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# *Multicomponent Plasmas in Penning-Malmberg Traps*

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The behavior of multicomponent plasmas in the Penning-Malmberg traps is discussed with the parameters corresponding to experiments with antiprotons and cryogenic electrons. The relaxation times for the energy transfer from antiprotons to electrons and between parallel and perpendicular components of electrons are estimated. It is shown that, depending on the values of parameters, both the former and the latter can be the bottleneck in the cooling process.

## I. INTRODUCTION

As a method to confine charged particles, the Penning-Malmberg trap has been widely applied not only to precise measurements with small number of particles (ions) but for analyses of properties of plasma bulk. [1] The assembly of trapped charged particles can be also used to host other particles introduced into the trap for various purposes. The cooling of high energy particles by cryogenic plasmas in the trap may be one of typical examples of such applications. [2] In these applications, the behavior of multicomponent plasmas in the trap is of essential importance. The purpose of this article is to analyze the thermal equilibrium and mutual relaxation processes in multicomponent plasmas in the Penning-Malmberg trap. We keep the application to the plasma composed of cryogenic electrons and high energy antiprotons in mind but present general expressions except for the cases where the assumptions on parameters are necessary.

## II. HAMILTONIAN

We consider the Penning-Malmberg trap with the magnetic field in the $z$-direction $\mathbf{B} = B\hat{z}$. For simplicity, we assume the cylindrical symmetry around the $z$-axis and denote the coordinates of particles $\mathbf{r}$ as

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$$\mathbf{r} = (\mathbf{R}, z) = (R, \theta, z). \tag{2.1}$$

In the rest frame, the Hamiltonian for charged particles is expressed as

$$H_0 = \sum_i \frac{1}{2} m_i \mathbf{v}_i^2 + U_{int}(\{\mathbf{r}_i\}) + \sum_i U_{ext}(R_i). \tag{2.2}$$

Here $U_{int}(\{\mathbf{r}_i\})$, $U_{ext}(R)$, $m_i$, and $\mathbf{v}_i$ are the interaction between particles, the external potential, the mass, and the velocity of the particle $i$, respectively. The canonical momentum of the particle $i$, $\mathbf{P}_i$, is given by

$$\mathbf{P}_i = m_i \mathbf{v}_i + q_i \mathbf{A}(\mathbf{r}_i), \tag{2.3}$$

where $q_i$ is the charge and $\mathbf{A}(\mathbf{r})$ is the vector potential at $\mathbf{r}$. In cylindrical coordinates, $\mathbf{A}(\mathbf{r})$ is written in the form

$$\mathbf{A}(\mathbf{r}) = (A_R, A_\theta, A_z) = (0, \frac{1}{2} BR, 0). \tag{2.4}$$

Due to our assumption of the rotational symmetry around the $z$-axis, we have the conservation of the $z$-component of total canonical angular momentum $M_z$ given by

$$M_z = \sum_i M_{z,i} = \sum_i (m_i R_i^2 \dot{\theta}_i + \frac{1}{2} q_i B R_i^2). \tag{2.5}$$

In thermal equilibrium, trapped particles perform a solid rotation around the $z$-axis. We denote its angular frequency by $\omega$. The Hamiltonian in the coordinate frame rotating with $\omega$ around the $z$-axis [3]

$$H = H_0 - \omega M_z \tag{2.6}$$

is rewritten into the form

$$H = H_0' + \frac{1}{2} \sum_i k_i R_i^2, \tag{2.7}$$

where

$$H_0' = \sum_i \frac{1}{2} m_i (\mathbf{v}_i - \omega \hat{z} \times \mathbf{R}_i)^2 + U_{int}(\{\mathbf{r}_i\}) + \sum_i U_{ext}(R_i) \tag{2.8}$$

and

$$\frac{1}{2} \sum_i k_i R_i^2 = -\frac{1}{2} \omega \sum_i (q_i B + m_i \omega) R_i^2. \tag{2.9}$$

The term (2.9) in $H$ serves as a potential which confines particles around the $z$-axis. Noting that

$$\mathbf{v}_i' = \mathbf{v}_i - \omega \hat{z} \times \mathbf{R}_i \tag{2.10}$$

is the velocity of the particle $i$ in the rotating frame, we observe that $H_0'$ has the same form as the Hamiltonian for particles in the potential field $U_{ext}(R)$ interacting with $U_{int}(\{\mathbf{r}_i\})$ in the rest frame. The canonical distribution in the rotating frame is thus given by that of a system where particles are in the external potential $\sum_i U_{ext}(R_i) + \frac{1}{2}\sum_i k_i R_i^2$ and are mutually interacting via the potential $U_{int}(\{\mathbf{r}_i\})$. [4]

When we have only one species of charged particles in the trap, the confining potential is given by

$$\frac{1}{2}kR^2, \tag{2.11}$$

where

$$k = -\omega(qB + m\omega). \tag{2.12}$$

In the case where $q = -e < 0$,

$$k = \omega(eB - m\omega), \tag{2.13}$$

and for confinement, the condition $\omega > 0$ is to be satisfied.

When we have several species of charged particles in the trap, the confining potential for the species $\alpha$ is given by

$$\frac{1}{2}k_\alpha R^2, \tag{2.14}$$

where

$$k_\alpha = -\omega(q_\alpha B + m_\alpha \omega). \tag{2.15}$$

For electrons and antiprotons, $q_\alpha = -e < 0$, and the Hamiltonian in the rotation frame is given by

$$H = H_0' + \frac{1}{2}\omega \sum_{electrons} (eB - m_e\omega)R_i^2 + \frac{1}{2}\omega \sum_{antiprotons} (eB - m_p\omega)R_i^2, \tag{2.16}$$

where $m_e$ and $m_p$ are the masses of an electron and an antiproton, respectively. For confinement of both species of particles, $\omega > 0$ and it is also necessary to have

$$\omega(eB - m_p\omega) > 0. \tag{2.17}$$

## III. EQUILIBRIUM DISTRIBUTION OF ONE-COMPONENT NONNEUTRAL PLASMA

We here consider the case of one-component plasma with the charge $-e$ and the mass $m$ in the trap and discuss the distribution in thermal equilibrium.

### A. Determination of $\omega$, $R$ and $n$

In thermal equilibrium, charged particles perform a solid rotation around the magnetic field with the angular frequency $\omega$. At low enough temperatures, their distribution has a sharp boundary and the density $n$ is uniform up to some radius $R$ where the density suddenly changes from $n$ to zero.

The $z$-component of the total canonical angular momentum (2.5) is conserved due to symmetry around the $z$-axis. Noting that ($\dot{} = d/dt$)

$$\sum_i m_i R_i^2 \dot{\theta}_i = \sum_i m_i \omega R_i^2, \tag{3.1}$$

for the solid rotation, we have

$$M_z = \sum_i (m_i\omega + \frac{1}{2}q_i B)R_i^2 = \frac{\pi}{2}n(m\omega - \frac{1}{2}eB)R^4 \tag{3.2}$$

per unit length along the $z$-axis.

Since the confining potential $\omega(eB - m\omega)$ is equivalent to the uniform background charge density $2\varepsilon_0 e\omega(eB - m\omega)$, the density $n$ is related to the confining potential by

$$n = m\omega(eB/m - \omega)2\varepsilon_0/e^2. \tag{3.3}$$

On the other hand, $n$ is related to the number of charges per unit length $N$ and $R$ by

$$\pi n R^2 = N. \tag{3.4}$$

When $M_z$ and $N$ are given, $\omega$ and $R$ are thus determined by the relations

$$\frac{(\omega - eB/2m)}{\omega(eB/m - \omega)} = \frac{4\pi\varepsilon_0}{e^2}\frac{M_z}{N^2}, \tag{3.5}$$

and

$$R^2 = \frac{e^2}{2\pi\varepsilon_0}\frac{1}{m}\frac{1}{\omega(eB/m - \omega)}N. \tag{3.6}$$

### B. Brillouin Limit

The minimum of $R$,

$$R_{min} = (2m/\pi\varepsilon_0)^{1/2}(N/B^2)^{1/2}, \tag{3.7}$$

is attained when

$$M_z = 0 \quad \text{and} \quad \omega = \frac{1}{2}\frac{eB}{m}. \tag{3.8}$$

This is called the Brillouin limit. [5] In this case, we have the maximum density

$$n_{max} = \frac{1}{2} m (eB/m)^2 \varepsilon_0 / e^2 = \frac{1}{2} (B^2/m) \varepsilon_0, \tag{3.9}$$

or

$$n_{max} e^2 / \varepsilon_0 m = \frac{1}{2} (eB/m)^2. \tag{3.10}$$

### C. Slow Rotation

When $|\omega| << eB/m$, relations (3.5) and (3.6) reduce to

$$-\frac{1}{2\omega} = \frac{4\pi\varepsilon_0}{e^2} \frac{M_z}{N^2}, \tag{3.11}$$

$$R^2 = \frac{e^2}{2\pi\varepsilon_0} \frac{1}{m} \frac{1}{\omega(eB/m)} N. \tag{3.12}$$

We also have

$$M_z = -\frac{\pi}{4} neBR^4, \tag{3.13}$$

and

$$\frac{n}{n_{max}} = \frac{4\omega}{eB/m}. \tag{3.14}$$

Since

$$\frac{ne^2}{\varepsilon_0 m} = 2\omega \frac{eB}{m}, \tag{3.15}$$

we have

$$\omega \ll \left( \frac{ne^2}{\varepsilon_0 m} \right)^{1/2} \ll \frac{eB}{m}. \tag{3.16}$$

We note that in this case

$$M_z < 0. \tag{3.17}$$

## IV. EXPERIMENTAL PARAMETERS

Here we list typical values of parameters expected in experiments to cool antiprotons by cryogenic electrons trapped in the Penning-Malmberg trap.

| | | |
|---|---|---|
| magnetic field | $B$ | 5T |
| trap length | | 1cm |
| trap radius | | 0.1cm |
| electron density | $n_e$ | $10^9\mathrm{cm}^{-3}$ |
| antiproton density | $n_p$ | $10^7\mathrm{cm}^{-3}$ |
| electron temperature | $T_e$ | 10K |
| antiproton temperature | $T_p$ | $10^4$K and higher |
| electron solid rotation | $\omega_e$ | $10^5\mathrm{s}^{-1}$ |

The cyclotron frequencies are given by

| | | |
|---|---|---|
| electron cyclotron frequency | $eB/m_e = 1.76 \cdot 10^{11} B[T][s^{-1}]$ | $9 \cdot 10^{11}\mathrm{s}^{-1}$ |
| antiproton cyclotron frequency | $eB/m_p = 9.58 \cdot 10^7 B[T][s^{-1}]$ | $5 \cdot 10^8\mathrm{s}^{-1}$ |

Some scales of lengths are

| | | |
|---|---|---|
| electron Debye length | $(\varepsilon_0 k_B T_e/n_e e^2)^{1/2} = 6.9(T_e[K]/n_e[cm^{-3}])^{1/2}[cm]$ | $7 \cdot 10^{-4}$cm |
| antiproton Debye length | $(\varepsilon_0 k_B T_p/n_p e^2)^{1/2} = 6.9(T_p[K]/n_p[cm^{-3}])^{1/2}[cm]$ | $2 \cdot 10^{-1}$cm |
| electron mean distance | $(3/4\pi n_e)^{1/3} = 0.620(n_e[cm^{-3}])^{-1/3}[cm]$ | $6 \cdot 10^{-4}$cm |
| antiproton mean distance | $(3/4\pi n_p)^{1/3} = 0.620(n_p[cm^{-3}])^{-1/3}[cm]$ | $3 \cdot 10^{-3}$cm |
| electron cyclotron radius | $2\pi v_{th,e}/(eB/m_e) = 1.39 \cdot 10^{-5}(T_e[K])^{1/2}/B[T][cm]$ | $9 \cdot 10^{-6}$cm |
| antiproton cyclotron radius | $2\pi v_{th,p}/(eB/m_p) = 5.96 \cdot 10^{-4}(T_p[K])^{1/2}/B[T][cm]$ | $1 \cdot 10^{-2}$cm |
| close collision radius | $e^2/4\pi\varepsilon_0 k_B T = 1.67 \cdot 10^{-3}/T[K][cm]$ | |

The coupling constants are given by

| | | |
|---|---|---|
| $\Gamma_e$ (electrons) | $e^2/4\pi\varepsilon_0 a_e k_B T = 2.70 \cdot 10^{-3}(n_e[cm^{-3}])^{1/3}/T_e[K]$ | $3 \cdot 10^{-1}$ |
| $\Gamma_p$ (antiprotons) | $e^2/4\pi\varepsilon_0 a_p k_B T = 2.70 \cdot 10^{-3}(n_p[cm^{-3}])^{1/3}/T_p[K]$ | $6 \cdot 10^{-5}$ |

Thermal velocities are given by

| | | |
|---|---|---|
| electron | $v_{th,e} = (k_B T_e/m_e)^{1/2} = 3.89 \cdot 10^5 (T_e[K])^{1/2}[cm/s]$ | $1 \cdot 10^6$cm/s |
| antiproton | $v_{th,p} = (k_B T_p/m_p)^{1/2} = 9.09 \cdot 10^3 (T_p[K])^{1/2}[cm/s]$ | $9 \cdot 10^7$cm/s |

We have an inequality for length scales

$$\begin{aligned} &\text{electron cyclotron radius} \ll \text{electron mean distance} \\ &\ll \text{antiproton mean distance} \ll \text{antiproton cyclotron radius} \end{aligned} \tag{4.1}$$

## V. RELAXATION OF ENERGY

Let us now consider the relaxation processes which occur when energetic antiprotons are introduced into cryogenic electrons.

### A. Relaxation of Antiproton Energy

When antiprotons are impinging cold electrons with the velocity $v_{p||}$, or with the energy $E_{p||} = m_p v_{p||}^2/2$, the loss rate of parallel energy is estimated as follows. We move to the frame where the antiproton is at rest and the electron is coming with the velocity $-v_{p||}$ from $z = \infty$. In the strong magnetic field, electrons move along the magnetic field lines gyrating with small cyclotron radius and the drift approximation may be applied. [6] In this approximation, electrons within the impact parameter

$$e^2/4\pi\varepsilon_0(m_e/2)v_{p||}^2 \tag{5.1}$$

are reflected. Electrons with the impact parameter larger than the above make a drift motion around the antiproton and eventually move to $z = -\infty$. After the collision, they have the parallel velocity $-v_{p||}$ and the perpendicular energy $E_\perp$ which is an adiabatic invariant.

The frequency of collisions with the impact parameter smaller than (5.1) is thus given by

$$n_e e^4/4\pi\varepsilon_0^2 m_e^2 v_{p||}^3 = 2^{1/2}\pi n_e \left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \frac{1}{m_e^{1/2} E_{p||}^{3/2}} \left(\frac{m_p}{m_e}\right)^{3/2}. \tag{5.2}$$

Since the momentum transfer to electron is $2m_e v_{p||}$, the change of the parallel velocity of antiproton (recoil) is given by $2m_e v_{p||}/m_p$. Noting that

$$\Delta E_{p||} = \frac{1}{2}m_p(v_{p||} - 2m_e v_{p||}/m_p)^2 - \frac{1}{2}m_p v_{p||}^2 = -4(m_e/m_p)E_{p||}, \tag{5.3}$$

we have

$$\frac{d}{dt}E_{p||} = -\frac{1}{\tau_1}E_{p||}, \tag{5.4}$$

where

$$\frac{1}{\tau_1} = 2^{5/2}\pi n_e \left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \frac{1}{m_e^{1/2} E_{p||}^{3/2}} \left(\frac{m_p}{m_e}\right)^{1/2}. \tag{5.5}$$

For $n_e = 10^9 cm^{-3}, E_{p||} = 10^4 K$,

$$\frac{1}{\tau_1} \sim 8 \cdot 10^5 s^{-1} \tag{5.6}$$

### B. Relaxation Between Parallel and Perpendicular Temperatures of Electrons

In strong magnetic field where cyclotron radius is smaller than the close collision radius, parallel and perpendicular components of energy of electrons relax separately to the Maxwell distributions with different temperatures: The relaxation between these components is a slow process limited by the many-body adiabatic invariance. [7–9] The relaxation time for the latter is written as

$$\frac{d}{dt}T_{e\|} = -\frac{1}{\tau_2}(T_{e\|} - T_{e\perp}), \tag{5.7}$$

$$\tau_2^{-1} = n_e(2e^2/4\pi\varepsilon_0 k_B T_{e\|})^2 (k_B T_{e\|}/(m_e/2))^{1/2} I(\kappa), \tag{5.8}$$

where $I(\kappa)$ is a function of

$$\kappa = (eB/m_e)(2e^2/4\pi\varepsilon_0 k_B T_{e\|})/(k_B T_{e\|}/(m_e/2))^{1/2}. \tag{5.9}$$

For $n_e = 10^9\mathrm{cm}^{-3}, T_{e\|} \sim T_{e\perp} \sim 10\mathrm{K}$, and $B = 5\mathrm{T}$, $\kappa = 1.7 \cdot 10^2$ and $\tau_2^{-1} \sim 10^1\mathrm{s}^{-1}$.

### C. Relaxation of Antiproton Energy

Let us now compare these relaxation times. We fix the magnetic field at 5T and the density and the temperature of electrons at $10^9\mathrm{cm}^{-3}$ and 10K, respectively. We note that both of the above process can limit the cooling of antiprotons: When $E_{p\|} > 10^8\mathrm{K}$ (10keV), $\tau_1 > \tau_2$ and the relaxation is limited by the process $E_{p\|} \to T_{e\|}$. When $E_{p\|} < 10^8 K$(10keV), $\tau_1 < \tau_2$ and the relaxation is limited by the process $T_{e\|} \to T_{e\perp}$.

## VI. CONCLUDING REMARKS

The analysis on the relaxation between the parallel and perpendicular energies of electrons have been done both theoretically and by numerical simulation and the result is shown to reproduce experiments in Malmberg trap. The transfer of antiproton energy to electron is partly confirmed by our numerical simulation. The analysis on the equilibrium distribution of each component in the trap and the relaxation processes in the real space are now in progress.

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