Some Statistical Considerations on Window Width and Matching Stability of Images.

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SYNOPSIS

In stereo matching of images, sample cross-covariances are used commonly as a criterion for deciding whether matched points are truly conjugate. Hereupon window width is a serious parameters to dominate matching stability. This paper argues about relation of matching stability with window width in terms of statistical behavior of sample covariances. For simple circumstances of analysis auto-covariances of a single image are considered instead of cross-covariances of stereo ones. First the mean and variance of sample auto-covariances are derived with parameters, window width and positional lag. Secondly they are evaluated from the correlation function estimated on an aerial image under the assumption of ergodicity to observe how they vary according as two parameters vary. From this result a variation factor is proved usefull to estimate appropriate window width.

1. INTRODUCTION

The major subject in stereo plotting study is to find the efficient way of identify conjugate points on stereo photographs. Among the methods proposed in present, the fundamental one is so-called area matching: i.e., a correlation window and the corresponding search

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window are settled on the left and right images respectively, and then
the former is shifted inside the latter to find the maximum correlation
peak.\(1,2\) For area matching method correlation window width is one
of the most predominant parameters. Improperly small width tends to
cause mismatches because of deficient textures, whereas improperly
large window tends to cause coarse matchings because of correlation
deterioration due to terrain reliefs. Then how large window is
appropriate for stereo matching? The appropriate window width depends
both on the spectral distribution property of terrain reliefs and of
image densities. If the terrain is flat, it is evident that larger
window make matching more precise and stable. Therefore the distri­
bution of image densities regulates permissible lower limit of window
width, and the distribution of terrain reliefs regulates upper limit.\(3\)
This paper offers some indications toward the former problem.

We consider the images to be samples of 2-dimensional stationary
process. By Imposing a few more essential assumptions on
this, the statistics about sample cross-covariances are derived, which
are used commonly as a matching criterion. And the appropriate
window width will be suggested from a viewpoint of a variation factor.
However it should be noted that the stochastic properties of images
viewing natural scenes are generally too complex to apply the results
obtained through the discussion quantitatively to them.

2. MEAN AND VARIANCE OF SAMPLE CROSS-COVARIANCES

If the terrain is flat and the ground objects do not posses the
prominent property of directional reflectivity, then we can consider
auto-correlations of a single image instead of cross-correlations
of stereo ones. Hereafter we regard an image as a sample of 2­
dimensional stationary stochastic field.

The image field is assumed noiseless, and averaged to zero. As
seen in Fig.1 we consider two windows which is of the same size
\(L \times L\) and have relative positional lag \(\tau_1, \tau_2\) in \(t_1, t_2\) coordinate
system respectively. Let a density at \((t_1, t_2)\) be denoted by \(x(t_1,
 t_2)\), and a sample cross-covariance (SSC) is defined as thus;

\[
\begin{align*}
C &= \nu_{12} - m_1 m_2 \\
\nu_{12} &= \frac{1}{4L^2} \int \int x(t_1 + \tau_1, t_2 + \tau_2) x(t_1, t_2) dt_1 dt_2 \\
\end{align*}
\]
Some Statistical Considerations on Window Width and Matching Stability of Images.

\[ m_1 = \frac{1}{4L^2} \int \int x(t_1, t_2) dt_1 dt_2 \]
\[ m_2 = \frac{1}{4L^2} \int \int x(t_1+\tau_1, t_2+\tau_2) dt_1 dt_2. \]

Integration domain is the inside of each window. The mean and variance of SCC can be obtained by calculation of the mean and variance of \( v_{12} \), \( m_1 m_2 \) as follows.

Mean of \( v_{12} \); \( E(v_{12}) \)

\[ E(v_{12}) = \frac{1}{4L^2} \int \int E(x(t_1+\tau_1, t_2+\tau_2) x(t_1, t_2)) dt_1 dt_2. \]

Eq. 2 is reduced to

\[ E(v_{12}) = \frac{1}{4L^2} \int \int R(\tau_1, \tau_2) dt_1 dt_2 \]
\[ = R(\tau_1, \tau_2) \quad (3') \]

Fig. 1 Correlation windows

from definition of auto-correlation function \( R(\tau_1, \tau_2) \). For simplicity auto-correlation function \( R(\tau_1, \tau_2) \) is assumed separable in \( t_1 \) and \( t_2 \) directions, i.e.,

\[ R(\tau_1, \tau_2) = R_1(\tau_1) R_2(\tau_2). \]

Variation of \( v_{12} \); \( V(v_{12}) \)

\[ V(v_{12}) = E(v_{12}^2) - (E(v_{12}))^2 \quad (4) \]

For calculating \( E(v_{12}^2) \), the 4-th moment of \( x(t) \) should be known. In general this is desperately difficult. But if densities are
distributed to normal, the 4-th moment can be decomposed to the product sum of 2-nd moments, that is, if a multi-variate \( x = (x_1, x_2, x_3, x_4) \) is distributed to normal with zero mean,

\[
E(x_1 x_2 x_3 x_4) = \mu_{12} \mu_{34} + \mu_{13} \mu_{24} + \mu_{14} \mu_{23},
\]

\[ \text{(5)} \]

where \( \mu_{ij} = E(x_i x_j) \). Hence \( E(v_{12}^2) \) becomes

\[
E(v_{12}^2) = \frac{1}{(4L^2)^2} \int \int \int \int E(x(t_1 + \tau_1, t_2 + \tau_2) \cdot x(t_1, t_2) \cdot x(t_3 + \tau_1, t_4 + \tau_2) \cdot x(t_3, t_4)) dt_1 dt_2 dt_3 dt_4.
\]

\[ \text{(6)} \]

\[
v_{12}^2 = \frac{1}{(4L^2)^2} \int \int (R(t_1 + \tau_1, \tau_2) + R(t_1 - t_3, t_2 - t_4) + R(t_1 - t_3, -t_4) + R(t_1 + \tau_1, -t_3)) dt_1 dt_2 dt_3 dt_4.
\]

\[ \text{(7)} \]

\[
V(v_{12}) \text{ in Eq.} 4 \text{ can be expressed as follows according to Eq.} 6 \text{ and Eq.} 2' \text{ after a simple variable transformation:}
\]

\[
V(v_{12}) = I_{11}(0) I_{22}(0) + I_{11}(\tau_1) I_{22}(\tau_2)
\]

\[ \text{(7)} \]

subject to

\[
\begin{align*}
I_{11}(\tau) &= \int_{-1}^{+1} (1 - |u|)R_1(2Lu + \tau)R_1(2Lu - \tau) du \\
I_{22}(\tau) &= \int_{-1}^{+1} (1 - |u|)R_2(2Lu + \tau)R_2(2Lu - \tau) du
\end{align*}
\]

\[ \text{(8)} \]

Mean of \( m_1 m_2 : E(m_1 m_2) \)

\[
E(m_1 m_2) = \frac{1}{(4L^2)^2} \int \int \int \int E(x(t_1 + \tau_1, t_2 + \tau_2) \cdot x(t_3, t_4)) dt_1 dt_2 dt_3 dt_4
\]

\[
= \frac{1}{4L^2} \int R_1(t_1 - t_3 + \tau_1) dt_1 dt_3 \cdot \frac{1}{4L^2} \int R_2(t_2 - t_4 + \tau_2) dt_2 dt_4
\]

\[ \text{(9)} \]
subject to
\[
\begin{align*}
    I_1(\tau) &= \int_{-1}^{+1} (1 - |u|) R_1(2Lu+\tau) \, du \\
    I_2(\tau) &= \int_{-1}^{+1} (1 - |u|) R_2(2Lu+\tau) \, du.
\end{align*}
\]  \tag{10}

Variance of $m_1 m_2 : V(m_1 m_2)$

We abbreviate the similar calculation process and show the results.

\[
V(m_1 m_2) = E(m_1^2 m_2^2) - (E(m_1 m_2))^2 = I_1^2(0) I_2^2(0) + I_1^2(\tau_1) I_2^2(\tau_2). \tag{11}
\]

Covariance of $v_{12}$ and $m_1 m_2 : \text{Cov}(v_{12}, m_1 m_2)$

Like the above the calculation results are,

\[
\text{Cov}(v_{12}, m_1 m_2) = E(v_{12}m_1 m_2) - E(v_{12}) E(m_1 m_2) = J_1(\tau_1) J_2(\tau_2) + J_1(0) J_2(0) \tag{12}
\]

subject to
\[
J_i(\tau) = \frac{1}{(2L)^3} \int_{-1}^{+1} \int_{-1}^{+1} \int_{-1}^{+1} R_i(t_1-t_5+\tau) R_i(t_1-t_3-\tau) dt_1 dt_3 dt_5
\]
\[
= \int_{-1}^{+1} (1 - |u|) R_i(2Lu+\tau) R_i(2Lv-\tau) dvdu + \int_{-1}^{+1} (1 - |v|) R_i(2Lv+\tau) R_i(2Lv-\tau) dvdu \quad (i=1, 2), \tag{13}
\]

where integration domains $I_1, II, III$ are illustrated in Fig 2.

Eventually the mean and variance of covariance $C$ can be expressed respectively using the above equations as thus;

\[
E(C) = E(v_{12}) - E(m_1 m_2)
= R_1(\tau_1) R_2(\tau_2) - I_1(\tau_1) I_2(\tau_2) \tag{14}
\]

\[
V(C) = V(v_{12}) + V(m_1 m_2) - 2 \text{Cov}(v_{12}, m_1 m_2)
= I_{11}(0) I_{22}(0) + I_{11}(\tau_1) I_{22}(\tau_2)
+ I_1^2(0) I_2^2(0) + I_1^2(\tau_1) I_2^2(\tau_2)
- 2 J_1(\tau_1) J_2(\tau_2) - 2 J_1(0) J_2(0) \tag{15}
\]

\[\text{Fig. 2 Integration domains for } J_i(\tau)\]
3. MATCHING STABILITY

On stereo images after orientation, matching can be executed one-dimensionally in the x direction based on Epipolar geometry. Therefore we can let $\tau_2 = 0$ in Eq.14,15. Further if an image signal might be assumed Markovian, it is well-known the correlation function $R_1(\tau)$ becomes exponential type (5);

$$R_1(\tau) = e^{-\alpha|\tau|} ; \quad \alpha > 0. \quad (16)$$

Since matching stability might be discussed vividly together with a numerical example, auto-correlation functions were estimated for typical flat and hilly regions in a middle scale (1:25,000) aerial photograph. The photograph is shown in Fig.3, the regions used in the experiment being enclosed with squares. The image are digitized with 50μm pixel width which corresponds to 1.2m on the ground. If the ergodic assumption associated with 2-nd moment might be permitted, the correlation functions can be estimated from SCC's as thus (6)

$$\hat{R}_1(\tau) = \frac{1}{N_1 N_2} \sum_{J=0}^{N_2-1} \sum_{I=0}^{N_1-\tau-1} x(I+\tau,J)x(I,J) \quad (17)$$

$$R_1(\tau) = \frac{\hat{R}_1(\tau)}{\hat{R}_1(0)}. \quad (18)$$

Fig.3 A photograph used for evaluation of correlation functions

The flat and hilly regions are enclosed with squares containing 100 x 100 pixels respectively.
The estimated correlation functions for two regions are shown in Fig. 4 (a)(b) respectively. In the flat region, the neighbouring pixels are highly correlated, whereas the correlation function of the hilly region looks near a delta function. By use of least squares method, the values of the coefficient $\alpha$ were determined, as $\alpha=0.13$ and $\alpha=0.80$ for the flat and hilly regions respectively. Since the correlation function in the hilly region fit poorly to exponential curve, the first 5 lags were used for fitting.

For these two cases, the mean $E(C)$ and standard deviation $\sqrt{V(C)}$ of auto-covariance were calculated for some values of parameters, window width $L$ and positional lag $\tau$, by numerical method using a computer. The results are shown in Fig.5(a)(b) and Fig.6(a)(b), from which the noticeable points follow:

- Fig. 5 shows that the means $E(C)$ converge to constant values as $L$ increases, especially the curves in the hilly region, where the image signal is close to white noise, converge rapidly for $L=10$. On the other hand associated with $\tau$, as the matter of course, $E(C)$ nears to 0 as $\tau$ increase.
- Fig. 5 shows the standard deviations $\sqrt{V(C)}$ increase from 0 as $L$ increases, reach the peak at $L=2$ and $L=8$ for two regions respectively, and decrease to 0 again at infinite $L$.

One criterion for stable matching is the values of the mean $E(C)$. in the vicinity of $\tau=0$, the larger $E(C)$ implies the more stable matching.

The other is the values of the standard deviation $\sqrt{V(C)}$. The larger $\sqrt{V(C)}$ implies the less stable matching. Therefore we can suggest to refer to a variation factor $\sqrt{V(C)}/E(C)$ as an unified criterion for stable matching. Fig.7(a)(b) shows the relation of variation factors for respective regions to window width $L$.

But attention should be paid the ordinate in Fig.7 denotes inverse of variation factor for illustrative convenience. The figure implies variation factors increase monotonously as $L$ increases, and decrease as $\tau$ increases. We can suggest that for sustaining variation factors under $1/3$ for $\tau$ less than 3 pixel, for example, more than 20 pixel of $L$ is needed.
Fig. 4 Estimated correlation functions $R_1(\tau)$

Fig. 5 Means of covariances $E(C)$
Some Statistical Considerations on Window Width and Matching Stability of Images.

![Graphs showing standard deviations of covariances \( \sqrt{V(C)} \) for flat and hilly regions.]

(a) flat region
(b) hilly region

Fig. 6 Standard deviations of covariances \( \sqrt{V(C)} \)

![Graphs showing variation factors of covariances \( \frac{1}{\sqrt{V(C)/E(C)}} \) for flat and hilly regions.]

(a) flat region
(b) hilly region

Fig. 7 Variation factors of covariances \( \frac{1}{\sqrt{V(C)/E(C)}} \)
4. CONCLUSION

This paper is intended to reveal a quantitative relation of appropriate window width to density distribution property of an image in area matching of stereo images. For this purpose statistical behavior of sample auto-covariances of a single image instead of stereo images was investigated. First the formulae to compute a mean and variance of sample cross-covariances using auto-correlation function were derived. Secondly the mean and variance were evaluated from an aerial photograph. It was shown both the mean and variance vary jointly depending on window width. As a result it was proved more useful to use the variation factor combining these statistics as designator of appropriate width of windows.

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REFERENCES