In this paper, we investigate a light-confinement phenomenon in the structure which has triangular lattice composed of Double NeGative Metamaterial (DNGM). In geometrical optics consideration, this structure is expected to confine lights completely by sequential refractions in the structure. We demonstrate it by using the two dimensional finite-difference time-domain simulations. We introduce Drude-Lorentz model for dielectric and magnetic dispersion of the material at optical frequencies. We analyze quantitatively the effects of energy loss in the DNGM on the light-confinement efficiency.

1. INTRODUCTION

The technology that confines lights and converts them to electricity have attracted much of attention [1]-[3]. If we can confine light strongly and long enough in a tiny resonator, hypersensitive sensors and tiny lasers can be fabricated. Needs for such devices have been emphasized in recent years due to the rapid increase of demand for energy-conscious applications. The metamaterial, that refracts wave at its surface in opposite direction to the way in ordinary dielectrics, is one of the candidates that make such devices possible.

In 1967, Veselago introduced materials with negative permittivity and negative permeability simultaneously that were shown to have a Negative Refractive Index (NRI) [4]. In 2000, such materials have been realized by arranging periodic arrays of small metallic wires and split-ring resonators for microwave frequencies [5]. This novel media is called Double NeGative Metamaterial (DNGM), and is expected to realize superlens [6] which exceed an optical limit. In recent years, DNGM operated at optical frequencies has been proposed and studied extensively [7]-[9].

In this paper, we demonstrate a structure which confines light by negative refraction. We analyze it by using the two dimensional
Finite-Difference Time-Domain (FDTD) method. Also, we extend the FDTD method to include interband transitions of electron at optical frequencies by introducing the Drude-Lorentz dispersion function. We investigate the effects of energy loss in the medium on the confinement efficiency.

2. METHODOLOGY

2.1 Dispersion Model

In many of the previous study on DNGM using numerical methods, negative values of permittivity and permeability have been modeled by introducing the Drude model in dielectric dispersion. In the Drude model permittivity is written as

$$\varepsilon(\omega) = \varepsilon_\infty \left(1 - \frac{\omega_p^2}{\omega(\omega + i\Gamma)}\right),$$

where $\omega_p$ is the plasma frequency and $\Gamma$ is the damping constant. This model describes dielectric dispersion of nearly free electrons in metal. On the other hand, it cannot reproduce the experimental spectrum of noble metals at the range of visible light. To overcome this drawback Drude-Lorentz (DL) model is introduced into our analysis at optical frequency. The DL model includes processes of interband transition of electrons in metals. In the DL model permittivity is written as

$$\varepsilon(\omega) = \varepsilon_\infty \left(1 - \frac{\omega_p^2}{\omega_0^2 + i\omega} + \sum_{j=1}^{k} \frac{\omega_j^2}{\omega_j^2 - \omega^2 + i\omega_j/\omega}\right),$$

where $\varepsilon_\infty$ is the permittivity at infinite frequency, $k$ is the number of oscillators with frequency $\omega_j$, and strength $f_j$. This model deals with the intraband effects separately from interband effects.

2.2 ADE-FDTD

We use the FDTD method with the auxiliary difference equation (ADE) to calculate the electromagnetic field in dispersive media. If an object has dispersion, we need to solve differential equations not only for the electric field and magnetic field, but also for the electric and magnetic polarization in the medium. In TM mode, the equations for Drude model are written as

$$\partial_t E_z = \frac{1}{\varepsilon_0} \left(\partial_x H_y - \partial_y H_x - J_z\right),$$
$$\partial_t J_z + \Gamma_0 J_z = \varepsilon_0 \omega_p^2 E_z,$$
$$\partial_t H_y = \frac{1}{\mu_0} \left(\partial_x E_z - K_y\right),$$
$$\partial_t H_x = \frac{1}{\mu_0} \left(\partial_y E_z - K_x\right),$$
$$\partial_t K_y + \Gamma_m K_y = \mu_0 \omega_{pm}^2 H_y,$$
$$\partial_t K_x + \Gamma_m K_x = \mu_0 \omega_{pm}^2 H_x,$$

where $J_z$ is the polarization current and $K_x$ and $K_y$ are the magnetic polarization current. On the other hand, the equations of electric field and electric polarization for DL model are written as

$$\partial_t E_z = \frac{1}{\varepsilon_0} \left(\partial_x H_y - \partial_y H_x - \sum_{j=0}^{M} J_{ej}\right),$$
$$\partial_t J_{e0} + \Gamma_0 J_z = \varepsilon_0 f_0 \omega_p^2 E_z,$$
$$\vdots$$
$$\partial_t J_{eM} + \Gamma_M J_z = \varepsilon_0 f_M \omega_p^2 E_z.$$

These equations are discretized with the standard staggered grid, leapfrog in time approach in the FDTD algorithm.

3. RESONATOR MODEL

Open resonator with a checkerboard structure composed of metamaterials and ordinary dielectrics has been proposed in Ref.[10] as depicted in Fig.1.

We extend this idea and propose the structure with triangular lattice of the composites as illustrated in Fig.2. In geometrical optics consideration, it is expected that the light emitted from the source returns back to the central region by sequential negative refractions at the interface between DNGM and dielectrics.
4. RESULT
4.1 Analysis on Drude Model (Lossless Case)
We set a source at the center of the structure, and analyze the electromagnetic wave propagating in the structure within the Drude model. Input frequency is 600THz. We adopt parameters in the model so that the refractive index be $-1$ at the frequency. Figures 3(a)-(c) show that the wave from the source returns to the center by sequential negative refractions.

We also analyze an optical confinement efficiency by calculating the sum of electromagnetic energy, $S$, accumulated in the triangular lattice at the center. $S(t)$ is written as

$$S(t) = \frac{1}{T} \int_{t}^{t+T} \sum_{i,j} (E_z(i,j))^2 \, dt.$$ 

Fig. 1 Open resonator which has lattice ((a) square (b) triangle) composed of medium with negative refractive index. The arrow indicates two possible paths which are closed with finite optical length.

Fig. 2. Proposed structure for light confinement. The arrows indicate two possible paths which are closed with finite optical length.

Fig. 3 Electromagnetic wave propagation in the light-confining structure. (a) $t=10\text{fs}$, (b) $40\text{fs}$, (c) $70\text{fs}$ after the beginning of emission.

We set a source at the center of the structure, and continue to excite the electromagnetic wave at the source. As a result of this calculation, amplification of $S(t)$ appears. This is caused by returning of electromagnetic wave propagating
within the resonator. The return wave and the excited wave are interfered and resonanced mutually. Until 50/fs, the value of $S(t)$ is kept 

Higher degree of amplification for the case of side position than the result for the center is shown. This result may be traced by short route between focus-refocus points of the traveling wave from the source. In the case of corner, it shows high degree of amplification similar to the side case except that $S(t)$ decreases once before the continuous amplification begins. This result may be attributed to the asymmetry in the route between the focus-refocus points. So we analyze the cases near the corner in more detail. We shift gradually the source position from side to corner (Fig. 7) or from center to corner (Fig. 8).

Figure 7 shows that energy gain decreases gradually as the source is moved from side to corner. This result implies the effect of the symmetry of route for the electromagnetic wave from the source. Figure 8 shows that energy gain increase gradually from center to corner under the condition that keeps symmetry of the route from the source.
Results so far show that the structure we demonstrate confines light by sequential refractions in the structure. These results are analyzed under the condition that the object is a lossless media. Real DNGM absorbs electromagnetic energy through the finite resistivity of the metals and also through a resonance due to the structure. Thus we investigate the effects of the energy loss on the confinement efficiency of light. Figures 9 and 10 show that effects of loss in the medium on amplification of electromagnetic energy by the structure. (The source position is located either at the center or at the side.)
5. CONCLUSION

We demonstrated that the structure proposed in this study confines light by a sequence of negative refractions. We analyze it by using the 2D-FDTD simulations. We introduced the Drude-Lorentz model for a quantitative analysis at optical frequencies. We have compared energy amplification for different source positions. We have also shown the effects of the materials loss on the light-confinement efficiency.

ACKNOWLEDGEMENTS

This work was supported by Grant-in-Aid for Scientific Research on Priority Areas “Nano Materials Science for Atomic Scale Modification 47” from MEXT of Japan.

REFERENCES