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Coupling-Network Dependence of Locking Phenomena in Microwave Oscillators

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Synopsis

A unified treatment of injection and mutual phase locking phenomena in microwave oscillators coupled by a network characterized by a Y-matrix is presented. Under certain simplifying assumptions, steady state solutions such as locking frequency and oscillation phase relation are given with emphasis placed on the coupling-network dependence of locking bandwidth. Also, some examples of locking system specified by $y_{21}=0$, $y_{21}=y_{12}$ and $y_{21}=-y_{12}$ are briefly discussed.

1. Introduction

In recent years, locking phenomena of oscillators [1]-[7] have found their fruitful applications to practical use in fields of microwave engineering : among them are microwave amplification by injection locking of an oscillator [8]-[9], frequency stabilization of a microwave oscillator by ordinary and self-injection locking [10]-[13], power combination of mutually phase-locked oscillators [14]-[21], and so forth. Through all such applications, the tightness of locking can be estimated in terms of "locking bandwidth", which is essentially important also for quick or broadband response to incoming signal and, though it is almost the same, for FM-noise suppression. However, quite few discussions have hitherto been made on how much the bandwidth depends on the coupling network used. In the present paper, a unified description of both injection and mutual phase locking is given with accent placed on the significance of the coupling network.

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2. Differential equations of coupled oscillators

Consider a coupled system of two microwave oscillators as shown in Fig.1.

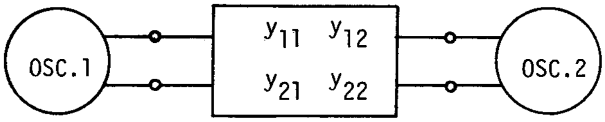

Fig. 1 Oscillators connected by a coupling network

The linear coupling network, which can be either reciprocal or non-reciprocal as the case may be, is characterized by its admittance matrix

$$Y = Y_0 \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} , \tag{1}$$

where Y_0 is the characteristic admittance of the output lines of each oscillator. The oscillator model used (see Fig.2) is a parallel connection of an active non-linear device to a parallel resonant circuit composed of L_n, C_n and G_n. The device-current which is a nonlinear function, $f(v)$, of the voltage across the device, v, can approximately be linearized as

$$i = f(v) \fallingdotseq -g(A)v \tag{2}$$

with

$$-g(A) = \frac{1}{\pi A}\int_0^{2\pi} f(A\cos\alpha)\cos\alpha \, d\alpha , \tag{3}$$

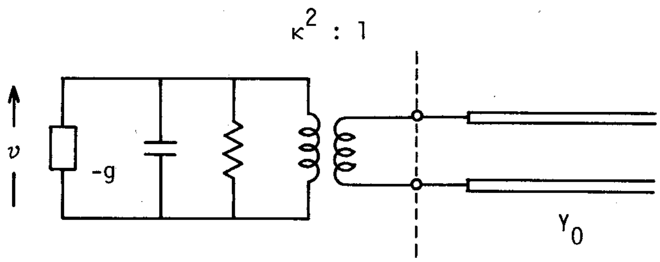

Fig. 2 Circuit model for a microwave oscillator. The broken line indicates the border between oscillator and coupling network.

when v is written as

$$v(t) = A(t)\cos\{\omega_s t + \theta(t)\} \tag{4}$$

using slowly varying functions $A(t)$ and $\theta(t)$. Throughout the paper, the symbol ω_s is used for the frequency of steady state oscillation.

Under the assumptions stated above, and by use of the complex amplitude representation for $v(t)$, $V(t)=A(t)e^{j\theta(t)}$, the whole system will be described by

$$\left.\begin{aligned} &\{-g(A_n) + Y_n(\omega)\}V_n + \kappa_n^2 Y_0(y_{nn}V_n + y_{nm}V_m) = 0 \\ &Y_n(\omega) \equiv j(\omega C_n - \frac{1}{\omega L_n}) + G_n \end{aligned}\right\} \tag{5}$$

$(n,m=1,2\ ;\ n\neq m)$

where κ_n^2 is the transformation ratio of admittance from the output line to the oscillator cavity. In the above, however, $Y_n(\omega)$ should be understood as

$$Y_n(\omega) = Y_n(\omega_s) + (\frac{d\theta_n}{dt} - j\frac{1}{A_n}\frac{dA_n}{dt})Y_n{}'(\omega_s)$$

in accordance with the assumption of slow time variation for $A_n(t)$ and $\theta_n(t)$. Thus, introducing a new time variable $\tau = \omega_s t$ and using notations $\omega_{0n} = 1/\sqrt{L_n C_n}$, $\omega_0 = (\omega_{01} + \omega_{02})/2$, $Q_{0n} = \omega_0 C_n/G_n$, $Q_{ex,n} = \omega_0 C_n/(\kappa_n^2 Y_0)$ and $y_{nm} = g_{nm} + jb_{nm}$, the governing equations of the system become

$$\left.\begin{aligned} 2\frac{dA_n}{d\tau} &= \frac{g(A_n)}{\omega_0 C_n}A_n - (\frac{1}{Q_{0,n}} + \frac{g_{nn}}{Q_{ex,n}})A_n \\ &\quad - \frac{1}{Q_{ex,n}}\{g_{nm}\cos(\theta_m - \theta_n) - b_{nm}\sin(\theta_m - \theta_n)\}A_m \\ 2\frac{d\theta_n}{d\tau} &= -\frac{2(\omega_s - \omega_{0n})}{\omega_0} - \frac{b_{nn}}{Q_{ex,n}} \\ &\quad - \frac{1}{Q_{ex,n}}\{g_{nm}\sin(\theta_m - \theta_n) - b_{nm}\cos(\theta_m - \theta_n)\}\frac{A_m}{A_n} \end{aligned}\right\} \tag{6}$$

where ω_{01}, ω_{02} and ω_s are assumed to be sufficiently close to each other. Furthermore, defining the phase lag, ϕ_{nm}, of the transadmittance y_{nm} by

$$y_{nm} = -|y_{nm}|e^{-j\phi_{nm}} \tag{7}$$ *

Eq.(6) can be simplified as

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* The negative sign before $|y_{nm}|$ appears because $-y_{nm}V_m$ indicates a current injected to Osc.n through the coupling network.

$$\left.\begin{aligned} 2\frac{dA_n}{d\tau} &= \frac{g(A_n)}{\omega_0 C}A_n - \left(\frac{1}{Q_0}+\frac{g_{nn}}{Q_{ex}}\right)A_n + \frac{|y_{nm}|}{Q_{ex}}A_m\cos(\theta_{mn}-\phi_{nm}) \\ 2\frac{d\theta_n}{d\tau} &= -\frac{2(\omega_s-\omega_{0n})}{\omega_0} - \frac{b_{nn}}{Q_{ex}} + \frac{|y_{nm}|}{Q_{ex}}\frac{A_m}{A_n}\sin(\theta_{mn}-\phi_{nm}) \end{aligned}\right\} \quad (8)$$

$(n,m=1,2\ ;\quad n\neq m)$

where $\theta_{nm}\equiv\theta_n-\theta_m$ and the suffix n was omitted for the cavity parameters except for ω_{0n}, for simplicity. Or, introducing the steady state solutions at $A_m=0$, which we denote as A_{n0} and ω_{n0}, by

$$\left.\begin{aligned} \frac{g(A_{n0})}{\omega_0 C} - \left(\frac{1}{Q_0}+\frac{g_{nn}}{Q_{ex}}\right) &= 0 \\ \frac{2(\omega_{n0}-\omega_{0n})}{\omega_0} + \frac{b_{nn}}{Q_{ex}} &= 0\ , \end{aligned}\right\}$$

Ex.(8) is alternatively expressed by

$$\left.\begin{aligned} \frac{dA_n}{d\tau} &= \frac{g(A_n)-g(A_{n0})}{2\omega_0 C}A_n + \frac{|y_{nm}|}{2Q_{ex}}A_m\cos(\theta_{mn}-\phi_{nm}) \\ \frac{d\theta_n}{d\tau} &= -\frac{\omega_s-\omega_{n0}}{\omega_0} + \frac{|y_{nm}|}{2Q_{ex}}\frac{A_m}{A_n}\sin(\theta_{mn}-\phi_{nm}) \end{aligned}\right\} \quad (9)$$

These equations form the basis for discussion on various locking phenomena and are called the reduced equations of the system.

3. Unilateral phase locking

When a unilateral (or oneway) coupling network characterized by $y_{21}=0$ is used, one is led to the case of injection phase locking of Osc.1 by Osc.2. The reduced equations for this case are

$$\left.\begin{aligned} \frac{dA_1}{d\tau} &= \frac{g(A_1)-g(A_{10})}{2\omega_0 C}A_1 + \frac{|y_{12}|}{2Q_{ex}}A_2\cos(\theta_{in}-\theta_1) \\ \frac{d\theta_1}{d\tau} &= -\frac{\omega_s-\omega_{10}}{\omega_0} + \frac{|y_{12}|}{2Q_{ex}}\frac{A_2}{A_1}\sin(\theta_{in}-\theta_1) \end{aligned}\right\} \quad (10)$$

where $\theta_{in}=\theta_2-\phi_{12}$ is the phase of the incoming signal from Osc.2 and ω_s here stands for its frequency.

The stable steady-state solution of Eq.(10) are given by

$$\left\{\frac{g(A_1)-g(A_{10})}{2\omega_0 C}\right\}^2 + \left(\frac{\omega_s-\omega_{10}}{\omega_0}\right)^2 = \left(\frac{|y_{12}|}{2Q_{ex}}\frac{A2}{A_1}\right)^2 \quad (11)$$

which gives a resonance curve for unilateral phase locking, together with the stability condition

$$\{g'(A_1)A_1 + g(A_1)-g(A_{10})\}\{g(A_1)-g(A_{10})\} + 4C^2(\omega_s-\omega_1^{(0)})^2 > 0 . \qquad (12)$$

For small incoming signal, that is, for

$$\frac{|y_{12}|}{Q_{ex}}\frac{A_2}{A_{10}} << 1,$$

Inequality (12) is written approximately as

$$A_1/\,A_{10} > 1$$

and, together with Eq.(11), give rise to the locking range

$$|\omega_s - \omega_{10}| < W \quad ,$$

where

$$W = \frac{\omega_0|y_{12}|}{2\,Q_{ex}}\frac{A_2}{A_{10}} . \qquad (13)$$

A remarkable result here is that the coupling network can affect the locking bandwidth, W, through the magnitude of its transadmittance, y_{12}. The value of $|y_{12}|$ can easily be calculated using the relation between a Y-matrix and the corresponding S-matrix :

$$y = (1 - S)(1 + S)^{-1}.$$

In the following, let us take a few examples to see how W depends on the coupling network.

(i) For a coupling network with a circulator (see Fig.3) whose S-matrix is given by $S_{12}=e^{-j\phi}$ and $S_{11}=S_{21}=S_{22}=0$, we have $y_{12}=-2e^{-j\phi}$ and an ordinary formula for the locking bandwidth results

$$W = \frac{\omega_0}{Q_{ex}}\frac{A_2}{A_{10}} \qquad (14)$$

(ii) If a loop for self-injection is added to the oscillator to be locked as shown in Fig.4, we have $S_{11}=re^{-j\psi_{11}}$, $S_{12}=e^{-j\psi_{12}}$ and $S_{21}=S_{22}=0$, where r is the feedback ratio and ψ is the phase shift of a returning signal, and obtain

$$y_{12} = \frac{-2\exp(-j\psi_{12})}{1 + r\exp(-j\psi_{11})}$$

which leads to

$$W = \frac{\omega_0}{Q_{ex}(1 + 2r\cos\psi_{11} + r^2)^{1/2}}\frac{A_2}{A_{10}} . \qquad (15)$$

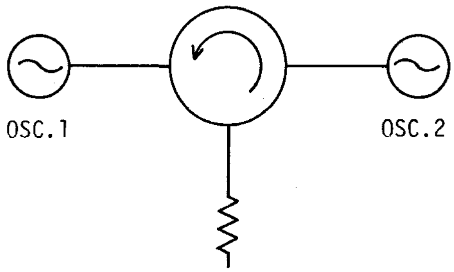

Fig. 3 A typical system for injection locking

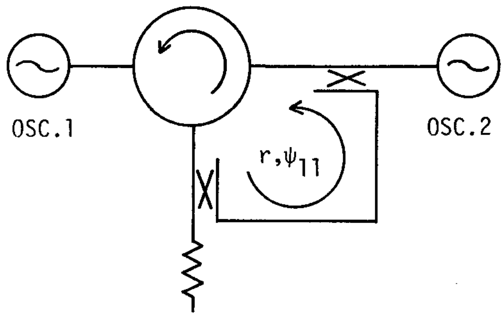

Fig. 4 Injection locking of a self-injection-locked oscillator

So, if ψ_n is taken an odd-integral multiple of π, the locking band can be broadend by a factor $(1-r)^{-1}$ compared with Eq.(14).

(iii) For the coupled system of Fig.5 which is used for parallel operation with one-way phase locking, using the notations indicated in the figure, we have $S_{11} = -k \exp[-j2(\psi_1+\psi_4)]$, $S_{12} = k \exp[-j(\psi_1+\psi_2+2\psi_4)]$ and $S_{21}=S_{22}=0$, and obtain

$$W = \frac{\omega_0}{Q_{ex}[1-2k\cos 2(\psi_1+\psi_4)+k^2]^{1/2}} \frac{kA_2}{A_{10}} . \tag{16}$$

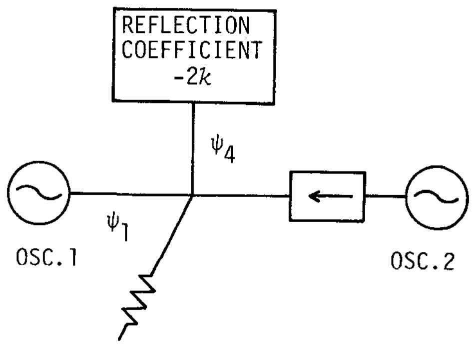

Fig. 5 Parallel-operating system with oneway phase locking

Then, if $2(\psi_1+\psi_4)$ is adjusted to take an even-multiple of π, W takes its maximum value $W_{max}=(\omega_0/Q_{ext})k/(1-k)$, and this phase adjustment proves to be preferable for attaining in-phase addition of the powers from two oscillators.

4. Mutual phase locking

Next, consider the case of a bilateral coupling network. The steady state in which two oscillators are phase-locked to each other must be described by

$$\left.\begin{aligned}
&\frac{g(A_1)-g(A_{10})}{\omega_0 C}A_1+\frac{|y_{12}|}{Q_{ex}}A_2\cos(\theta_{21}-\phi_{12})=0\\
&-\frac{2(\omega_s-\omega_{10})}{\omega_0}+\frac{|y_{12}|}{Q_{ex}}\frac{A_2}{A_1}\sin(\theta_{21}-\phi_{12})=0\\
&\frac{g(A_2)-g(A_{20})}{\omega_0 C}A_2+\frac{|y_{21}|}{Q_{ex}}A_1\cos(\theta_{21}+\phi_{21})=0\\
&-\frac{2(\omega_s-\omega_{20})}{\omega_0}-\frac{|y_{21}|}{Q_{ex}}\frac{A_1}{A_2}\sin(\theta_{21}+\phi_{21})=0
\end{aligned}\right\} \tag{17}$$

In the following, two typical cases of reciprocal and anti-reciprocal coupling network specified by $y_{21}=y_{12}$ and $y_{21}=y_{12}$, respectively, will be discussed. As we are primarily concerned with the effect of coupling network, the simple case of $A_{10}=A_{20}$ is considered here. Then, under the approximation $A_1 \doteqdot A_2$, which is valid for $|y_{nm}|/Q_{ex} \ll 1$, the following results can be derived for the frequency of synchronizing oscillation, the locking bandwidth and the phase difference between two oscillators. First, in case of $y_{21}=y_{12}$, putting $\phi_{12}=\phi_{21}=\phi$, they are

$$\omega_s=\frac{\omega_{10}+\omega_{20}}{2}-\frac{1}{2}W\tan\phi\left[1-\left(\frac{\omega_{20}-\omega_{10}}{W}\right)^2\right]^{1/2} \tag{18}$$

$$W\equiv\frac{\omega_0}{Q_{ex}}|y_{12}|\cdot|\cos\phi|=\frac{\omega_0|g_{12}|}{Q_{ex}} \tag{19}$$

and

$$\sin\theta_{21}=\begin{cases}(\omega_{20}-\omega_{10})/W & \text{for } |\phi|<\pi/2\\ -(\omega_{20}-\omega_{10})/W & \text{otherwise}\end{cases}. \tag{20}$$

The stability condition for the steady state solution is obtained from

$$\frac{d\theta_{21}}{d\tau}=\frac{\omega_{20}-\omega_{10}}{\omega_0}-\frac{|y_{12}|}{Q_{ex}}\cos\phi\cdot\sin\theta_{21} \tag{21}$$

and is given by

$$\cos\phi\cdot\cos\theta_{21}>0, \tag{22}$$

which was taken into consideration to derive Eq's (18)∿(20). Note that, this time, the locking bandwidth is proportional to the magnitude of transconductance of the coupling network. This means that ϕ_{12} should be taken as an integral multiple of π in order to get a maximum locking bandwidth under a given coupling strength.

For anti-reciprocal coupling, $y_{12}=-y_{21}$, putting $\phi=\phi_{12}=\phi_{21}+\pi$, similar calculation leads to

$$\omega_s = \frac{\omega_{10}+\omega_{20}}{2} + \frac{1}{2}W\cot\phi\left[1-\left(\frac{\omega_{20}-\omega_{10}}{W}\right)^2\right]^{\frac{1}{2}} \tag{23}$$

$$W = \frac{\omega_0}{Q_{ex}}|y_{12}|\cdot|\sin\phi| = \frac{\omega_0|b_{12}|}{Q_{ex}} \tag{24}$$

and

$$\cos\theta_{21} = \begin{cases} -(\omega_{20}-\omega_{10})/W & \text{for } 0<\phi<\pi \\ (\omega_{20}-\omega_{10})/W & \text{otherwise,} \end{cases} \tag{25}$$

together with the stability condition

$$\sin\phi \cdot \sin\theta_{21} > 0, \tag{26}$$

which comes form

$$\frac{d\theta_{21}}{d\tau} = \frac{\omega_{20}-\omega_{10}}{\omega_0} + \frac{|y_{12}|}{Q_{ex}}\sin\phi\cdot\cos\theta_{21}. \tag{27}$$

Comparing with Eq's(18) ∿ (22), it is noticed that all the solution for the anti-reciprocal coupling case is obtained from that for the reciprocal coupling case if both ϕ and θ_{21} are simultaneously replaced by $\phi+\pi/2$ and $\theta_{21}+\pi/2$, respectively.

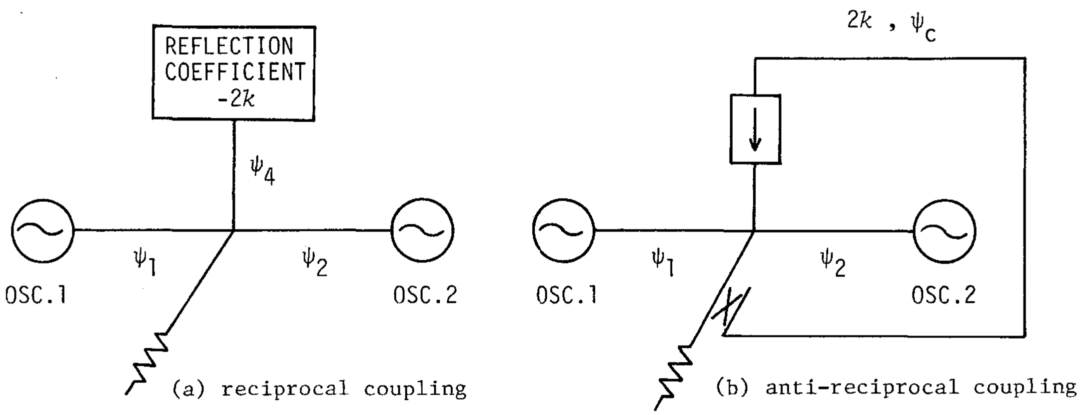

Fig. 6 Mutually coupled parallel-operating systems

Next, let us consider two important examples as shown in Fig.6. For the system of Fig.6(a), which is a familiar magic T-coupled system useful for power combination, we have

$$y_{12} = y_{21} = \frac{-2k\exp(-j\psi)}{1-2k\exp[-j(\psi-n\pi)]}$$

with

$$\psi \equiv \psi_1+\psi_2+2\psi_4 ,$$

provided that $\psi_2 - \psi_1 = n\pi(n=0,1,1,\cdots\cdots)$, which gives $y_{11}=y_{22}$. Then, the locking bandwidth can be maximized by taking $\psi = n'\pi (n' = 0, 1, 2, \cdots)$ which corresponds to $\phi = n'\pi$,

$$W_{max} = \frac{\omega_0}{Q_{ex}} \frac{2k}{1-2k} \quad \text{for} \quad n' = n (\text{mod. } 2) . \tag{28}$$

It is easy to see that the choice of $n'-n$ also ensures in-phase addition of powers from two oscillators.

In the second example of Fig.6(b) which was first noticed as a parallel-operating system effective for FM-noise suppression [21], the coupling is anti-reciprocal :

$$y_{12} = - y_{21} = \frac{-2k \exp(-j\psi)}{1 + 2k \exp[-j(\psi - (2n+1)\pi/2)]}$$

with

$$\psi \equiv \psi_1 + \psi_2 + \psi_c \quad ,$$

when $\psi_2 - \psi_1 = (2n+1)\pi/2$ is assumed to give $y_{11} = y_{22}$. Although the locking bandwidth takes its maximum value given by the same expression as Eq.(28) for $\psi = (2n'+1)\pi/2$ with $n'-n =$ odd integer, the powers from each oscillators add just in the opposite phase under this condition. For $n' = n$ (mod.2), on the other hand, in-phase addition can be established, but the maximum locking bandwidth reduces to a little smaller value of

$$W_{max} \Big|_{\text{anti-reciprocal}} = \frac{\omega_0}{Q_{ex}} \frac{2k}{1+2k} . \tag{29}$$

5. Conclusion

A coupled system of two microwave oscillators with coupling network characterized by Y-matrix has been analyzed to give a unified description of both injection and mutual phase locking. It has been shown that the locking bandwidth depends largely on the coupling network : it is proportional to $|y_{12}|$, $|g_{12}|$ and $|b_{12}|$ according as the case of oneway, reciprocal and anti-reciprocal coupling.

Experimental confirmation of the theoretical results remains for future work. From a practical point of view, it is also interesting to clairify how the locking bandwidth, FM-noise depression and the condition for in-phase power addition correlate with each other. Another important subject to be studied should be extention of the discussion of this paper to a coupled system of more oscillators or to the case of stronger coupling.

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