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Akira Inoue
Okayama University

Shiro Masuda
Okayama University

Valeri T. Kroumov
Okayama University

Kenji Sugimoto
Okayama University

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Multivariable MRACS for Systems with Rectangular Transfer Matrix Using Coprime Factorization Approach

A. INOUE, S. MASUDA, V. T. KROUMOV, and K. SUGIMOTO
Department of Information Technology,
Okayama University, Okayama, Japan, 700
inoue@suri.it.okayama-u.ac.jp

Abstract

A multivariable model reference adaptive control system (MRACS) design method for a plant with m inputs and p outputs is proposed ($m \neq p$). Using an interactor matrix the coprime factorization of the plant for (1) $m > p$ case, and (2) $m < p$ case is derived. Further, using this coprime factorization the structure of MRACS is established.

1. Introduction

The model reference adaptive control system (MRACS) deals with plants whose characteristics are not completely known. In the multivariable case couplings exist between inputs and outputs, and this leads to difficulties with the structural problem in parametrizing the controller. Especially, the structural problem for plants with rectangular transfer matrix is quite difficult task, because any desired trajectories cannot be followed when the number of outputs is greater than the number of inputs.

In [1] the adaptive control algorithms for the systems with rectangular transfer matrix are developed based on the predictor form for the output vector, and in this method the control objective is to follow the prescribed trajectories instead of outputs of reference model transfer functions. However it is difficult to give straightforward description of the class of prescribed admissible trajectories, and it is necessary to modify the control objective when the number of outputs is greater than the number of inputs, in order to follow any desired trajectories. These drawbacks are caused by the control objective to follow the prescribed trajectories. And difficulties will be eliminated by using the exact model matching scheme. In [2] the exact model matching scheme is used, and it is based on the description of the plant over the ring of polynomials. Here arbitrary pole placement control is applied to the design of MRACS, and the model matching is achieved by a cancellation of open-loop zeros assigning them as closed-loop poles. However, when the number of the inputs m differs from that of the outputs p, the way to choose closed-loop poles is still unclear[2], and there is not so much development in these control design problems here.

The solution of the exact model matching problem can be given clearly by using coprime factorization approach over the ring of proper stable rational functions[6]. In this approach multivariable plants can be generally treated irrespective of the number of inputs and outputs. Therefore, in this paper, we propose a design method of multivariable MRACS for plants with different number of inputs and outputs by using the coprime factorization approach over the ring of proper stable rational functions. Such a method for the SISO case is already proposed in [4], as well as for multivariable systems when $p = m$ [5]. Here the relation between coprime factorization of plant and the interactor matrix is given and solutions of Bezout identity in the rational form are rewritten into polynomial form in order to construct MRACS using factorization approach. The main contribution of this paper is to give such a relation in the case of the system with rectangular transfer matrix. By using the proposed method we can eliminate the difficulties of the construction of MRACS with rectangular transfer functions.

The paper is organized as follows. Section 2 presents the problem statement the paper deals with. Section 3 presents the coprime factorization of the plant model in terms of the interactor matrix for the two cases, i.e. $m > p$ and $m < p$. Section 4 then gives the structure of the proposed MRACS. The last section concludes the paper.

The following notations from [5] are used. $\boldsymbol{C}^+$ is the right half complex plane. $\boldsymbol{RH}_\infty$ represents the ring of proper stable rational functions in indeterminate variable s with coefficients in the field of the real numbers ($\boldsymbol{R}$), The ring of polynomials with real coefficients is given by $\boldsymbol{R}[s]$. For the size of given matrix, the notations $\boldsymbol{RH}_\infty{}^{p\times l}$, $\boldsymbol{R}(s)$ of $\boldsymbol{R}[s]$ respectively. Further, $\partial_{ci}[\cdot]$ and $\partial_{ri}[\cdot]$ express the column and raw degrees of given polynomial matrix, and $[\cdot]_C$ ($[\cdot]_R$) is for the highest column (raw) degree coefficient matrix notation.

2. Problem Statement

Consider an unknown linear, time-invariant, finite dimensional, plant with m-inputs and p-outputs characterized by a transfer matrix $P(s)$:

$$y_p(t) = P(s)u(t) \tag{2.1}$$

where $u(t) \in \mathbf{R}^m$, $y_p(t) \in \mathbf{R}^p$ are the plant input and output vectors, respectively. It is assumed that $P(s)$ is full rank (rank $P(s) = \rho = \min\{m,p\}$) and strictly proper. $P(s)$ is factored as:

$$P(s) = N_p(s)D_p(s)^{-1} \tag{2.2}$$

where $N_p(s) \in \boldsymbol{RH}_\infty{}^{p\times m}$, $D_p(s) \in \boldsymbol{RH}_\infty{}^{m\times m}$ are relatively right coprime over $\boldsymbol{RH}_\infty$.

The transfer matrix of the reference model, denoted as $P_M(s)$, is strictly proper and asymptotically stable,

$$y_m(t) = P_M(s)r(t), \quad P_M(s) \in \boldsymbol{RH}_\infty{}^{p\times l} \tag{2.3}$$

where $r(t) \in \mathbf{R}^l$, $y_m(t) \in \mathbf{R}^p$, piecewise continuous and uniformly bounded, are the reference input and model output vectors respectively.

The following assumptions are made for the plant[2].

(A.1) The interactor matrix $L[s]$ is known, that is, a matrix $L[s] \in \mathbf{R}[s]^{p\times p}$ such that

$$\lim_{s\to\infty} L[s]P(s) = A \tag{2.4}$$

where A is nonsingular matrix.

(A.2) The plant maximum observability index ν is known.

(A.3) The plant is minimum phase, that is, $N_p(s)$ is of full rank for any $s \in \boldsymbol{C}^+$.

The control problem is to determine a differentiator free controller which generates a bounded control input signal vector, so that all the signals in the closed loop system remain bounded and the following equation is satisfied.

$$\lim_{t\to\infty} \|e(t)\| = \lim_{t\to\infty} \|y_p(t) - y_m(t)\| = 0. \tag{2.5}$$

3. Coprime Factorization Using the Interactor Matrix

Let the interactor matrix $L[s]$ be such that $\det(L[s])$ is an asymptotically stable polynomial. Such the interactor will be called a stable interactor matrix. If a given interactor matrix is not stable, then another stable interactor matrix can be obtained from the given one. Hence, it can be assumed without loss of generality that a given interactor is stable. In this section a relation between coprime factorization of the plant over $\boldsymbol{RH}_\infty$and the given interactor matrix is derived. This is used later to construct an MRACS. Two cases are considered:

1. Number of the inputs m is greater than the number of the outputs p ($m > p$).
2. Number of the outputs p is greater than the number of the inputs m ($m < p$).

3.1. Case I — $m > p$

Let the plant be described as in eq.(2.2). There exists a unimodular $U(s) \in \boldsymbol{RH}_\infty{}^{m\times m}$ matrix satisfying the expression

$$N_p(s)U(s) = [W(s),\ 0] \tag{3.1}$$

where $W(s) \in \boldsymbol{RH}_\infty{}^{p\times p}$. The next lemma characterizes $W(s)$.

Lemma 3.1 *Let the plant satisfy assumptions (A.1) - (A.3). Then $W(s)$ in eq.(3.1) has the full rank for the arbitrary $s \in \boldsymbol{C}^+$.*

Proof: Because the plant is minimum phase, for any $s \in \boldsymbol{C}^+$, $N_p(s)$ is of full rank. Also, $U(s)$, being unimodular, is of full rank for arbitrary $s \in \boldsymbol{C}^+$. Hence from eq. (3.1) the lemma holds □

The next proposition considers the interactor matrix of $W(s)$

Lemma 3.2 *Let $L[s] \in \mathbf{R}[s]^{p\times p}$ be a stable interactor matrix of $W(s)$, i.e. $L[s]$ satisfy the equation*

$$\lim_{s\to\infty} L[s]W(s) = B \tag{3.2}$$

where B is nonsingular, and $\det(L[s])$ is a stable polynomial. Then $L[s]$ is a stable interactor of the plant $P(s)$.

Conversely, an arbitrary interactor $L[s]$ of the plant $P(s)$ is an interactor of $W(s)$, too.

Proof: From (3.1)

$$N_p(s) = [W(s),\ 0]U(s)^{-1}. \tag{3.3}$$

From (2.2) and (3.3) the following equation is given.

$$\lim_{s\to\infty} L[s]P(s) = \lim_{s\to\infty}[L[s]W(s),\ 0]U(s)^{-1}D_p(s)^{-1}. \quad (3.4)$$

From the definition of $L[s]$, $\lim_{s\to\infty} L[s]W(s)$ is a nonsingular matrix. Further, since $U(s)$ is a unimodular, $U(s)^{-1}$ is a unimodular, and $U(s)$ tends to a nonsingular matrix as t goes to infinity.

Also, because of the assumption for strictly properness of $P(s)$, $\lim_{s\to\infty} D_p(s)^{-1}$ is a nonsingular. Hence eq.(3.4) converges to a nonsingular matrix and $L[s]$ is an interactor matrix of $P(s)$.

To prove the second part, note that because $L[s]$ is an interactor matrix of $P(s)$, $\lim_{s\to\infty} L[s]P(s)$ is a nonsingular matrix. From this it follows that $\lim_{s\to\infty} L[s]N_p(s)$ is also of full rank and nonsingular. Now, using eq.(3.1)

$$L[s]N_p(s)U(s) = [L[s]W(s),\ 0]. \quad (3.5)$$

If one takes a limit of (3.5) for $s \to \infty$ it is clear that the right hand side of (3.5) is of full rank, too, because $L[s]N_p(s)$ and $U(s)$ converge to proper constant matrices. Hence the proposition holds. □

The following results give an expression of the coprime factorization of the plant in terms of the interactor matrix.

Proposition 3.1 *Let the plant interactor matrix be denoted by $L[s]$. Then a right coprime factorization over $\boldsymbol{RH}_\infty$ of the plant is given by*

$$\begin{aligned} N_1(s) &= [L[s]^{-1},\ 0] \\ D_1(s) &= D_p(s)U(s)\begin{bmatrix} L[s]W(s) & 0 \\ 0 & I \end{bmatrix}^{-1}. \end{aligned} \quad (3.6)$$

Proof: From eq.(3.1)

$$\begin{aligned} N_p(s) &= [W(s),\ 0]U(s)^{-1} \\ &= [L[s]^{-1},\ 0]\begin{bmatrix} L[s]W(s) & 0 \\ 0 & I \end{bmatrix}U(s)^{-1}. \end{aligned} \quad (3.7)$$

From eq.(3.2) $L[s]W(s)$ and its inverse are proper. Further, from Lemma 3.1 and the definition of $L[s]$ it is seen that $L[s]W(s)$ does not have unstable zeros. Hence, $L[s]W(s) \in \boldsymbol{RH}_\infty{}^{p\times p}$, and moreover, $\begin{bmatrix} L[s]W(s) & 0 \\ 0 & I \end{bmatrix}$ is unimodular over $\boldsymbol{RH}_\infty$.

Hence $(N_1(s), D_1(s))$ as given by (3.6) are right coprime over $\boldsymbol{RH}_\infty$. □

Proposition 3.1 allows to express the plant factorization in terms of the interactor matrix.

3.2. Case II — $m < p$

We consider the case when the number of outputs is greater than the number of inputs.

Let the plant transfer function be factored over the ring of the polynomial matrices.

$$P(s) = Z_p[s]R_p[s]^{-1} = \begin{bmatrix} Z_p^m[s] \\ Z_p^{p-m}[s] \end{bmatrix} R_p[s]^{-1} \quad (3.8)$$

where $Z_p^{p-m}[s] \in \mathbf{R}[\mathrm{s}]^{(p-m)\times m}$, and $Z_p^m[s], R_p[s] \in \mathbf{R}[\mathrm{s}]^{m\times m}$. Then the interactor matrix $L[s]$ is given as follows[3].

$$L[s] = \begin{bmatrix} L^m[s] & 0 \\ -\gamma_1[s] & \gamma_2[s] \end{bmatrix} \quad (3.9)$$

where $L^m[s]$ is a stable interactor matrix of the $Z_p^m[s]R_p[s]^{-1} \in \mathbf{R}(s)^{m\times m}$. And $\gamma_1[s]$ and $\gamma_2[s]$ are relatively left prime and $\gamma_2[s]$ is a nonsingular lower left triangular matrix in Hermite normal form with monic diagonal entries. Furthermore, the following equation is satisfied in terms of $\gamma_1[s]$ and $\gamma_2[s]$.

$$Z_p^{p-m}[s]Z_p^m[s]^{-1} = \gamma_2[s]^{-1}\gamma_1[s]. \quad (3.10)$$

Here in addition to the assumption (A.1) the following assumption (A.1)' is imposed on the interactor matrix.

(A.1)' A lower left triangular interactor matrix defined by eq.(3.9) is known.

The following preliminary result is important for a further discussion.

Lemma 3.3 *If the plant satisfies the assumption (A.3), then* $\det(Z_p^m[s])$ *in eq.(3.8) is a stable polynomial.*

Proof: From the definition of the interactor matrix $L[s]$ (eq.(3.9) and (3.10)), it follows that:

$$L[s]Z_p[s] = \begin{bmatrix} L^m[s]Z_p^m[s] \\ 0 \end{bmatrix}. \quad (3.11)$$

Since $P(s)$ satisfies the assumption (A.3) and $L[s]$ is a stable interactor matrix, the left hand side of eq.(3.11) is of full rank for any $s \in \boldsymbol{C}^+$. Furthermore, $L^m[s]$ is of full rank for any $s \in \boldsymbol{C}^+$. Hence, it follows that $Z_p^m[s]$ is of full rank for any $s \in \boldsymbol{C}^+$. □

The following proposition gives a factorization of the plant for the case of $m < p$.

Proposition 3.2 *Let the interactor matrix be given by eq.(3.9) and (3.10). Then a right coprime factor-*

ization over $\boldsymbol{RH}_\infty$ of the plant is given by

$$\begin{aligned} N_p(s) &= \begin{bmatrix} I & 0 \\ -\gamma_1[s] & \gamma_2[s] \end{bmatrix}^{-1} \begin{bmatrix} L^m[s]^{-1} \\ 0 \end{bmatrix}, \\ D_p(s) &= R_p[s] Z_p^m[s]^{-1} L^m[s]^{-1}. \end{aligned} \tag{3.12}$$

Proof: It is clear from the following equation that the above $(N_p(s), D_p(s))$ represents a factorization of the plant $P(s)$.

$$\begin{aligned} &\begin{bmatrix} I & 0 \\ -\gamma_1[s] & \gamma_2[s] \end{bmatrix}^{-1} \\ &= \begin{bmatrix} I & 0 \\ \gamma_2[s]^{-1}\gamma_1[s] & \gamma_2[s]^{-1} \end{bmatrix} \\ &= \begin{bmatrix} I & 0 \\ Z_p^{p-m}[s] Z_p^m[s]^{-1} & \gamma_2[s]^{-1} \end{bmatrix}. \end{aligned} \tag{3.13}$$

$N_p(s) \in \boldsymbol{RH}_\infty$ follows the fact that $\gamma_2[s]$ is a stable matrix and $L^m[s]$ is a stable interactor matrix. Further from Lemma 3.3 it follows that $D_p(s) \in \boldsymbol{RH}_\infty$, because det $(Z_p^m[s])$ is a stable interactor matrix.

At the end, because $N_p(s)$ does not have finite unstable roots and $\lim_{s\to\infty} D_p(s)$ is a nonsingular matrix, it can be concluded that $N_p(s)$ and $D_p(s)$ are right coprime over $\boldsymbol{RH}_\infty$. □

4. Multivariable MRACS for Systems with Rectangular Transfer Matrix

In this section a design of multivariable MRACS is presented. The design procedure is based on the control law of EMM for the general rectangular transfer matrix.

In order to realize the MRACS design, first, a control law is constructed under the assumption that the plant $P(s)$ is known. Using the coprime factorization approach, the law for the general rectangular transfer matrix is given as,

$$\begin{aligned} u(t) &= (I - G_p^{-1} Y_p(s)) u(t) - G_p^{-1} X_p(s) y_p(t) \\ &\quad + G_p^{-1} K(s) r(t) \end{aligned} \tag{4.1}$$

where $G_p \in \mathbf{R}^{m\times m}$ is a nonsingular matrix given by

$$G_p = Y_p(\infty) = D_p(\infty)^{-1} \tag{4.2}$$

and $K(s) \in \boldsymbol{RH}_\infty{}^{m\times l}$ satisfies the equation

$$P_M(s) = N_p(s) K(s) \tag{4.3}$$

$X_p(s) \in \boldsymbol{RH}_\infty{}^{m\times p}$ and $Y_p(s) \in \boldsymbol{RH}_\infty{}^{m\times m}$ are solutions of the Bezout identity:

$$X_p(s) N_p(s) + Y_p(s) D_p(s) = I. \tag{4.4}$$

Since the only *a priori* information about the plant is given in assumptions (A.1)–(A.3), the unknown parameters must be identified in order to realize the control law (4.1). In the control law (4.1), matrix $K(s)$ can be obtained by solving (4.3) since $N_p(s)$ is derived from the known interactor $L[s]$ using proposition 3.1 or proposition 3.2. However since $D_p(s)$ is not known, matrices G_p, $X_p(s)$ and $Y_p(s)$ are unknown matrices. Hence these unknown parameters are to be identified.

To identify these unknown parameters, the following identification model is considered[5]. Multiplying eq.(4.4) from the left-hand side by $N_p(s)$ and from the right-hand side by $D_p(s)^{-1}u$ gives:

$$y_p(t) = N_p(s)[Y_p(s)u(t) + X_p(s)y_p(t)]. \tag{4.5}$$

The following proposition gives the conditions for the orders of $X_p(s)$ and $Y_p(s)$. Then the expressions of these matrices are derived in a similar way as [5] which is for systems with square transfer matrix.

Proposition 4.1 *Let the plant maximal observability index be denoted by ν, and let $\Xi_2[s] = \xi_2[s]I$ for some monic stable polynomial $\xi_2[s]$ of order $\nu - 1 + q$ ($q \geq 0$). Then there exists a solution $X_p(s) \in \boldsymbol{RH}_\infty{}^{m\times p}$, $Y_p(s) \in \boldsymbol{RH}_\infty{}^{m\times m}$ of Bezout identity (4.4) of the form*

$$X_p(s) = \Xi_2[s]^{-1} Z_x[s], \quad Y_p(s) = \Xi_2[s]^{-1} Z_y[s]. \tag{4.6}$$

Proof: The proof is omitted because it can be done in a similar way to proposition 2 in [5]. □

Using the above proposition eq. (4.5) can be rewritten as:

$$\begin{aligned} &y_p(t) \\ &= N_p(s)\left[Z_y[s]\frac{1}{\xi_2[s]}u(t) + Z_x[s]\frac{1}{\xi_2[s]}y_p(t)\right] \\ &= N_p(s)\left[(Z_y[s] - \xi_2[s]G_p)\frac{1}{\xi_2[s]}u(t) + G_p u(t)\right. \\ &\quad \left. + Z_x[s]\frac{1}{\xi_2[s]}y_p(t)\right]. \end{aligned} \tag{4.7}$$

From the proof of Proposition 4.1

$$\begin{aligned} Z_y[s] - \xi_2[s]G_p &= H_{\nu-2+q}s^{\nu-2+q} + \cdots + H_0, \\ Z_x[s] &= J_{\nu-1+q}s^{\nu-1+q} + \cdots + J_0. \end{aligned} \tag{4.8}$$

Form a state variable filter

$$\begin{aligned} \omega(t)^T = &\left[\frac{s^{\nu-2+q}}{\xi_2[s]}u(t)^T, \cdots, \frac{1}{\xi_2[s]}u(t)^T,\right. \\ &\left.\frac{s^{\nu-1+q}}{\xi_2[s]}y_p(t)^T, \cdots, \frac{1}{\xi_2[s]}y_p(t)^T\right]. \end{aligned} \tag{4.9}$$

and define the unknown parameters vector as

$$\Theta = [H_{\nu-2+q}, \cdots, H_0, J_{\nu-1+q}, \cdots, J_0]. \quad (4.10)$$

Using the above definitions equation (4.7) can be rewritten now as

$$y_p(t) = N_p(s)[\Theta\omega(t) + G_p u(t)]. \quad (4.11)$$

Depending on the number of the inputs and the outputs two cases are considered below.

Case I – $m > p$

By proposition 3.1 take $N_p(s) = [L[s]^{-1},\ 0]$. Substitute $N_p(s)$ in eq.(4.11) and multiply the latter from the left-hand side by $L[s]$, which gives

$$L[s]y_p(t) = [I,\ 0][\Theta\omega(t) + G_p u(t)]. \quad (4.12)$$

Denoting

$$y_E(t)^T = [L[s]y_p(t)^T,\ 0] \quad (4.13)$$

yields

$$y_E(t) = \Theta\omega(t) + G_p u(t). \quad (4.14)$$

To avoid the differentiation a standard filter arguments can be used:

$$f[s]^{-1}y_E(t) = \bar{\Theta}\bar{\zeta}(t) \quad (4.15)$$

where

$$\bar{\Theta} = [\Theta,\ G_p] \quad (4.16)$$
$$\bar{\omega}(t)^T = [\omega(t)^T,\ u(t)^T] \quad (4.17)$$
$$\bar{\zeta}(t) = f[s]^{-1}\bar{\omega}(t). \quad (4.18)$$

$f[s]$ have to be chosen in such a manner that $f[s]^{-1}L[s]$ is proper. Eq.(4.15) is a linear relation between the filtered input and output of the plant and the matrix of unknown parameters $\bar{\Theta}$.

In order to estimate $\bar{\Theta}$ a suitable method can be chosen [2],[7], [8].

For the control law (4.1), using proposition 4.1, one can write

$$u(t) = -G_p^{-1}\Theta\omega(t) + G_p^{-1}K(s)r(t). \quad (4.19)$$

By using the identified parameters Θ and G_p the control law (4.1) becomes realizable.

Thus the structure of MRACS is established.

Case II – $p > m$

For the left inverse matrix of $N_p(s)$ from proposition 3.2 one can write

$$N_p(s)^L = [\ L^m[s]\quad 0\ ]. \quad (4.20)$$

Multiplying eq.(4.11) from the left-hand side by $N_p(s)^L$ gives

$$N_p(s)^L y_p(t) = \Theta\omega(t) + G_p u(t) \quad (4.21)$$

Now, introduce a filter in order to avoid the differentiation

$$f[s]^{-1}N_p(s)^L y_p(t) = \bar{\Theta}\bar{\zeta}(t) \quad (4.22)$$

Thus the MRACS for this case is established, too.

5. Conclusion

A new design method for multivariable MRACS with different number of inputs and outputs was proposed in this paper. It is based on the coprime factorization in the matrix ring of proper stable rational functions. The proposed method is an extension to the existing design methods. The MRACS structure is based on the relation between the interactor matrix and the factorization of the plant in $\boldsymbol{RH}_\infty$. The contribution of the paper is not only the proposed structure but the latter relation as well.

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