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Performance and Sensitivity in Visual Servoing

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Abstract

This paper describes the relationship between the control performance and the number/configuration of the image features in feature-based visual servoing. The performance is evaluated by two ways: accuracy and speed. A quantitative definition of the sensitivity is given and the relationship among the sensitivity, the speed of convergence and the accuracy are discussed by using image Jacobian. It is proved that these performance indices are increased effectively by using a point with different height. Experiments on Puma 560 are given to show the validity of these performance measures.

1 Introduction

The performance of the feature-based visual servoing depends on the selection of image features. From an image recognition point of view, features must be robust and unique. On the other hand, the features must be sensitive against the object pose, i.e., the image features must change if the object position or orientation changes. Also the features must be controllable, i.e., the reference features must be selected so that they are attained by applying a sequence of control actions.

Feature selection problem has been discussed in e.g., [1, 2]. However they consider only minimum number of features and the effect of redundant features is not discussed. A lot of reference points are used to calibrate hand-eye systems [3, 4]. However, this is a result of least square estimation and the effect of feedback is not considered. In visual servoing, if the number of feature points are changed, one have to consider the change of dimension of feedback variable, the controllability and the closed loop stability. In consequence, the evaluation methods of calibration accuracy and visual servoing accuracy are different. For visual servo problem, two measures have been proposed so far. One is the resolvability introduced by Nelson and Khosla to find optimal sensor placement [5, 6]. And the other is the motion perceptibility proposed by Sharma and Hutchinson to solve the motion planning problem [7]. These indices are the same in a sense that they measures the norm of the feature change caused by unit motion of the object. The difference is that the resolvability is used to obtain directional properties for guiding the robot during the task execution but the perceptibility is a scalar quantity used to optimize the robot performance for an entire task [7].

This paper gives another quantitative measure, sensitivity, for controlling all the degree of freedom of a robot by visual servoing. The sensitivity is used to select the feature points so as to minimize the joint error and increases the response speed. Also discussed is the relations among the sensitivity and the control performance, i.e., accuracy and quickness. The Jacobian introduced by Weiss et al. [2], which is called image Jacobian in this paper, plays important roles. The sensitivity is defined by the smallest singular value of the image Jacobian.

It is shown, in this paper, that if the image Jacobian is not full rank, i.e., if the sensitivity is equal to zero, then the closed loop system becomes internally unstable. In other words, the input-output stability seems to be satisfied but the internal variable becomes unstable. On the other hand, if the sensitivity is increased, both the accuracy of the joint control and the quickness of the response are increased. Also proved is that adding proper features, i.e., using redundant features, increases the sensitivity and a feature point that has different height is effective to improve the sensitivity. Moreover, it is proved that the image Jacobian becomes full rank by using redundant features. To investigate the speed and accuracy of the redundant visual servo system, real time experiments on the PUMA 560 are carried out. Translational step response with three (which is minimum), four and five (which are redundant) features are examined. The results...
results exhibit the quick and accurate performance of the visual servoing with redundant features.

2 Sensitivity

2.1 Definition

From the control theory point of view, robust and accurate performance can be expected by utilizing the measurements which contain rich information. For feature-based visual servo system we propose sensitivity for a measure of the richness. As discussed in [5] and [7] the image Jacobian can be used to evaluate the perceptibility of motion. To control m degree of freedom robot one need m features (m/2 feature points). If the features are under-observed, there always exists camera (equivalently, object) motions that are not perceptible by the observer. Thus we consider only for minimum and redundant cases (m \leq 2n).

Consider the singular value decomposition of image Jacobian

\[ J = U \Sigma V^T \]  \hspace{1cm} (1)

where

\[ U = [u_1, \ldots, u_m], \quad V = [v_1, \ldots, v_n], \]
\[ \Sigma = \begin{bmatrix} \Sigma' & 0 \\ 0 & 0 \end{bmatrix}, \quad \Sigma' = \text{diag} \{\sigma_1, \ldots, \sigma_m\}. \]  \hspace{1cm} (2)

The matrices U and V are orthogonal, \( u_i \) and \( v_j \) are, respectively, i-th left singular vector and j-th right singular vector, and \( \sigma_i \) is the i-th singular value. It is easy to verify that

\[ J v_i = \sigma_i u_i. \]  \hspace{1cm} (3)

Thus the unit motion of joint angle \( v_i = \Delta \theta_i / \| \Delta \theta \|_2 = 1 \) is transformed to the feature motion in \( u_i \) direction and scaled by \( \sigma_i \), i.e., \( \sigma_i u_i = \Delta \xi \). Thus if the image Jacobian is not full rank, there exist joint motion that is not perceptible through the image features. Also, since the minimum singular value satisfies

\[ \sigma_m = \sigma_{\text{min}} = \inf \{ \| \Delta \xi \|_2 : \| \Delta \theta \|_2 = 1 \}, \]  \hspace{1cm} (4)

\( \sigma_m \) is the magnification for the most insensitive direction. Thus the minimum singular value is adequate for a measure of (worst case) sensitivity.

2.2 Internal Instability

Fig. 1 shows a block diagram of the visual servo system. The feature \( \xi \) is fed back and compared with the reference \( \xi_d \). The feature error \( e = \xi_d - \xi \) is multiplied by the controller gain \( K \) and generates the joint motion command \( \Delta \theta \). The robot is driven by this command and the camera moves. The camera motion generates the feature motion \( \Delta \xi = J \Delta \theta \), which is integrated to yield the feature \( \xi \). In this closed loop system, suppose that the minimum singular value \( \sigma_m \) is vanishingly small. If the generated joint command includes \( \omega_m \) direction component, this component is hardly reflected to image motion and the joints will keep moving in this direction. This means that joint variable \( \theta \) (integration of joint displacement command \( \Delta \theta \)) inside the "System Model" box is internally unstable while the external signal \( \xi \) seems stable. We call this internal instability.

2.3 Improving Sensitivity

Feddema et al. selected feature points so as to minimize the condition number. Since the condition number is the ratio of maximum and minimum singular values \( (\sigma_{\text{max}} / \sigma_{\text{min}}) \), their approach is also appropriate for increasing the sensitivity. However we propose another approach. Suppose that there are \( n \) feature points and we add one more feature to the already existing features. Then we obtain the following theorem.

**Theorem 1** The sensitivity strictly increases by adding a feature point if and only if the minimum singular vector of the image Jacobian corresponding to the already existing points does not belong to the kernel of the image Jacobian corresponding to the newly added feature point.

**Proof:** Let \( J_n \) and \( J_{n+1} \) be the image Jacobian with \( n \) and \( n+1 \) feature points. Since we have added a feature point to the already existing \( n \) feature points, we have

\[ J_{n+1} = \begin{bmatrix} J_n \\ \Omega_{n+1} \end{bmatrix} \]  \hspace{1cm} (5)

where \( J^{(n+1)} \) is the \( 2 \times m \) image Jacobian corresponding to the \( (n+1) \)-st feature point. Thus for any vector \( v \), the following inequality holds

\[ \| J_{n+1} v \|_2 \geq \| J_n v \|_2. \]  \hspace{1cm} (6)

From the definition of minimum singular value (4), we have

\[ \sigma_{\text{min}}(J_{n+1}) \geq \sigma_{\text{min}}(J_n). \]  \hspace{1cm} (7)
The equality holds if and only if \( J^{(n+1)} v_n = 0 \), where \( v_n \) is the singular vector corresponding to the minimum singular value of \( J_n \), i.e., the vector that attains the infimum of (4).

Since the first three columns of \( J^{(i)}_{\text{image}} \) are proportional to \( 1/Z_i \), changing the depth \( (Z_i) \) is effective to increasing the linear dependency of the image Jacobian.

2.4 Sensitivity and Response Speed

Let \( K \) be the controller gain matrix. Then the transfer function of the closed loop system (from \( \xi_d \) to \( \xi \)) can be approximated by \( (sI+JK)^{-1}K \). Most visual servoing schemes for redundant features [8, 9, 10] use the generalized inverse or transpose of the image Jacobian multiplied by a scalar \( k \) for the controller gain. Suppose that we fix the scalar \( k \) and add a feature point. If the generalized inverse is used, the closed loop poles do not depend on \( J \). Thus the response speed will not be affected by the number of feature points. If the transpose is used, it is easy to show by doing mode decomposition that the closed loop poles are \( -k \sigma_i^2, \ldots, -k \sigma_m^2 \). Thus the slowest mode \( (-k \sigma_m^2) \) will be fasten by increasing the sensitivity. For the case of the discrete time optimal controller [11, 12], one can not fix the controller \( K \) by adjusting \( Q \) and \( R \) because the system model depends on \( J \) and the dimension of the controller changes. However if \( Q, R \) are adjusted to yield the same matrix gain of the controllers \( \|K\|_{\infty} \), it is confirmed by simulations that the slowest pole shifts to the origin in the complex plane by adding feature points (see Table 2).

3 Rank Condition

In this section we consider a six degree of freedom robot. Suppose that one want to control the position and orientation of the camera in 3D space. Then three features are necessary but not sufficient because there is a singular cylinder [13]. However, if one can use one more point on a plane that the three points lie on, then we have the following theorem.

**Theorem 2** Suppose that there are four points on a plane in 3D space and four feature points corresponding to these points are selected as the feature vector. Then the image Jacobian is full rank provided that the robot configuration is not singular and any three feature points out of four feature points are not collinear in the image plane.

The proof is straightforward and omitted [14]. The assumption on feature point configuration is natural because it is satisfied if the four feature points make a quadrangle (not necessarily square or rectangle but any four-sided plane) in the image plane.

4 Experiments

4.1 Sensitivity

Examples of smallest singular values for various feature sets are computed. As shown in Fig. 2, six cases with three to six feature points and two object position \( p_1 = [-50, -50, -1000, 0, 0, 0] \) and \( p_2 = [5, 10, -1000, -8, -5, -3] \) are tested (position is expressed in [mm] and orientation is expressed by Euler angles [degree]). The robot configuration is the same as the experiments (see the following section and Fig. 3). Table 1 shows the minimum singular values. For \( p_1 \), since the camera is on the singular cylinder, the sensitivity for three feature points is zero (a). The sensitivity is improved by increasing the number of feature points (b-f). Four feature points are necessary to guarantee the internal stability (b). Since the improvement is not very significant for five and six features on the same plane (c, d), one must consider the tradeoff between the performance and the image processing time. If a point with different height is available, the sensitivity is increased considerably (e, f). Even for the position \( p_2 \), which is outside of the singular cylinder, the sensitivity of three points is small. Four, five and six points give better sensitivity compared with \( p_1 \). By changing the height of the center
4.2 Setup
Real time experiments were carried out on the visual feedback control system with a PUMA 560 to compare the performance within three feature sets (a, b, e). The objects are attached to a PUMA 550. The world coordinate system is at the base of the PUMA 560. A nominal camera position is in front of the plane on which the marks are and the distance is 1000mm ($p_2$). The nominal positions of the object and camera are shown in Fig. 3. The $X_w - Y_w - Z_w$ coordinate system is the world coordinate system. We carried out vertical step tests to verify that sensitivity is an appropriate measure to select the features.

4.3 Control Law
In this paper we adopt a discrete time optimal control law [12]. The discrete time state equation is $z_{k+1} = z_k + Bu_k$, where $z_k = JT_\xi_k$, $u_k = \Delta \theta_k$ and $B = JTJ$. Then, for positive definite matrices $Q$ and $R$, the optimal control law is given by

$$u_k = -K \xi^T e_k, \quad K = (R + B^T PB)^{-1}B^T P$$

where $e_k = \xi_k - \xi_0$ and $P$ is the positive solution of the Riccati equation $Q = PB(R + B^T PB)^{-1}B^T P$. The closed loop poles are det$(zI - I + BK)$.

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4.4 Vertical Step
The object is moved up 100mm in vertical axis $Z_w$. The camera moves to keep the features at the initial position. Thus the initial values and the reference values are the same. The object motion is considered as a disturbance for the plots of the features in the image plane. On the other hand, the object motion becomes the step change of the reference position for the position of the camera in the world coordinate system. The reference orientation is the same as the initial orientation.

4.4.1 Three Points
Fig. 4 has six curves which show the $x$ and $y$ coordinates of the feature point in the image plane. The horizontal axis is the time. The curves disturbed largely are the $y$ coordinates and the others are the $x$ coordinates. They are almost stabilized in 6 seconds. Thus the response in the image plane is very good. However the plots in Fig. 5, which depicts the position errors of the camera in the world coordinate system $X_w - Y_w - Z_w$, is diverging. The features are kept in the neighborhood of the reference position due to the rotation of the robot wrist. Fig. 6 shows the orientation errors of the camera expressed in the Euler angles, say $\psi, \eta, \phi$. The plot of $\psi$ also becomes unstable. It shows that the camera keeps rotating. These plots exhibit the internal instability.

4.4.2 Four Points
Fig. 7 shows the response of the features in the image plane with four feature points. It takes 5 seconds to stabilize the disturbance. The response in the image plane is not improved very much compared with the response of three points. However, as shown in Fig. 8, the response of the camera position in the world coordinate system is stabilized, though it is sluggish. It takes over 20 seconds to stabilize the disturbance. The steady state errors are within 5mm. Thus the accuracy is fairly good. These plots show that the feature errors are reduced by the camera rotation as well as the camera translation in 5 seconds.
orientation errors are gradually reduced, but the speed is slow because the sensitivity is small. The response in the image plane seems quick due to the fast poles (0.900) but the response is actually slow because of the very slow pole (0.995).

### 4.4.3 Five Points

Fig. 9 depicts the features in the image plane for the experiment with five points. The disturbance is stabilized in 5 seconds. The response in the image plane is similar to those with three and four points. Fig. 10 shows the response of the camera position in the world coordinate system. It is improved very much for both speed and accuracy because the sensitivity and the slowest pole are improved. The steady state errors are smaller than 5mm for all directions. These plots demonstrate the effectiveness of the redundant features for improvements of both speed and accuracy of the feature-based visual servoing.
5 Conclusions

Discussions on the performance improvement due to redundant features were presented. Real-time experiments on PUMA 560 were carried out to evaluate the improvement of the accuracy and speed by utilizing the redundant features. The results have shown the quickly converging stable performance. The accuracy of the camera position control in the world coordinate system was increased by utilizing redundant features. Also, the convergence speed was improved considerably by adding the extra feature point. Moreover, the experiments also verified that the minimum singular value of the extended image Jacobian plays an important role to evaluate the performance of the feature-based visual servoing.

References


