Memoirs of the School of Engineering, Okayama University Vol.13, February 1979

Basic Characteristics of Squarewave Inverter Circuit with Series R-L Load

Takeshi FUJITSUKA* and Tōru AGUSA**

(Received October 20, 1978)

Synopsis

In this paper, the steady state operations of the squarewave inverter circuit with a series R-L load are discussed. The circuit consists of transistors and feedback diodes. The basic equation is derived from its equivalent circuit. Solving the basic equation with steady state conditions, the instaneous value of the load current i is derived. The period t_2 for which the current flows from the supply to the load and the period t_1 for which the current feedbacks through diodes from the load to the supply are calculated from i, and the ratio of t_1 to t_2 is illustrated using power factor of fundamental wave, pf, as a variable. The ratios of transistor mean current I_{tr}, diode mean current I_D, supply mean current I_s to the load current I are illustrated using pf as a variable, too. In result, each current ratios to I is shown in simple expressions. The load current can be calculated simply using the coefficient reading off the figure. In addition, it becomes clear from the figure that the load current is scarcely influenced by the harmonic voltage in less than 0.8 of pf. The ratio $t_2/(t_1+t_2)$ calculated in squarewave voltage, shows the limit of pulse width control whose out put voltage is the squarewave.

* Department of Electrical Engineering.
** Graduate School of Electrical Engineering.

1. Introduction

Since the thyristor inverter improved commutation is published in 1961[1], the square wave inverter with Feed-back diodes are widely used. The load of these inverter are inductively for the many cases, that is, induction heating, discharge lamp, induction moter, welding and melting etc.

The study to compensate for the inductive load have been carried on in several[2) 3) 4)]. But on the fundamental characteristics of the square wave inverter circuit with series *R-L* load, that is, the load current in steady state expressed by mathematics and the ratioes of currents flow through several elements, it is not discussed in detail.

In this paper, the circuit movement is discussed in hte case of connected the inductive load to the square wave inverter circuit.

2. Circuit analysis

2-1 Equivalent circuit

In the circuit in Fig.1, it is assumed that the power source and every elements are ideal, and the ideal square voltage is induced on the terminal A, B of the load.

The equivalent circuit is shown in Fig.2.

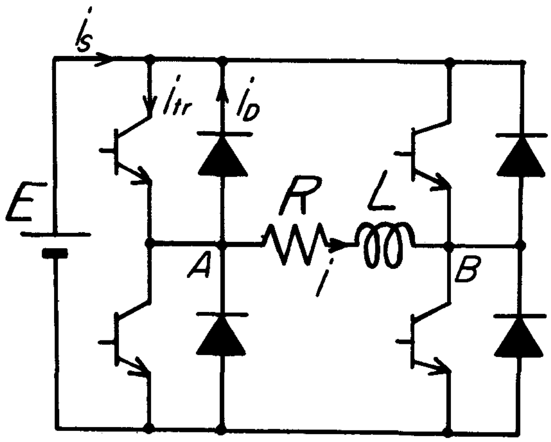


Fig.1. Bridge inverter circuit with *R-L* load

2-2 Steady state solution of load current

The waveforms of the load voltage v and the load current i are shown in Fig.3.

The fundamental differential equation of the circuit is shown in Equ.(1), and the solution is in Equ.(2).

$$Ri + Ldi/dt = E \quad \text{-------} \quad (1)$$

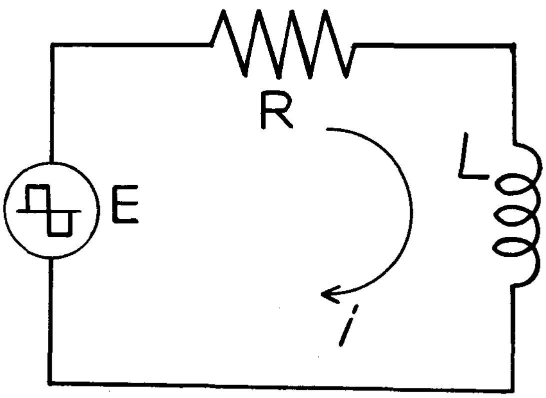


Fig.2. Equivalent circuit

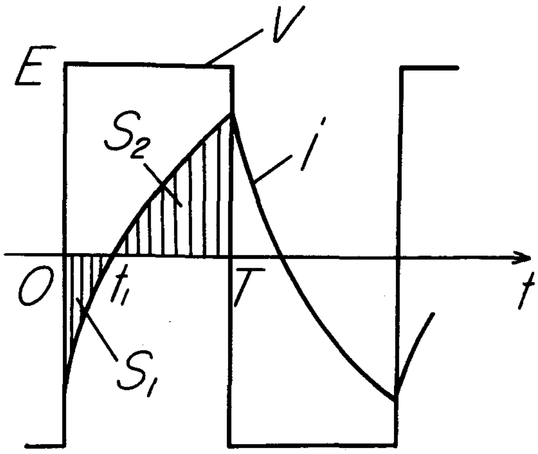


Fig.3. Load voltage and load current waveforms

$$i(t) = \frac{E}{R}(1-\varepsilon^{-\frac{R}{L}t}) + i(0)\cdot\varepsilon^{-\frac{R}{L}t} \qquad \text{--------- (2)}$$

where, $i(t)$ is the load current and $i(0)$ is initial vlaue of the load current.

Under steady state conditions, the following condition is satisfied

$$-i(0) = i(T) \qquad \text{--------- (3)}$$

From Equ.(2) and (3), we get

$$i(0) = -\frac{E}{R}\cdot\frac{1-\varepsilon^{-\frac{R}{L}T}}{1+\varepsilon^{-\frac{R}{L}T}} \qquad \text{--------- (4)}$$

Substituting the value of $i(0)$ from Equ.(4), we get

$$i(t) = \frac{E}{R}(1-\varepsilon^{-\frac{R}{L}t}) - \frac{E}{R}\cdot\frac{1-\varepsilon^{-\frac{R}{L}T}}{1+\varepsilon^{-\frac{R}{L}T}}\cdot\varepsilon^{-\frac{R}{L}t} \qquad \text{--------- (5)}$$

By substituting $i(t_1)=0$ into Equ.(5), the period t_1 for which the load current is fed back to the source is given,

$$t_1 = -\frac{L}{R}\, In[\frac{1}{2}(1+\varepsilon^{-\frac{R}{L}T})] \qquad \text{------ (6)}$$

The period t_2 for which the current is fed to the load from the source, and the period t_1 for which the current is fed back to the source from the load are shown in Fig.4. Where, the variable is the power factor of the fundamental frequency.

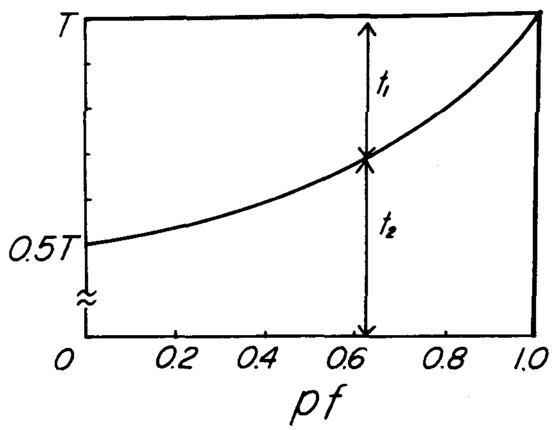


Fig.4. Period t_1 flowing through diodes and period t_2 flowing through transistors

2-3 Mean current in various parts

In Fig.3, the area S_1 is the electric charge which is fed back through the diodes, the area S_2 is the electric charge which is fed through the transistors.

$$S_1 = \int_0^{t_1} i(t)dt$$

$$= -\frac{EL}{R^2}\left\{In\frac{1}{2}(1+\varepsilon^{-\frac{R}{L}T}) + \frac{1-\varepsilon^{-\frac{R}{L}T}}{1+\varepsilon^{-\frac{R}{L}T}}\right\} \quad \text{-----} \ (7)$$

$$S_2 = \int_{t_1}^{T} i(t)dt$$

$$= \frac{EL}{R^2}\left\{\frac{R}{L}T + \frac{2}{1+\varepsilon^{-\frac{R}{L}T}}\cdot\varepsilon^{-\frac{R}{L}T} + In\frac{1}{2}(1+\varepsilon^{-\frac{R}{L}T}) - 1\right\} \quad \text{-----} \ (8)$$

The mutual relation of the load current I, the transistor current I_{tr}, the diode current I_D and the source current I_s in mean value is follows;

$$I = I_{tr} + I_D \quad \text{-----} \ (9)$$

$$I_s = I_{tr} - I_D \quad \text{-----} \ (10)$$

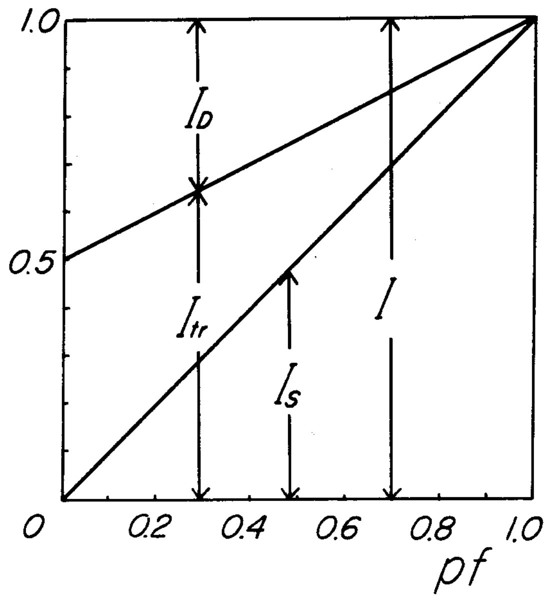


Fig.5. Ratios of each current versus I

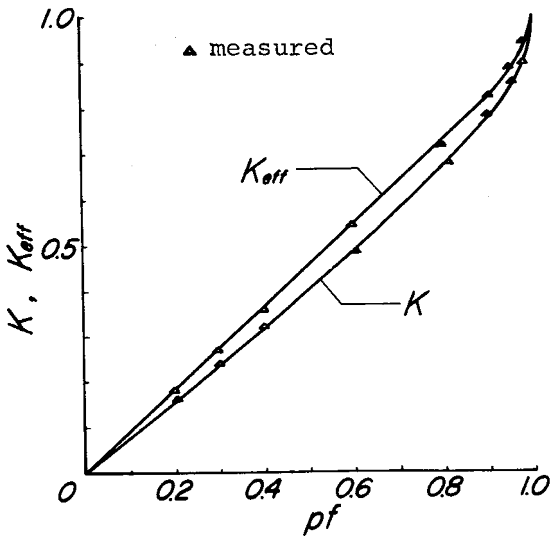
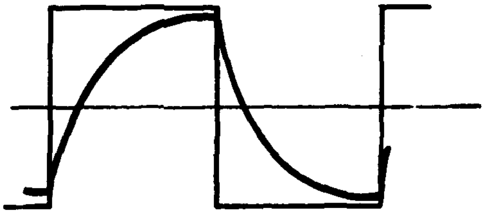
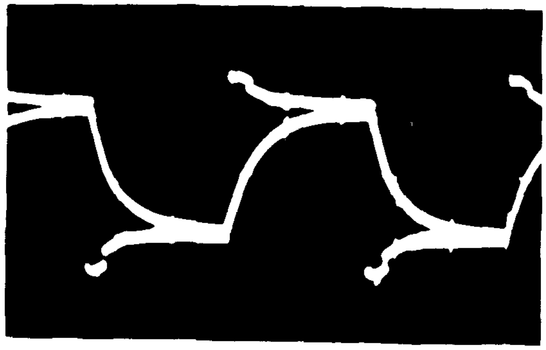


Fig.6. Coefficient K on mean value, and K_{eff} on effective value

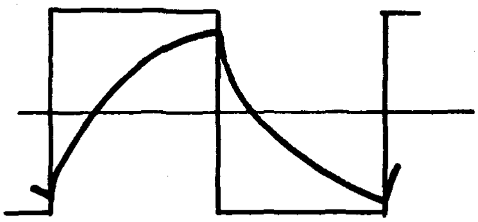
From Equ.(7)~(10), the relation of each currents shows in Fig.5 used a variable pf.



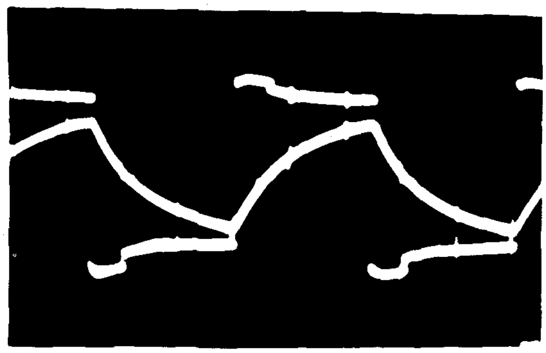
(a) pf=0.8



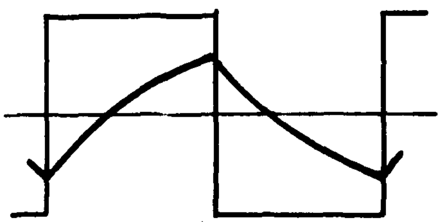
(a) pf=0.8



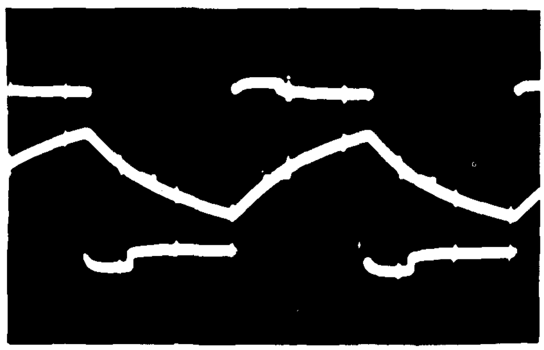
(b) pf=0.6



(b) pf=0.6



(c) pf=0.4



(c) pf=0.4

Fig.7. Comparison between calculated waveforms and measured waveforms of v, v_R
(1) Calculated v, v_R waveforms
(2) Measured v, v_R waveforms
Time scale: 0.2ms/div
Amplitude scale: v, v_R 10V/div

2-4 Mean and effective value of load current

The load currents in mean value I and effective value I_{eff} are follows;

$$I = (|S_1| + |S_2|)/T \qquad \text{--------- (11)}$$

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T (i(t))^2 dt} = \frac{E}{R}\sqrt{1 - \frac{2L}{RT}\cdot\frac{1-\varepsilon^{-\frac{R}{L}T}}{1+\varepsilon^{-\frac{R}{L}T}}} \qquad \text{--------- (12)}$$

Using Equ.(13), (14) the normalized terminal voltage of the load resistance shows in Fig.6.

$$K = V_R/E = IR/E \qquad \text{----- (13)}$$

$$K_{eff} = V_{eff}/E = I_{eff}\cdot R/E \qquad \text{----- (14)}$$

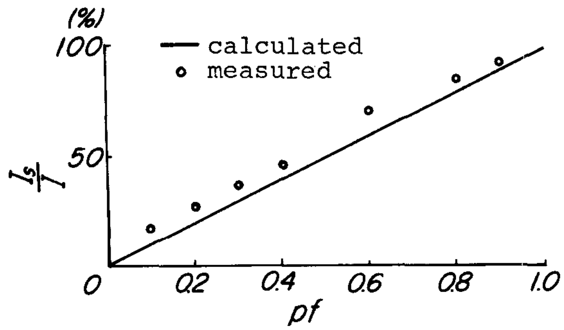


Fig.8. Comparison between calculated and measured value of I_S/I

3. Experimental result

The load current waveforms are shown in Fig.7. The ratio of the source current, the transistor current and diode current to the

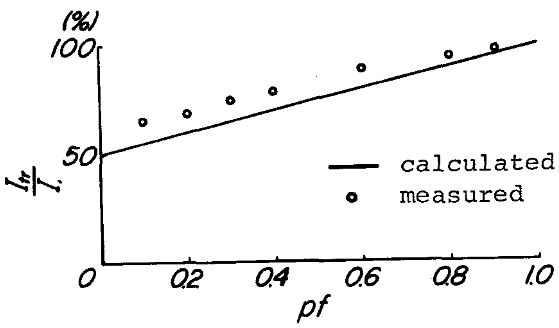


Fig.9. Comparison between calculated and measured value of I_{tr}/I

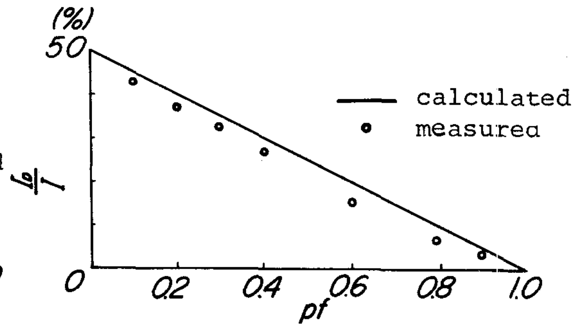


Fig.10. Comparison between calculated and measured value of I_D/I

load current are shown in Fig.8, Fig.9 and Fig.10.

4. Discussion

I_s/I, I_{tr}/I and I_D/I change in proportional to pf. Therefore, we get the next simple relations.

$$\frac{I_{tr}}{I} = \frac{1+pf}{2} \qquad \text{----- (14)} \qquad\qquad \frac{I_D}{I_{tr}} = \frac{1-pf}{1+pf} \qquad \text{----- (15)}$$

The load current in mean or effective value is given easily using K or K_{eff}.
In the case of the sine-wave source, the circuit current I_L is shown as follows;

$$I_L = \frac{E}{pf \cdot R}$$

In the same way as the sine-wave source, in the case of the square-wave source, the load current shows as follows;

$$I = \frac{E}{R} \cdot pf \cdot K' \quad \begin{cases} K' \simeq 0.8 & (pf<0.8) \\ K = 0.8 \sim 1.0 & (pf>0.8) \end{cases} \qquad \text{--------- (16)}$$

$$I_{eff} = \frac{E}{R} \cdot pf \cdot K_{eff}' \begin{cases} K_{eff}' \simeq 0.9 & (pf<0.9) \\ K_{eff} = 0.9 \sim 1.0 & (pf>0.9) \end{cases} \qquad \text{--------- (17)}$$

where,

$$K' = K/pf \qquad \text{--------- (18)}$$

$$K_{eff}' = K_{eff}/pf \qquad \text{--------- (19)}$$

When the ideal square wave voltage is induced on the R-L load, the ratio of the period t_1 for which the power is fed back to the source through diodes and the period t_2 the power is fed to the load from the source through transistors are shown in Fig.4.

For the period t_1, transistors need not to be on and the voltage is induced by the load inductance. We get the square wave load voltage when the transistors is on for the period t_2. Therefore Fig.4 shows the limit to occur the square wave voltage in the case of doing the pulse width control.

5. Conclusion

The analysis and the experimental result of the square wave inverter with the *R-L* load are discussed. In result, the steady state solution of the load current is derived and the mean and effective value of the load current can be easily given. It becomes clear that I_s/I, I_{tr}/I and I_D/I change in proportional to pf and each current has simple relations. In addition, the range to give the square wave voltage is shown doing the pulse width control.

References

(1) W.McMurray & P.Shattuck, *AIEE Trans.* Vol.80 Part I p.531~542, (1961)

(2) G.N.Revankar, *IEEE Trans.* Vol.IECI-20, No.3, p.178~182, Aug.(1973)

(3) G.N.Revankar, *IEEE Trans.* Vol.IECI-21, No.1, p.18~21, Feb.(1974)

(4) G.N.Revankar, *IEEE Trans.* Vol.IECI-21, No.4, p.226~229, Nov.(1974)