Price Determination Method Based on Price Elasticity

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(Received February 25, 1997)

In this paper, we propose the price determination method using the parameter of the price elasticity that shows the relation between price and demand. Firstly, the state of the price elasticity is examined under the condition that the relation between price and demand are assumed by the inverse proportional function, the linear function and the quadratic function. Secondly, the profit is estimated for each product by the break even point analysis. And the price is determined under the condition that the relation between the demand and price is shown by one of three demand–price functions above mentioned.

1. INTRODUCTION

The price determination plays an important role in the management strategy to yield a larger profit and to increase the market share. Many authors have been forecasted on the demand,[1][2][3][4]. In these researches, the amount of quantity was mainly improved to maximize the profit under the condition that the price was fixed and due data was given. Recently the price of product changes depending on the consumers demand, and drastic price cut is required to compete in the market. However, there is no method for determining the price under these circumstances.

In this paper, we propose the price determination method based on the price elasticity function [5] and the break even point analysis under the condition that the demand–price function is given. [6] The price elasticity function is the index of the relation between price and demand. Firstly, we formulate the price elasticity function in three cases, that is, inverse proportional function, linear function and quadratic function and evaluate their characteristics precisely. Next, we estimate the profit of each product comparing with break even point and the price is determined to bring the profits.

2 PROPOSED METHOD

2.1 Symbols

We use the following symbols for determining the price based on the relation between price and demand.

$S_i$: The product's series, $i = 1, 2, \ldots, k$

$DC_i$: Limited price showing price zone of series $S_i$

$C_{ij}$: Price of products in series $S_i, j = 1, 2, \ldots, n$

$D_i$: Fixed cost for the products in series $S_i$

$V_i$: Variable cost for the products in series $S_i$

$Q_i$: Demand of products in series $S_i$ : $Q_i = f(C_{ij})$

$\eta_i$: Price elasticity function of the product

2.2 Restrictions

Until now, the relation between demand and price varies fairly in wide area, but now it is changing as follows. Firstly, the price of product is classified into some series ($S_i$). Secondly, the price in series ($S_i$) varies in the price zone ($DC_i$) showing in Fig. 1. In each series ($S_i$), the order of price is $C_{1i} \leq C_{2i} \leq \cdots \leq C_{ni}$, and we assumed that the variable cost ($V_i$) and the fixed cost ($D_i$) of the product are constant.

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Fig. 1 Demand–price function in each series
Further we assumed that the demand–price function (Q=f(C)) is given. The price is determined based on Price Elasticity Function (PEF) and the break even point analysis.

2.3 The Demand–price Function
In this study, three types of demand–price function are assumed. The inverse linear model is the most famous one, and the demand varies inversely for price (Fig.2). The linear model is a simple relation, and the demand varies linearly (Fig.3). For the quadratic function, the demand shows the maximum or minimum in the price zone (Fig.4).

2.4 Price Elasticity Function (PEF)
In the latter, the suffix i of variable is omitted to show simply the equation. The price elasticity function (PEF) can be shown as equation (1).

\[ \eta = \frac{dq}{dC} \frac{C}{Q} \]

PEF is the parameter of the sensitivity of change of the demand per unit price (Q/C) at (Q/C). As the demand is increased (decreased) for the price, PEF (\( \eta \)) is positive (negative). The price elasticity function is calculated to three functions.

(1) Case 1: Inverse proportional function (Q=a/C, a>0)
As the price is high/low, the demand becomes low/high. This is a natural phenomenon in the production. In this case, PEF (\( \eta \)) in the series (S.) becomes constant as \(-1\) by the equation (2).

\[ \eta = \frac{dq}{dC} \frac{C}{Q} = -1 \]

(2) Case 2: Linear function (Q=aC + b)
As the price is equal to zero, the demand becomes infinity by the inverse proportional function. From the view point of customer satisfaction, this phenomenon is not natural. So, the relation between price and demand is considered under the condition that the demand is constant, as price is equal to zero. As the relation between C and Q is given by the linear function, PEF (\( \eta \)) in the series (S.) is given by equation (3).

\[ \eta = -b/(aC+b) \]

(a) \( a>0 \) and \( aC + b > 0 \)

Fig. 6–(i) shows the relation between the price and \( \eta \). The value of \( \eta \) is always positive, as the price is increased. In the “Bubble period in Japan” from 1985–1988 [7], as the price of land was increased, the demand was also increased. This phenomenon is explained by this equation. \( \eta \) is
always positive, and as the price C is increased infinity, the limited value of η approaches to 1. In the price zone [DC → DC], η increased monotonously from η → η to η → η. Where η → η and η → η are calculated by the equation (4).
\[
\eta = \frac{1 - b}{(aDC - b)}
\]
\[
\eta = \frac{1 - b}{(aDC + b)}
\]

(b) a<0 and aDC → 0 < b>0

As the price is lowered, the demand is increased conversely. This relation is shown in Fig.6→(i). This price is belonged in the range of 0 ≤ η ≤ b. As the demand—price function is decreased, η is negative and belongs in the area in η → η → η.

(3) Case3: Quadratic function Q=a(C - b)² + d (DC → b ≤ DC → b

As the price is increased, the demand is changed the state, that is, decreasing from increasing or vice versa at the price b. This situation is modelled by the quadratic equation of Q=a(C - b)² + d. PEF (η) is given by the equation(5).
\[
\eta = \frac{2a(C - b)}{a(C - b)² + d}
\]
And the range of η is dependent on the parameters of a and d. For the price values DC → and DC →, η → and η → are calculated by the equation (6).
\[
\eta = \frac{2a(DC - b)}{a(DC - b)² + d}
\]
(a) a>0 and d>0
1. (b)2 ≤ DC →

η is increased monotonously from η → η to η → η in the area of price zone [DC → DC → DC → ] in Fig.7→(i). And η is negative, as C is less than b and η is positive, as b ≤ C ≤ DC →.
2. DC → <(b)2<DC →

The numerator of the equation (5) is attained the minimum value at C=b/2 and η → is put as the value of η at C=b/2 by the equation (7),(Fig.7→(i)).
\[
\eta = \frac{2a(C - b)(DC - b)}{a(DC - b)² + d}
\]
η is decreased from η → η to η → η as C is increased from DC → to b/2, and η is increased monotonously η → η to η → η, as C is greater than b/2.

(b) a<0, a(DC → - b)² + d>0, a(DC → - b)² + d>0, and DC → ≤ b ≤ DC →.
1. (b)2<DC →

η is positive in the area of [DC → DC]. And η is decreased from η → η (η → η >0) to 0. Further η is negative and decreased from 0 to η → η (η → η <0), as C is increased from b to DC →.
2. DC → ≤ (b)2 ≤ DC →

η is positive in the area of [DC → b]. But η is increased from η → η to η → η, as C is increased from DC → to (b)2. And η is decreased monotonously from η → η to η → η (<0), as C is greater than (b)2.
Table 1 shows the range of $\eta$ in each demand–price function.

<table>
<thead>
<tr>
<th>Function</th>
<th>PEF $\eta$</th>
<th>Range of $\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q=a/C$</td>
<td>$\eta=1-b(aDC+b)$</td>
<td>$a&gt;0$, $a(DC-1+b)&gt;0$ $0&lt;\eta \leq \eta_{20}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$a&lt;0$, $a(DC-1+b)&gt;0$ $0&gt;\eta \leq \eta_{20}$</td>
</tr>
<tr>
<td>$Q=a(C-b)^2+d$</td>
<td>$\eta=\frac{2aC(C-b)}{(a-C)^2+d}$</td>
<td>$a&gt;0$, $b&gt;0$ $0&lt;\eta \leq \eta_{20}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(b/2)&lt;DC-1$ $\eta_{10} \leq \eta \leq \eta_{20}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$a&lt;0$, $b&gt;0$ $0\leq \eta \leq \eta_{20}$</td>
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<td>$(b/2)&lt;DC-1$ $\eta_{10} \leq \eta \leq \eta_{20}$</td>
</tr>
</tbody>
</table>

2.5 Cost Determination Method

We can estimate the profit of each product by break even point analysis. The amount of break even point (TP) is given by the equation (8).

\[
TP-b/(C-V) \tag{8}
\]

If $Q$ is greater than TP, the product brings the profits and if $Q$ is less than TP, we can't yield a profit. So we should change the price or fixed/variable cost.

1. Case 1: $Q=aC$, $a>0$

If it is satisfied that \{aC-(D/(C-V))>0, the profit can be brought, and we can get the limit price $C_\circ$ by the equation (9).

\[
C_\circ=aV/(a-b) \tag{9}
\]

1. $V \leq DC_{1-1}$

(a) $C_\circ < DC_{1-1}$

As price $C_\circ$ is less than $DC_{1-1}$, we can determine the price at any value in the price zone $[DC_{1-1}, DC_\circ]$.

(b) $DC_{1-1} \leq C_\circ \leq DC_\circ$

The price area is restricted to $[C_\circ, DC_\circ]$. So the price of a product should be determined in this area.

2. $DC_{1-1} \leq V \leq DC_\circ$

The price area is determined as $[C_\circ, DC_\circ]$ if $C_\circ$ is less than $DC_\circ$. As the price area $[C_\circ, DC_\circ]$ becomes narrow, so we make effort to decrease the variable or fixed cost. In the other case, we can not determine the price area for the series.

2. Case 2: $Q=aC+b$

(a) $a>0$ and $aDC_{1-1}+b>0$

There are two prices ($C_{1-0}$ and $C_{2-0}$) satisfying $D/(C-V)=aC+b$ and they are given by equation (10).

\[
C_{1-0}=\frac{(aV-b)+\sqrt{DD}}{2a} \tag{10}
\]

\[
C_{2-0}=\frac{(aV-b)-\sqrt{DD}}{2a}
\]

Where, $DD=(aV+b)^2-4aD$

And these prices satisfy the following relation.

$C_{1-0} < V < C_{2-0}$

1. $V \leq DC_{1-1}$

(i) $C_{2-0} < DC_{1-1}$: The price zone $[DC_{1-1}, DC_\circ]$ is used to determine the price of a product.

(ii) $DC_{1-1} \leq C_{2-0} \leq DC_\circ$: As the price $C$ is greater than $C_{2-0}$, we can yield the profit. But in this case, the price area $[C_{2-0}, DC_\circ]$ becomes narrow, and the price should be determined highly.

![Fig 8 Price area of Q=aC+b](image-url)
(2) DC$_{10}$ < $V < $DC$_{20}$.

If C$_{20}$ is less than DC$_{10}$, the price can be determined in the narrow area of [C$_{20}$, DC$_{10}$].

(b) a<0 and aDC$_{10}$ + b>0.

In this case, we have two prices (C$_{10}$, C$_{20}$) under the condition that DD in equation (10) is positive. And V $\leq$ C$_{10}$ $\leq$ C$_{20}$ is always satisfied. The price area can be determined in the following cases.

1) V<DC$_{11}$

(i) If C$_{10}$ < DC$_{11}$ and DC$_{11}$ < C$_{20}$, then the price is determined in the price zone [DC$_{11}$, DC$_{10}$].

(ii) If DC$_{11}$ < C$_{10}$ < C$_{20}$, then the price area is determined as [C$_{10}$, DC$_{11}$].

(iii) If DC$_{11}$ < C$_{10}$ < C$_{20}$ < DC$_{10}$, the price area is put as [C$_{10}$, C$_{20}$].

2) DC$_{11}$ < V<DC$_{10}$.

As the relation of V $\leq$ C$_{10}$ $\leq$ C$_{20}$ is satisfied, then the price area becomes [C$_{10}$, DC$_{10}$] or [C$_{10}$, C$_{20}$].

(3) Case 3: Q=a(C-b)$^2$ +d, DC$_{11}$ $\leq$ b $\leq$ DC$_{10}$

(a) a>0 and d>0:

The equation a(C-b)$^2$ +d=D/(C-V) has one or two or three roots. These roots are greater than V.

Case of one root (C$_{12}$): V and C$_{12}$ are compared with DC$_{11}$ and DC$_{10}$. The price area is one of [DC$_{11}$, DC$_{10}$] or [C$_{10}$, DC$_{11}$].

Case of two roots (C$_{10}$, C$_{20}$): Q=a(C-b)$^2$ +d and TP=D/(C-V) contacted at C$_{20}$. The price area is determined as one of [DC$_{11}$, DC$_{10}$] or [C$_{10}$, DC$_{11}$].

Case of three roots (C$_{10}$, C$_{20}$, C$_{30}$):

We compare C$_{10}$ and C$_{20}$, C$_{30}$ with V, DC$_{11}$ and DC$_{10}$, and determine the price area which satisfies the relation of Q<TP.

(b) a<0, a(DC$_{11}$ - b)$^2$ +d>0, and a(DC$_{11}$ - b)$^2$ +d>0

There is always one root C$_{12}$ (<V) in any case. But this root cannot be used for the price determination. So the price area is determined, in the case that the equation Q=TP has three roots (C$_{10}$, C$_{20}$, C$_{30}$).

These roots satisfy the relation C$_{10}$ < V < C$_{20}$ < C$_{30}$. As we compare two roots C$_{20}$ and C$_{30}$ with V, DC$_{11}$ and DC$_{10}$, the price area that satisfies the relation of Q<TP is determined as one of [DC$_{11}$, DC$_{10}$], [C$_{20}$, DC$_{10}$] or [C$_{20}$, C$_{30}$].

3. EXAMPLE

We apply the proposed method to determine the price of automobiles. There are four series (S$_1$, S$_2$, S$_3$, S$_4$) of automobiles. The price and demand in each series is given by Fig.10.

(1) Demand–price function in each series. Demand–price function in each series is given as follows

S$_1$: Q=0.3C-20 $\times$ 10$^4$ : Price zone: (DC$_1$ =100 $\times$ 10$^4$ $\$, DC$_1$ =150 $\times$ 10$^4$ $\$
S$_2$: Q=(-0.0024(C/10$^4$-220)$^2$+30) $\times$ 10$^4$ : Price zone: (DC$_2$ =150 $\times$ 10$^4$ $\$, DC$_2$ =250 $\times$ 10$^4$ $\$
S$_3$: Q=0.07C-0.5 $\times$ 10$^4$ : Price zone: (DC$_3$ =250 $\times$ 10$^4$ $\$, DC$_3$ =350 $\times$ 10$^4$ $\$
S$_4$: Q=-0.07C+44.5 $\times$ 10$^4$ : Price zone: (DC$_4$ =350 $\times$ 10$^4$ $\$, DC$_4$ =400 $\times$ 10$^4$ $\$

(2) The price area in each series can be determined as follows.

Series 1(S$_1$): The variable cost is equal to V =90 $\times$ 10$^4$ $\$ and the fixed cost D$_1$ is 260 $\times$ 10$^4$ $\$

As the demand–price function is linear and parameter a is positive, we calculate only one root C$_{20}$ by the equation (10) as follows. The profit will be yielded as the price is belonged in the price area of [110 $\times$ 10$^4$, 150 $\times$ 10$^4$ $\$].
The variable cost is equal to $V = 120 \times 10^4 \$\), and the fixed cost $D_s$ is $290 \times 10^4 \$\).

As $a = -0.0024$ is negative, one root $C_s \approx (150 \times 10^4 \$, 250 \times 10^4 \$)$ is always existed. $TP = (290 \times 10^4 )/C = 120 \times 10^4 $ in the price zone of $[150 \times 10^4 \$, 250 \times 10^4 \$]$. $Q = (-0.0024(C - 220)^2 + 30) \times 10^4$ is greater than $18.4 \times 10^4$ in price zone of $[150 \times 10^4 \$, 250 \times 10^4 \$]$. Then $Q$ is greater than $TP$ in this price zone. So the two roots of $Q = TP$ are satisfied the following conditions:

$$V = 120 \times 10^4 < C_s < DC_2 = 150 \times 10^4 \$, \text{ and } DC_3 = 250 \times 10^4 \$. $C_s \approx (150 \times 10^4 \$, 250 \times 10^4 \$)$.

From these results, the price area is determined as $[150 \times 10^4 \$, 250 \times 10^4 \$]$. 

Series 3 (S$_3$): In this series, the variable cost is equal to $V = 230 \times 10^4 \$, and the fixed cost $D_3$ is $310 \times 10^4 \$. $C_3$ is less than $250 \times 10^4 \$, than we can yield the profit, as the price is belonged in the price zone of $[250 \times 10^4 \$, 350 \times 10^4 \$]$. 

$$\sqrt{DD} = 18.17 \times 10^6 \quad C_m = 248 \times 10^4 \$. 

Series 4 (S$_4$): In this series, the variable cost is equal to $V = 280 \times 10^4 \$, and the fixed cost $D_4$ is $370 \times 10^4 \$. The profit will be brought, as the price is belonged in the area of $[350 \times 10^4 \$, 372 \times 10^4 \$. 

$$\sqrt{DD} = 22.72 \times 10^6 \quad C_m = 296 \times 10^4 \$, 620 \times 10^4 \$. 

### Conclusion

In this paper, we proposed the price determination method based on the price elasticity function and the break even point analysis under the condition that the demand–price function is given. The price elasticity function is the index of the relation between the price and demand. Firstly, we formulated the price elasticity function in three cases of inverse proportional function, linear function and quadratic function and evaluated their characteristics precisely. Next, we estimated the profit of each product comparing with break even point and the price is determined to bring the profit.

### References