. The plant and model are given by

\[ P(s) = \frac{s + 3.0}{s^2 - 2.0s + 3.0}, \quad P_M(s) = \frac{s + 7.0}{s^2 + 7.0s + 12.0}. \quad (5.1) \]

Since the plant is the second-order plant, the characteristic polynomial of the first filter \( \lambda[s] \) and the one of the second filter \( \beta_m[s] \) are monic Hurwitz polynomials of order 1. In the simulation they are given by

\[ \lambda[s] = s + 4.0, \quad \beta_m[s] = s + 6.0. \quad (5.2) \]

The reference signal is given by the square wave with the period of 2s. In the adaptive adjusting law (4.18) \( \sigma, \kappa \) and \( \Gamma \) are given by

\[ \kappa = 1.0^{-5}, \quad \Gamma = \text{diag}[100, 100, 100]. \quad (5.3) \]

Here the adaptive gain and the poles of the filters are chosen to be relatively small so that the adaptive system may not be sensitive to noises or disturbances. Therefore the performances of the standard MRACS are not so good.

The design parameters of the switching function \( F(\rho) \) is given as follows.

\[ f = 5.0 \quad \delta = 1.0e^{-10}, \quad M = 1.0e^2. \quad (5.4) \]

Figure 3 shows that the influence of parameter uncertainties on the plant output is reduced by the switching compensator in transient response comparing the results of Figure 6 in the standard MRACS. And Figure 5 and Figure 8 shows that the estimated parameter converges to a constant value more rapidly than one in the standard MRACS. Furthermore Figure 4 and Figure 7 shows that the proposed VS-MRAC doesn’t bring about undesirable high-frequency oscillation in the plant input as well as one in the standard MRACS.

**6. Conclusion**

In this paper we propose a new variable structure model reference adaptive control for a single-input single-output linear time invariant system. The boundedness of all signals of the system is proved. The structure is based on a
Figure 5: The control parameters in the proposed VS-MRAC

Figure 6: The plant and model output in the standard MRACS

Figure 7: The plant input in the standard MRACS

Figure 8: The control parameters in the standard MRACS

parametrization of stabilizing compensator obtained from 2 d.o.f. compensator scheme, and the switching compensator corresponds to a free parameter specifying feedback properties which can be designed independently of tracking properties. Therefore the estimated parameter converges to a constant value more rapidly than one in the standard MRACS and the undesirable high-frequency signals and excessive control activity due to the switching function can be avoided. The development of a more general switching function remains as a future research.

References