Transient Analysis of Two-Phase Induction Motor driven by Voltage Source Inverter with Current Limiter

Shigeyuki FUNABIKI*, Masakatsu IYASU*, Tsutomu KAMURA* and Toyoji HIMEI*

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Synopsis

In this paper, the analysis of transient performance of two-phase induction motor driven by a voltage source inverter with current limiter is described. The motor is represented by using a two-axis model, that is, the Kron primitive machine. The state equations of mechanical-electrical system are derived. The calculation method with a discrete point of time is employed in order to decrease the CPU time of computer. The calculated results gained from this method agree well with the measured. Then, with the aid of this analytical method, the discussions on transient performance of this system are also performed.

1. Introduction

An induction motor has many strong points, e.g. a simpler and firmer construction, and then easy maintenance, compared with a DC motor. Its variable speed control system by means of a variable frequency power converter is going to take the place of DC motor driving system. From a view of saving energy, it has attracted special interest recently1).

A two-phase induction motor, e.g. a condenser motor, is available for a small capacity one. However, when a two-phase induction motor is driven by a voltage source inverter, a peak waveform of current
flows through the inverter circuit and motor windings. Therefore, for
the sake of protection of power transistors in the inverter circuit
and reduction of capacity of its driving circuit, it is necessary to
suppress a peak value of current. Then, the effects of current limiter
on the steady state characteristics have been discussed\(^2\). In this
Literature, the limit of peak value of current is found to be
effective for not only the countermeasure mentioned above but also
the improvement of motor characteristics in the steady state. And
also, it is necessary to discuss the effect of current limiter on
the transient performance when the motor speed is varied.

In this paper, the transient performance analysis of two-phase
induction motor driven by a voltage source inverter with current
limiter is described. In this analysis, the motor is represented by
the Kron primitive machine\(^3\), and the state equations of mechanical-
electrical system are derived. Thus, the calculated results closely
agree with the measured. Therefore, this analysis method is found to
be very useful for the discussion of transient performance. Then,
with the aid of this analytical method the transient performance is
discussed.

2. Driving System of Two-Phase Induction Motor

2.1 Voltage Source Inverter

The voltage source inverter driving a condenser motor as a two-
phase induction motor is shown in Fig.1. Two square waveform voltages
with \(\pi/2\) displacement are impressed to the main winding and the
auxiliary winding of this motor, respectively. At the same time the
thyristors are fired in order that \(V_m/f\) may be constant. Where, \(V_m\)
is the voltage of the main winding and \(f\) is the inverter frequency.
The current limiter in this system limits the peak value of currents
through the main winding and the auxiliary winding, and each limiting
value is independent.

2.2 Control Method

The control block diagram in this system is shown in Fig.2. The
inverter frequency is decided by the speed command signal. The
transistor base signals of main and auxiliary winding are with \(\pi/2\)
displacement.

As mentioned above, the current limiter limits the peak value of current so as to be constant. Namely, when the current reaches its limiting value, the transistors in the lower arm of inverter circuit are open during a definite period of time (\(=\Delta t_{off}\)) and during this period the current circulates through a motor winding, diodes, and a transistor. Then, the current value is detected by the shunt resistor. The thyristors in the converter circuit are fired so that \(V_m/f\) may be constant.
3. Performance of Motor Driving System

3.1 Performance of System

The voltage and current waveforms in this system are shown in Fig.3. The motor available here is a condenser motor and has the performance of unbalance two-phase induction motor. To make its performance clear, the idealized waveforms shown in Fig.4 are considered. In this figure each value in the motor windings is as follows:

\[ v_D = v_C + 2V_{D\text{drop}}, \]
\[ v_W = v_C - 2(V_{T\text{drop}} + V_{D\text{drop}}), \]
\[ v_L = V_{T\text{drop}} + 2V_{D\text{drop}}. \]

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**Fig.3. Waveforms.**

**Fig.4. Idealized Waveforms.**
where, \( V_C \) is the condenser voltage, \( V_{T r \text{drop}} \) and \( V_{D \text{drop}} \) are the forward voltage drops in transistor and diode, respectively.

After the transistors \( T_{r2} \) and \( T_{r4} \) are closed at time \( a \), the current through the motor winding increases. When the current reaches the current limiting value (time \( b \)), the transistor \( T_{r4} \) is open. However, the transistor \( T_{r1} \) remains closed, and then the current circulates through a motor winding, diodes, and a transistor (between time \( b \) and \( c \)). The off time of transistor \( T_{r4} \), \( \Delta t_{\text{off}} \) is fixed to be constant. Furthermore, the transistor \( T_{r4} \) is closed again at time \( c \) and then the current increases. After then, the operations mentioned above are continued till time \( f \) and the peak value of current is limited to be constant.

The transistors \( T_{r1} \) and \( T_{r4} \) are open at time \( f \). The current circulates through a motor winding, diodes, and a condenser and decreases to be zero at time \( g \).

After time \( g \) the motor winding is electrically separated from this circuit and the induced voltage from the rotor occurs across the motor winding.

3.2 States of Inverter Circuit

In this section the states of inverter circuit are clarified according to the performance of two-phase induction motor mentioned in the previous section. The circuit in the main winding has four sorts of state on account of transistor switching.

![Fig.5. Classification of Circuit Operating States.](image)
A: Transistors $T_{p1}$ and $T_{p4}$ (or $T_{p2}$ and $T_{p3}$) are closed.
B: The current limiter operates, that is, a transistor $T_{p4}$ (or $T_{p3}$) is open.
C: All transistors are open and the current flows through diodes.
D: The current doesn't exist.
In the auxiliary winding circuit four states exist as well as the main winding. These states are illustrated in Fig.5.

4. Analysis of Transient Performance

4.1 Equivalent Circuit and State Equation of Two-Phase Induction Motor

The equivalent circuit of two-phase induction motor is shown in Fig.6. As illustrated the cage rotor is replaced on $d$-$q$ axis with the balanced two-phase windings. Then, the performance of this circuit is divided into four modes in accordance with the main winding current and the auxiliary winding current. The equation in each mode is as follows:

(1) MODE I ($i_m \neq 0$, $i_a \neq 0$)

\[
\begin{bmatrix}
  v_m \\
  v_a \\
  0
\end{bmatrix}
= \begin{bmatrix}
  R_m + L_m p & 0 & M_d p & 0 \\
  0 & R_a + L_a p & 0 & M_q p \\
  M_d \omega & -M_q \omega & R_r + L_r p & -L_r \omega \\
  0 & M_q \omega & L_r \omega & R_r + L_r p
\end{bmatrix}
\begin{bmatrix}
  i_m \\
  i_a \\
  i_d \\
  i_q
\end{bmatrix}
\] (4)

(2) MODE II ($i_m = 0$, $i_a \neq 0$)

\[
\begin{bmatrix}
  v_a \\
  0 \\
  0
\end{bmatrix}
= \begin{bmatrix}
  R_a + L_a p & 0 & M_q p \\
  -M_q \omega & R_r + L_r p & -L_r \omega \\
  M_q p & L_r \omega & R_r + L_r p
\end{bmatrix}
\begin{bmatrix}
  i_a \\
  i_d \\
  i_q
\end{bmatrix}
\] (5)
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The induced voltage from the rotor is as follows:

\[ e_m = M_d (p i_d). \]

(3) MODE III \((i_m \neq 0, i_a = 0)\)

\[
\begin{bmatrix}
\nu_m \\
0 \\
0
\end{bmatrix} =
\begin{bmatrix}
R_m + L_m p & M_m p & 0 \\
M_a p & R_p + L_p p & -L_\omega \\
M_d p & L_\omega & R_p + L_p p
\end{bmatrix}
\begin{bmatrix}
i_m \\
i_d \\
i_a
\end{bmatrix}.
\]

The induced voltage from the rotor is as follows:

\[ e_a = M_q (p i_q). \]

(4) MODE IV \((i_m = i_a = 0)\)

\[
\begin{bmatrix}
0 \\
0
\end{bmatrix} =
\begin{bmatrix}
R_p + L_p p & -L_\omega \\
L_\omega & R_p + L_p p
\end{bmatrix}
\begin{bmatrix}
i_d \\
i_q
\end{bmatrix}.
\]

Where, \( p=\frac{d}{dt}, \omega; \) an angular velocity in electrical angle, \( n_\omega n_\omega; \) the number of pairs of poles, \( \omega_n; \) a rotational angular velocity.

4.2 State Equation in Transient

The torque produced in this motor is defined as follows:

\[ T_M = n (M_d i_m i_q - M_a i_a i_d). \]

(8)

Furthermore, this produced torque \( T_M \) is related to the load torque \( T_L \) and the inertia moment \( J. \)

\[ T_M = J d\omega_n / dt + T_L. \]

(9)

Using a DC separately excited generator as a load, the load torque \( T_L \) is expressed as follows:

\[ T_L = (a\omega_n - b)/(R_A + R) + T_{LO}. \]

(10)

Where, \( a \) and \( b \) are constants, \( R_A; \) an armature resistance, \( R; \) a load
resistance, and $T_{LO_1}$ a torque on no-load.

The state equations of mechanical-electrical system in the transient state are derived from the equations (4)-(10) as follows:

(1) MODE I ($i_m \neq 0, i_a \neq 0$)

\[
\begin{align*}
\frac{d}{dt} \begin{bmatrix} i_m \\ i_a \\ i_d \\ i_q \\ v_m \\ v_a \\ \omega_r \\ T_{LO_1} \end{bmatrix} &= \begin{bmatrix}
-L R_m / k_1 & L M q / k_2 & M d R_r / k_1 & -M d L_r / k_1 \\
M d M_q / k_2 & -R_r R_a / k_2 & L M q / k_2 & M q R_r / k_2 \\
M d R_m / k_1 & L M q / k_1 & -L M R_r / k_2 & M L r / k_1 \\
-L M a / k_2 & M q a / k_2 & L M q / k_2 & -L M R_r / k_2
\end{bmatrix} \begin{bmatrix} i_m \\ i_a \\ i_d \\ i_q \end{bmatrix} \\
\begin{bmatrix} v_m \\ v_a \\ \omega_r \\ T_{LO_1} \end{bmatrix} &\begin{bmatrix} 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_m \\ i_a \\ i_d \\ i_q \end{bmatrix} \begin{bmatrix} \omega_r \\ v_m \\ v_a \\ T_{LO_1} \end{bmatrix}
\end{align*}
\]

(11)

(2) MODE II ($i_m = 0, i_a \neq 0$)

\[
\begin{align*}
\frac{d}{dt} \begin{bmatrix} i_a \\ i_d \\ i_q \\ v_a \\ \omega_r \\ T_{LO_1} \end{bmatrix} &= \begin{bmatrix}
-L R_a / k_2 & L M q / k_2 & M q R_r / k_2 \\
M q a / k_2 & -R_r R_a / k_2 & \omega \\
M q R_r / k_2 & -L a R_r / k_2 & -L a R_r / k_2
\end{bmatrix} \begin{bmatrix} i_a \\ i_d \\ i_q \\ v_a \\ \omega_r \\ T_{LO_1} \end{bmatrix}
\end{align*}
\]
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\[
\begin{pmatrix}
L_r/k_2 & 0 & 0 \\
0 & 0 & 0 \\
M_q/k_2 & 0 & 0 \\
0 & 0 & 0 \\
0 & -k_2/J & -1/J \\
0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
i_a \\
i_d \\
i_q \\
v_a \\
\omega_r \\
T_{L01}
\end{pmatrix}
= \begin{pmatrix}
\frac{d}{dt} \begin{pmatrix}
i_m \\
i_d \\
i_q \\
v_m \\
\omega_r \\
T_{L01}
\end{pmatrix}
\end{pmatrix}
\]

(12)

(3) MODE III \( (i_m \neq 0, i_a = 0) \)

\[
\begin{pmatrix}
-\frac{L_r R_m}{k_1} & -\frac{M_d R_r}{k_1} & -\frac{M_d L_r \omega}{k_1} \\
\frac{M_d R_m}{k_1} & \frac{L_m R_r}{k_1} & \frac{L_m L_r \omega}{k_1} \\
0 & 0 & 0 \\
\frac{M_d \omega}{L_r} & -\omega & -\frac{R_r}{L_r} \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
i_m \\
i_d \\
i_q \\
v_m \\
\omega_r \\
T_{L01}
\end{pmatrix}
= \begin{pmatrix}
\frac{d}{dt} \begin{pmatrix}
i_m \\
i_d \\
i_q \\
v_m \\
\omega_r \\
T_{L01}
\end{pmatrix}
\end{pmatrix}
\]

(13)

(4) MODE IV \( (i_m = i_a = 0) \)

\[
\begin{pmatrix}
\frac{d}{dt} \begin{pmatrix}
i_d \\
i_q \\
\omega_r \\
T_{L01}
\end{pmatrix}
\end{pmatrix}
= \begin{pmatrix}
-\frac{R_r}{L_r} & \omega & 0 & 0 \\
-\omega & -\frac{R_r}{L_r} & 0 & 0 \\
0 & 0 & -\frac{k_2}{J} & -1/J \\
0 & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
i_d \\
i_q \\
\omega_r \\
T_{L01}
\end{pmatrix}
\]

(14)

Where, \( k_1 = L_r L_m - M_a^2 \), \( k_2 = L_r L_a - M_a^2 \), \( k_3 = a/(R_A + R) \), \( T_{L01} = T_{L0} - b/(R_A + R) \).
4.3 Assumption in Analysis and Analytical Procedure

The assumptions used in this analysis are as follows:

(1) Electrical system
   (1-1) The capacitance voltage in the converter circuit is a constant DC one.
   (1-2) The forward voltage drop in transistors and diodes is constant.
   (1-3) The transistor is an ideal switching element.

(2) Dynamic system
   (2-1) The motor constants and the elements of load are fixed.
   (2-2) The iron loss is neglected.
   (2-3) The torque from friction is neglected, and the torque on no-load is constant without the motor speed.
   (2-4) The brush voltage drop of the DC generator is constant.

The frequency command in this system is expressed as follows:

\[ f = f_{up} + (f_0 - f_{up}) e^{-\frac{t}{\tau}}. \]  \hspace{1cm} (15)

Where, \( f_0 \) and \( f_{up} \) are the frequencies before and after the speed variation, and \( \tau \) is a time constant of the control circuit.

In this analysis the next method is taken in order to decrease the CPU time of computer. Now, it is assumed that at time \( t_i \) the period of the inverter is \( \tau_i \), and the motor torque and the load torque are \( T_{Mi} \) and \( T_{Li} \). Furthermore, the time of next calculation is assumed to be \( t_j \). If there are assumed to be \( n_k \) half cycles between \( t_i \) and \( t_j \), the next equations are valid.

\[
\begin{align*}
\omega_{r_{i+1}} &= \frac{1}{j} \left[ \frac{\tau_i}{2} T_i dt + \omega_{ri} \right], \\
\omega_{r_{i+2}} &= \frac{1}{j} \left[ \frac{\tau_{i+1}}{2} T_{i+1} dt + \omega_{r_{i+1}} \right], \\
& \quad \vdots \\
\omega_{r_{k+1}} &= \frac{1}{j} \left[ \frac{\tau_k}{2} T_k dt + \omega_{r_{k-1}} \right], \\
& \quad \vdots \\
\end{align*}
\]  \hspace{1cm} (16)
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\[ \omega_{rj} = \frac{1}{J} \int_0^{\frac{\tau}{J}} T_j dt + \omega_{rj-1} \cdot \]

Where, \( T_k \) is an average torque in a cycle, \( T_k = T_{Mk} - T_{Lk} \), and \( \tau_k \) is a period of \( k \)th cycle. From the equation (16), the next relation is obtained,

\[ \omega_{rj} = \omega_{rj-k} = (n_{k-1})/J \int_0^{\frac{T}{\tau}} Tdt + \omega_{rj} \cdot \]

(17)

Where, \( \tau \) is an average period of one cycle, \( T \) is an average torque between \( t_i \) and \( t_j \).

If it is assumed that \( \tau \) equals to \( \tau_j \) and \( T \) equals to \( T_j = T_{Mi} - T_{Li} \), the equation (14) is rearranged as follows:

\[ \omega_{rj} = 2(t_j - t_i)/J \int_0^{\frac{\tau}{J}} T_i dt + \omega_{rj} \cdot \]

(18)

5. Analyzed Results

5.1 Comparison of the Calculated and the Measured

Fig. 7 shows the responses in the motor speed and the load torque \( T_L \) with the measured and the calculated when the inverter frequency

<table>
<thead>
<tr>
<th>Table 1. Circuit Constants.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_m )</td>
</tr>
<tr>
<td>( L_m )</td>
</tr>
<tr>
<td>( M_d )</td>
</tr>
<tr>
<td>( R_A )</td>
</tr>
<tr>
<td>( L_A )</td>
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<tr>
<td>( M_q )</td>
</tr>
<tr>
<td>( R_r )</td>
</tr>
<tr>
<td>( L_r )</td>
</tr>
<tr>
<td>( R_A )</td>
</tr>
<tr>
<td>( J )</td>
</tr>
</tbody>
</table>
is varied from 60Hz to 80Hz at $\tau=3.5$. From this figure both results are coincident well, and this analytical procedure is found to be appropriate.

In this calculation, however, the fluctuation in motor speed is observed in several per cent when the inverter frequency reaches $f_{up}$. The cause of this performance is considered that the torque during the acceleration of motor speed is estimated to be large by using the approximation in the equation (17) when $df/dt \approx 0$. In order to decrease this fluctuation in motor speed, the calculation period is set to be small when $df/dt = 0$. However, this technique results in the increase of CPU time. Therefore, it is not on practice in this analysis.

The circuit constants in this system are shown in Table 1.

5.2 Transient Characteristics

Fig.8 shows the transient response in the motor speed when the inverter frequency is varied from 60Hz to 80Hz. Where, duty $N=50$ per cent, duty $A=65$ per cent, and the slip in 60Hz is 0.07. As shown in this figure the accelerative response without the current limiter is faster than that with the current limiter. So, it is shown that the motor cannot be accelerated with the current limiter when $\tau$ is smaller than 2.5. On the other hand the motor can be accelerated even at $\tau=0.5$ without the current limiter.

Fig.9 shows the variation of motor slip in Fig.8. As the time

![Figure 8: Effect of $\tau$ on Transient Response in Motor Speed.](image)

![Figure 9: Transient Response in Slip.](image)
constant \( \tau \) becomes small, the motor slip increases because of the fast variation of the inverter frequency. In this figure, the motor slip is nearly constant at \( \tau = 3.5 \). Fig.10 shows the torque response with and without the current limiter at \( \tau = 1.0 \) and 0.5, respectively.

Consequently, it is necessary that the motor torque is large enough to be able to accelerate the motor, especially when the motor slip is large. That is, the maximum torque is to be large enough for the motor to be accelerated. However, because the peak value of current is limited in this system, the maximum torque becomes small. From a viewpoint of the transient response the system is undesirable to have the current limiter. Namely, in order to decrease the transient time, the current limiting value and the duty ratio must be as large as possible. For example, the transient response in motor speed with some current limiting values are shown in Fig.11. From this figure it is found that the transient time with \( I_{mL} = 10 \text{(A)} \) and \( I_{aL} = 12 \text{(A)} \) is smaller than that with \( I_{mL} = I_{aL} = 10 \text{(A)} \).

6. Conclusions

The analytical procedure of the speed control of two-phase induction motor driven by the voltage source inverter with current limiter is described. Then, with the aid of this analysis the effects of the limiting value, the duty ratio, and the time constant of control circuit on the transient characteristics of this system
are investigated. These results are summarized as follows:
(1) The two-phase induction motor is represented by the Kron primitive
machine. The state equations of this mechanical-electrical system
are derived. Then, the calculated results of the transient response
by this calculation agree well with the measured.
(2) Due to the reduction of the maximum torque the transient response
time of the system with current limiter becomes longer than that
without current limiter.
(3) If the duty ratio is set smaller, the transient response time
becomes longer because of the reduction of the maximum torque, that
is, as well as the lowering of the limiting value.
(4) In case of the reduction of the time constant of control circuit
in the system with current limiter, the motor revolution cannot
follow the variation of the inverter frequency and then the motor
stalls. Therefore for these countermeasures it is necessary to make
the limiting value and the duty ratio large.
(5) From the mentioned above, though the characteristics in the
steady state may be improved by limiting the current, from a view of
the transient response the system without current limiter is better.
Therefore, it is necessary to set the adequate value of them in the
steady state and the transient, respectively. In consequent, the
method obtained in this paper is useful for the evaluation of these
factors' effects on the transient performance and the decision of
their appropriate values.

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