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Abstract

A design method of multivariable model reference adaptive control system (MRACS) using coprime factorization approach is proposed. First, a relation between the coprime factorization over $\boldsymbol{RH}_\infty$ and the interactor matrix is derived. Using this relation, a method for design of multivariable MRACS over the ring of proper stable rational functions is obtained. Since the proposed method is based on the coprime factorization approach, the control structure in this paper is more simple. It is shown that the control structure of multivariable MRACS by other authors is included in the structure proposed in this paper. A robust MRAC scheme which achieves asymptotically rejection of unmeasurable disturbance is derived also.

1. Introduction

The multivariable model reference adaptive control systems have been extensively studied by many theorists and practitioners, and has been established as one of the mostly used design methods in the field of control. The solution of the MRACS problem consists of two parts: the algebraic part deals with the controller structure with which given plant matches the reference model exactly. And, the other is the analytic part which deals with the manner the control parameters are to be adjusted, i.e. with the adaptive adjusting law. In the algebraic part the model matching design problem is simply and straightforwardly solved by the coprime factorization in the set of proper stable rational functions. In the multivariable MRAC case the issue of the parametrization of the controller becomes a dominant problem which is solved by using the interactor matrix. In order to apply the general solution, it is necessary to bridge between coprime factorization and the interactor matrix. The contributions of this paper are to give such a relation and to apply the model matching scheme based on the coprime factorization to a MRACS design.

Since Wolovich and Falb [1] developed the concept of interactor it has had an important role in multivariable adaptive control. Since then a considerable amount of important contributions to the control literature has done. Elliott and Wolovich [9] proposed a design method of multivariable MRACS based on the exact model matching (EMM) approach, where the prior knowledge of the interactor was explicit. In their method factorization of the plant over polynomial ring was used for the construction of EMM. Later, they extended the method [10] to the case that only the order of the interactor was *a priori* known. In [3] the previous results on multivariable discrete-time adaptive control were generalized for a linear systems with a stable inverse. However, relations to coprime factorization have not been stated explicitly. In [15] is given an algorithm for a discrete-time robust regulation system structure described in the ring of proper stable rational functions, but the MRACS design problem was not discussed there. The problem of designing robust multivariable control systems was further examined in [6], but they considered only a plant factored over polynomial ring.

The factorization approach allows to parametrize all stabilizing compensators for given plant and, moreover, the solution of model matching problem can be made more straightforward. Therefore, by using the factorization approach over a ring of proper stable rational functions in the algebraic part of MRACS design, the synthesis and analysis can be expected to be more simple and clear.

In this paper, first, the *a priori* information about the plant is specified. Then an EMM control structure based on factorization over proper stable rational functions is proposed. The control parameters needed to be identified in order to achieve the EMM control law are established. Further relations between the interactor matrix and the coprime factorization of the plant are derived and solution of the Bezout identity in the ring of rational functions is derived. Next, the EMM control law, described in terms of the adjusting parameters and the known interactor matrix, is given. Finally, this paper shows that a design method of robust multivariable MRACS in presence of deterministic disturbances can be easily derived. Based on the factorization approach, the structure is more clear and simple than Elliot and Wolovich's one [9].

2. Preliminary Assumptions

The following notations from [12] are used. $\boldsymbol{R}^n$ denotes an n dimensional Euclidean space. $\boldsymbol{RH}_\infty$ represents the ring of proper stable rational functions in indetermined variable s with coefficients in the field of the real numbers ($\boldsymbol{R}$). $\boldsymbol{R}(s)$ denotes the field of rational functions in s with real coefficients. The ring of polynomials with real coefficients is given by $\boldsymbol{R}[s]$. For the size of given matrix, the notations $\boldsymbol{RH}_\infty{}^{p\times l}$, $\boldsymbol{R}(s)^{p\times l}$, and $\boldsymbol{R}[s]^{p\times l}$ mean that the elements of the $(p \times l)$ matrix are from $\boldsymbol{RH}_\infty$, $\boldsymbol{R}(s)$, or $\boldsymbol{R}[s]$ respectively. Further, $\partial_{ci}[\cdot]$ and $\partial_{ri}[\cdot]$ express the column and raw degrees of given polynomial matrix, and $[\cdot]_C$ ($[\cdot]_R$) is for the highest column (raw) degree coefficient matrix notation.

Consider an unknown linear, time-invariant, finite dimensional, plant with m-inputs and m-outputs characterized by a transfer matrix $T(s)$. It is assumed that $T(s)$ is full rank and strictly proper, that is, the relative degree of each transfer function component is positive. $T(s)$ can always be factored as:

$$y(t) = T(s)u(t), \quad T(s) = R[s]P[s]^{-1} = \tilde{P}[s]^{-1}\tilde{R}[s], \tag{1}$$

where $R[s]$, $P[s] \in \boldsymbol{R}[s]^{m\times m}$ are relatively right prime, and $P[s]$ is column proper. Also $\tilde{R}[s]$, $\tilde{P}[s] \in \boldsymbol{R}[s]^{m\times m}$ are relatively left prime and $\tilde{P}[s]$ is raw proper matrix [8]. $u(t)$, $y(t) \in \boldsymbol{R}^m$ are the plant input and output vectors respectively.

The transfer matrix of the reference model, denoted as $T_M(s)$, is strictly proper and asymptotically stable

$$y_M(t) = T_M(s)v(t), \quad T_M(s) = R_M[s]P_M[s]^{-1}. \tag{2}$$

where $R_M[s]$, $P_M[s] \in \boldsymbol{R}[s]^{m\times m}$ are relatively right prime. $v(t)$, $y_M(t) \in \boldsymbol{R}^m$, piecewise continuous and uniformly bounded, are the model input and output vectors respectively.

The next lemma gives an extension to the interactor matrix definition.

Lemma 1 *[11] One expression of the interactor matrix of a nonsingular plant (1) is the nonsingular $m \times m$, lower left triangular polynomial matrix $L[s]$ of the form*

$$L[s] = \Sigma[s]\mathrm{diag}[r_i(s)], \quad i = 1, \ldots, m \tag{3}$$

where $r_i(s)$ is a polynomial and

$$\Sigma[s] = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ \sigma_{21}(s) & 1 & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ \sigma_{m1}(s) & \cdots & \cdots & 1 \end{bmatrix}$$

where $\sigma_{ij}(s)$ is a polynomial (or is zero), such that

$$\lim_{s\to\infty} L[s]T(s) = G \tag{4}$$

with G nonsingular.

The following three assumptions are made for the plant [9]:

(A1) $\det(R[s])$ is asymptotically stable polynomial;

(A2) The system maximum observability index is known;

(A3) The interactor matrix $L[s]$ of the form (3) is known.

The control problem is to determine a differentiator free controller which generates a bounded control input signal vector, so that all the signals in the closed loop system remain bounded and the following equation is satisfied:

$$\lim_{t\to\infty} e(t) = \lim_{t\to\infty}(y(t) - y_M(t)) = 0. \tag{5}$$

3. Multivariable MRACS based on the coprime factorization over RH_∞

Coprime factorization representation of EMM system

Consider a coprime factorization of the plant (1) over a ring of proper stable rational functions $\boldsymbol{RH}_\infty$.

$$T(s) = N(s)D(s)^{-1}. \tag{6}$$

The following theorem gives an expression for plant factorization over $\boldsymbol{RH}_\infty$ in terms of the known (assumption **(A3)**) interactor matrix.

Theorem 1 *Let the interactor matrix $L[s]$ be such that $\det[L[s]]$ is an asymptotically stable polynomial. Then the coprime factorization of the plant transfer matrix (6) expressed in terms of the interactor matrix $L[s]$ is given by:*

$$N(s) = L[s]^{-1}G, \quad D(s) = T(s)^{-1}L[s]^{-1}G \tag{7}$$

Proof. From Lemma 1, taking the polynomials $r_i(s)$ stable an interactor matrix such that $\det[L[s]]$ is stable can be chosen. From expression (3) it directly follows that $N(s) \in \boldsymbol{RH}_\infty$ and $D(\infty) = I$. Also, from assumption (A1) and, again, from the above lemma it is clear that $D(s)$ does not have poles in $\boldsymbol{C}^+$. Also, from the definition of $N(s)$, it does not have unstable zeroes. Hence, it can be shown that $N(s)$, $D(s)$ $\in \boldsymbol{RH}_\infty$ are coprime. From eq. (7) it is clear that $T(s) = N(s)D(s)^{-1}$.

∎

Now using the above theorem a solution of the EMM problem will be given. The EMM problem is to find a stabilizing compensator for a given plant (1) such that the closed loop transfer function matrix $T_{yv}(s)$ of the system is equal to given model transfer function matrix $T_M(s)$. The next proposition gives a generalized EMM control law based on two parameter compensation scheme in $\boldsymbol{RH}_\infty$.

Proposition 1 *[4] Consider a plant transfer matrix $T(s)$ factored as in (6) and let all notations are as in equations (4) and (7). If the transfer matrix of the reference model $T_M(s)$ is given as*

$$T_M(s) = N(s)K(s) \text{ for some } K(s) \in \boldsymbol{RH}_\infty \quad (8)$$

then the control law which achieves the EMM is:

$$u(t) = Y^{-1}(s)K(s)v(t) - Y^{-1}(s)X(s)y(t), \quad (9)$$

where $K(s)$ is a solution of (8) and $X(s)$, $Y(s)$, $\in$ $\boldsymbol{RH}_\infty{}^{m\times m}$ satisfy the Bezout identity:

$$X(s)N(s) + Y(s)D(s) = I. \quad (10)$$

Remark 1 *Theorem 1 also allows to express the condition for EMM eq. (8) in terms of the interactor matrix $L[s]$:*

$$T_M(s) = L[s]^{-1}GK(s) \quad K(s) \in \boldsymbol{RH}_\infty. \quad (11)$$

The description of the reference model here is same as in, e.g. [9].

Remark 2 *Proposition 1 gives a generalized expression for the EMM control law for a multivariable plant. This follows directly from the fact that eq.(10) is the condition for stabilizing compensator, and ,also, from the reference model transfer matrix equation (8).*

Rewriting eq. (9) in observer-controller form yields

$$u(t) = (I - Y(s))u(t) - X(s)y(t) + K(s)v(t) \quad (12)$$

a general description of the control law in $\boldsymbol{RH}_\infty$.

The structure of the system is shown in Fig. 1.

Identification model

Because the only *a priori* information about the plant is that, given by assumptions (A1)-(A3), there is a need to identify the unknown parameters, included in the control law (12). In this section the identification model is derived.

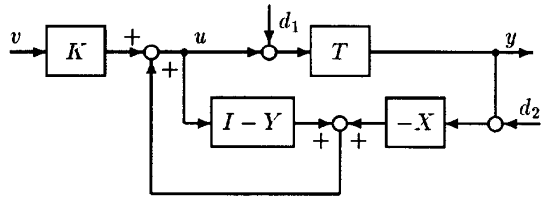

Figure 1: Exact Model Matching System

Multiplying eq. (10) from the left-hand side by $N(s)$ and from the right-hand side by $D(s)^{-1}u(t)$ gives:

$$y(t) = N(s)[Y(s)u(t) + X(s)y(t)] \quad (13)$$

The following proposition gives the conditions for the orders of $X(s)$ and $Y(s)$.

Proposition 2 *Let the plant maximal observability index is denoted by ν, and let $\Xi[s] = \xi(s)I$ for some monic stable polynomial $\xi(s)$ of order $\nu-1+p$ ($p \geq 0$). Then there exists a solution $X(s)$, $Y(s) \in \boldsymbol{RH}_\infty$ of the Bezout identity (10) of the form:*

$$X(s) = \Xi[s]^{-1}Z_x[s], \quad Y(s) = \Xi[s]^{-1}Z_y[s] \quad (14)$$

Proof. Since $P[s]$ and $R[s]$ are right coprime there exist polynomial matrices $U[s]$, $V[s] \in \boldsymbol{R}[s]^{m\times m}$ such that

$$U[s]R[s] + V[s]P[s] = I \quad (15)$$

is satisfied. Denote $\Xi_1[s] = G^{-1}L[s]R[s]$. Multiplying (15) from the left side by $\Xi[s]$ $\Xi_1[s]$ gives:

$$\Xi[s]\Xi_1[s]U[s]R[s] + \Xi[s]\Xi_1[s]V[s]P[s] = \Xi[s]\Xi_1[s] \quad (16)$$

Performing polynomial matrix division [13] of $\Xi[s]\Xi_1[s]U[s]R[s]$ by $\tilde{P}[s]$

$$\begin{aligned} \Xi[s]\Xi_1[s]U[s] &= Q[s]\tilde{P}[s] + E[s] \\ \partial_{ci}[E[s]] &< \partial_{ci}[\tilde{P}[s]], \end{aligned} \quad (17)$$

where $Q[s]$, $E[s] \in \boldsymbol{R}[s]^{m\times m}$. Since the observability indexes are coincident with the raw degrees of $\tilde{P}[s]$, for ν we have $\nu \geq \partial_{ri}[\tilde{P}[s]]$, $i = 1, \cdots, m$. Also, because $\nu \geq \partial_{ci}[\tilde{P}[s]]$, $i = 1, \cdots, m$, then $\nu - 1 \geq \partial_{ci}[E[s]]$, $i = 1, \cdots, m$. After substitution of eq. (18) in (16) and rearrangement the following yields

$$E[s]R[s] + (\Xi[s]\Xi_1[s]V[s] + Q[s]\tilde{R}[s])P[s] = \Xi[s]\Xi_1[s]. \quad (18)$$

Multiplying eq. (18) from left by $\Xi[s]^{-1}$ and from the right side by $\Xi_1[s]^{-1}$ gives

$$\Xi[s]^{-1}Z_x[s]N(s) + \Xi[s]^{-1}Z_y[s]D(s) = I, \quad (19)$$

where $Z_x[s] = E[s]$, $Z_y[s] = \Xi[s]\Xi_1[s]V[s] + Q[s]\tilde{R}[s]$. From the definition of $\Xi[s]$ and the fact that the order of $Z_x[s]$ is less than $\nu - 1$, it directly follows that $X(s) \in \boldsymbol{RH}_\infty$. Also, for eq. (19) when $s \to \infty$, $Y(s) = I$, and noting the definition of $\Xi[s]$, it can be concluded that $Y(s) \in \boldsymbol{RH}_\infty$.

∎

From Theorem 1 and Proposition 2 the identification model (13) now can be rewritten as:

$$\begin{aligned} y(t) &= L[s]^{-1}G[\Xi[s]^{-1}Z_y[s]u(t) + \Xi[s]^{-1}Z_x[s]y(t)] \\ &= L[s]^{-1}G\left[Z_y[s]\frac{1}{\xi(s)}u(t) + Z_x[s]\frac{1}{\xi(s)}y(t)\right] \\ &= L[s]^{-1}G\left[(Z_y[s] - \xi(s)I)\frac{1}{\xi(s)}u(t) + u(t)\right. \\ &\left. + Z_x[s]\frac{1}{\xi(s)}y(t)\right]. \end{aligned} \quad (20)$$

From the proof of Prop. 2

$$\begin{aligned} Z_y[s] - \xi(s)I &= H_{\nu-2+p}s^{\nu-2+p} + \cdots + H_0 \\ Z_x[s] &= J_{\nu-1+p}s^{\nu-1+p} + \cdots + J_0. \end{aligned} \quad (21)$$

Form a state variable filter

$$\omega(t)^T = \left[\frac{s^{\nu-2+p}}{\xi(s)}u(t)^T, \cdots, \frac{1}{\xi(s)}u(t)^T, \frac{s^{\nu-1+p}}{\xi(s)}y(t)^T, \cdots, \frac{1}{\xi(s)}y(t)^T\right] \quad (22)$$

and define the unknown parameters vector as

$$\Theta = [H_{\nu-2+p}, \cdots, H_0, J_{\nu-1+p}, \cdots, J_0]. \quad (23)$$

Hence the output model (20) becomes:

$$y(t) = L[s]^{-1}G[\Theta\omega(t) + u(t)]. \quad (24)$$

Further, premultiplying (24) by $L[s]$ gives

$$L[s]y(t) = \bar{\Theta}\bar{\omega}(t), \quad (25)$$

where

$$\begin{aligned} \bar{\Theta} &= [G\Theta,\ G] \\ \bar{\omega}(t)^T &= [\omega(t)^T,\ u(t)^T]. \end{aligned}$$

To avoid the differentiation a standard filter arguments can be used:

$$f^{-1}(s)L[s]y(t) = \bar{\Theta}\bar{S}(t), \quad (26)$$

where

$$\bar{S}(t) = f^{-1}(s)\bar{\omega}(t).$$

$f(s)$ is chosen to be a stable monic polynomial of order same as the order of $L[s]$'s element of maximum order. Equation (26) is a linear relation between the filtered input and output of the plant and the matrix of unknown parameters $\bar{\Theta}$.

In order to estimate Θ and G a suitable method can be chosen [9], [12].

For the control law (12), using equations (22) and (23) we obtain

$$u(t) = -\Theta\omega(t) + G^{-1}L[s]T_M(s)v(t). \quad (27)$$

By using the identified parameters Θ and G the control law (9) becomes realizable.

Thus the structure of MRACS is established.

4. Relations to other methods

The relation to the structure given by Elliott and Wolovich [9] is briefly shown here. The control law in [9] is

$$\begin{aligned} u(t) &= K[s]q_1^{-1}(s)u(t) + H[s]q_1^{-1}(s)y(t) \\ &+ G^{-1}L[s]y_M(t), \end{aligned} \quad (28)$$

where $q_1(s)$ is a stable monic polynomial of order $\nu-1$, and $K[s]$ and $H[s]$ satisfy the Diophantine equation:

$$K[s]P[s] + H[s]R[s] = q_1(s)[P[s] - G^{-1}L[s]R[s]]. \quad (29)$$

This control law is derived from the control law in the previous section through the following substitutions:

$$\begin{aligned} \Xi[s] &= -q_1(s)I \qquad (\xi[s] = -q_1(s)) \quad &(30) \\ Z_x[s] &= H[s] \quad &(31) \\ Z_y[s] &= K[s] - q_1(s)I \quad &(32) \end{aligned}$$

The above shows that the control structure proposed in this paper is more general than the structure given by Elliott and Wolovich [9].

5. Robust MRACS in presence of Deterministic Disturbances

There exist several schemes for design of a MRACS when the disturbance model is known [5][6][7]. In this section a MRACS including the model of external disturbances is constructed by using the coprime factorization over $\boldsymbol{RH}_\infty$.

Let the disturbance $d_1(t)$ applied to the plant input is described as:

$$d_1(t) = G_D(s)d_0, \quad (33)$$

where d_0 is a scalar constant vector, and $G_D(s)$ is an unstable disturbance signal generator having left coprime factorization as

$$G_D(s) = \tilde{D}_D(s)^{-1}\tilde{N}_D(s). \quad (34)$$

The solution to the problem of disturbance rejection is given next.

Theorem 2 *[4] Let $\phi(s)$ denote the largest invariant factor of $\tilde{D}_D(s)$. Then there exists a compensator C(s) which achieves disturbance rejection asymptotically if $N(s)$ and $\phi_D(s)I$ are left-coprime. Moreover, the set of all C(s) which achieve disturbance rejection is given by*

$$\{C_1(s)/\phi_D(s) : C_1(s) \in S(T(s)/\phi_D(s))\} \quad (35)$$

where $S(T(s))$ denotes the set of all compensators C $\in \boldsymbol{R}(s)^{m\times m}$ that stabilize T(s).

Because the plant $T(s)$ is a minimum-phase (assumption **(A1)**) it satisfies the condition of Theorem 2.

Now, construct the following augmented system:

$$T_E(s) = T(s)/\phi_D(s) \quad (36)$$

Theorem 2 shows that a compensator which stabilizes the extended system $T_E(s)$ can be found. First, consider a factorization of $T_E(s)$ and a Bezout identity as follows:

$$\begin{array}{l} T_E(s) = N_E(s)D_E(s)^{-1} = \tilde{D}_E(s)^{-1}\tilde{N}_E(s) \\ X_E(s)N_E(s) + Y_E(s)D_E(s) = I. \end{array} \quad (37)$$

In [2] the relation between the factorization of the original plant $T(s)$ and the factorization of the augmented system $T_E(s)$ is given as

$$\begin{array}{ll} N_E(s) = N(s), & D_E(s) = \phi_D(s)D(s) \\ \tilde{N}_E(s) = \tilde{N}(s) & \tilde{D}_E(s) = \phi_D(s)\tilde{D}(s). \end{array} \quad (38)$$

Then the EMM control law which achieves disturbance rejection, using eq. (12) can be written as:

$$\begin{aligned} \tilde{u}(t) &= (I - Y_E(s))u(t) - X_E(s)y(t) \quad (39) \\ &\quad + K(s)v(t) \\ u(t) &= \frac{1}{\phi_D(s)}\tilde{u}(t), \end{aligned}$$

where $K(s) \in \boldsymbol{RH}_\infty$ is the solution of eq. (8).

The next proposition considers the expressions for $X_E(s)$ and $Y_E(s)$.

Proposition 3 *Let $(n_D[s], d_D[s])$ be right coprime factors of $\phi_D(s)$ over algebraic polynomial ring, where $d_D[s]$ and $n_D[s]$ are monic Hurwitz polynomials of order q, and $d_D[s]$ is stable. And let $\Xi_E[s] = \xi_E(s)I$, where $\xi_E(s)$ is a monic stable polynomial of order $\nu - 1 + p + q (p \geq 0)$. Then there exists a solution $X_E(s)$, $Y_E(s) \in \boldsymbol{RH}_\infty$ of Eq. (37) given as*

$$X_E(s) = \Xi_E[s]^{-1}Z_{x_E}[s], \quad Y_E(s) = \Xi_E[s]^{-1}Z_{y_E}[s]. \quad (40)$$

Proof. The proof is contained in the appendix.

From this proposition It is seen that for the case of presence of deterministic disturbance, the MRACS is constructed in a similar fashion as it was done in the previous section.

6. Conclusion

In this paper a relation between the coprime factorization over $\boldsymbol{RH}_\infty$ and the interactor matrix was derived. Using this relation, a method for design of multivariable MRACS over the ring of proper stable rational functions was derived. The prior information needed for the realization of the control law, given in the first section, is almost the same as in [9] where the proposed MRACS algorithm is based on presentation over polynomial rings. It is shown that the control structure of MRACS in [9] is included in the structure proposed in this paper. The condition for the *a priori* knowledge of the interactor matrix is a key for the developing the proposed scheme. A robust MRAC scheme which achieves asymptotically rejection of unmeasurable disturbance was derived also. The proposed scheme is an extention to the results given in [5] and it is hoped that it will lead to utilizing the recent results of the area of linear multivariable control systems using factorization approach.

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Appendix A

Proof of Theorem 3. Let the factorization of the augmented system over the ring of polynomial matrices is

$$\begin{aligned} T_E(s) &= T(s)/\alpha_D(s) \quad (41)\\ &= (R[s]d_D[s])(P[s]n_D[s])^{-1}\\ &= (\tilde{P}[s]n_D[s])^{-1}(\tilde{R}[s]d_D[s]). \end{aligned}$$

Note that the factorization may not be coprime. Denote $R_E[s] = R[s]d_D[s]$, $P_E[s] = P[s]n_D[s]$, $\tilde{R}_E[s] = \tilde{R}[s]d_D[s]$, $\tilde{P}_E[s] = \tilde{P}[s]n_D[s]$. Now the augmented system can be rewritten as

$$T_E(s) = R_E[s]P_E^{-1}[s] = \tilde{P}_E^{-1}[s]\tilde{R}_E[s], \quad (42)$$

and the column and raw degrees of P_E are given by next relations

$$\begin{aligned} \nu + q &\geq \partial_{ri}[\tilde{P}_E[s]], \quad i = 1,\dots,m \quad (43)\\ \nu + q &\geq \partial_{ci}[\tilde{P}_E[s]], \quad i = 1,\dots,m. \quad (44) \end{aligned}$$

Since $P_E[s]$ and $R[s]$ are right prime (because $\det R[s]$ is asymptotically stable polynomial) then there exist polynomial matrices $U_E[s]$, $V_E[s] \in \boldsymbol{R}[s]^{m\times m}$ satisfying the Bezout identity

$$U_E[s]R[s] + V_E[s]P_E[s] = I. \quad (45)$$

Let $\Xi_1[s]$ be the same as in the proof of Proposition 2, that is, $\Xi_1[s] = G^{-1}L[s]R[s]$. After multiplying the last equation from the left-hand side by $d_D[s]\Xi_E[s]\Xi_1[s]$ and rearrangement the following yields

$$\begin{aligned} &\Xi_E[s]\Xi_1[s]U_E[s]R_E[s] + \quad (46)\\ &\quad d_D[s]\Xi_E[s]\Xi_1[s]V_E[s]P_E[s] = d_D[s]\Xi_E[s]\Xi_1[s]. \end{aligned}$$

Now a polynomial matrix division gives

$$\Xi_E[s]\Xi_1[s]U_E[s] = Q_E[s]\tilde{P}_E[s] + E_E[s] \quad (47)$$

$$\partial_{ci}[E_E[s]] < \partial_{ci}[\tilde{P}_E[s]], \quad i = 1,\dots,m \quad (48)$$

$$\text{and } \nu - 1 + q \geq \partial_{ci}[E_E[s]] \quad (49)$$

for some $Q_E[s]$, $E_E[s] \in \boldsymbol{R}[s]^{m\times m}$. Substituting (47) in (46) and after some rearrangements it can be written

$$\begin{aligned} &E_E[s]R_E[s] + Q_E[s]\tilde{P}_E[s]R_E[s]\\ &\quad + d_D[s]\Xi_E[s]\Xi_1[s]V_E[s]P_E[s]\\ &\quad = d_D[s]\Xi_E[s]\Xi_1[s]. \quad (50) \end{aligned}$$

From eq. (42) $\tilde{P}_E[s]R_E[s] = \tilde{R}_E[s]P_E[s]$. Using this, from (50) the following equation can be derived

$$\begin{aligned} &E_E[s]R_E[s] \quad (51)\\ &\quad + [Q_E[s]\tilde{R}_E[s]n_D[s] + d_D[s]\Xi_E[s]\Xi_1[s]V_E[s]]P_E[s]\\ &= \Xi_E[s]\Xi_1[s]d_D[s]. \end{aligned}$$

Now, substitute

$$\begin{aligned} Z_{x_E}[s] &= E_E[s],\\ Z_{y_E}[s] &= Q_E[s]\tilde{R}_E[s]n_D[s] + d_D[s]\Xi_E[s]\Xi_1[s]V_E[s], \end{aligned}$$

and multiply eq. (51) from left-hand side by $\Xi_E[s]^{-1}$, and from right-hand side by $(\Xi_1[s]d_D[s])^{-1}$. This gives as a result the following

$$\Xi_E^{-1}[s]Z_{x_E}[s]N_E[s] + \Xi_E^{-1}[s]Z_{y_E}[s]D_E[s] = I, \quad (52)$$

where the relation $N_E(s) = N(s) = L[s]^{-1}G$ is used. Now applying eq. (40) yields (37).

What remains is to show that $X_E[s]$, $Y_E[s] \in \boldsymbol{RH}_\infty$. $X_E[s] \in \boldsymbol{RH}_\infty$ follows directly from the definition of $\Xi_E[s]$ and the order of $Z_{x_E}[s]$. From eq. (52), when $s \to \infty$, $Y_E(\infty)$ becomes equal to I, and from the definition of $\Xi_E[s]$ it can be concluded that $Y_E(s) \in \boldsymbol{RH}_\infty$. ∎