Analysis of the A-C Voltage Control Circuit with Parallel Connection of SCR and Reactor

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The a-c voltage control circuit, composed of parallel connection of a SCR and a reactor, gives the similar performance as the control circuit of a back to back SCR pair. However, the control characteristics of this circuit is affected by magnetizing characteristics of reactor. In this paper, the circuit with the resistive load has been analyzed by using the approximated models of hysteresis loop of core materials, where the SCR's forward voltage drop is taken into account.

The results of the analysis have revealed the facts as follows,
1) The control characteristics of the a-c voltage in average value is independent on the magnetizing characteristics of reactors except in the vicinity of minimum output point. But, in effective value, it is not valid.
2) A SCR's forward voltage drop results in the reduction of not only SCR's current but also reactor's current.
3) The a-c output current does not include the d-c component, if the winding resistance of the reactor is negligible.

§ 1. Introduction

The silicon controlled rectifier (SCR) is a sort of synchronous switch and the circuit of a back to back SCR pair in parallel is an a-c voltage control circuit. On the other hand, the circuit with saturable reactors i.e., a magnetic amplifier, is too a similar a-c voltage control circuit. The circuit introduced in this paper is a new type of the a-c voltage control circuit that, though composed of parallel connection of a SCR and a reactor, has the similar control characteristics.

Here, in order to investigate the dependence of the control characteristics upon the magnetizing characteristics of reactor, the circuit with the resistive load has been analyzed by using the four models of hysteresis loop, where the SCR's voltage drop in conducting is taken into account.

§ 2. Performance of the Circuit

The circuit is shown in Fig. 1. To illustrate the performance of the circuit, it is assumed that the reactor has the ideal magnetizing characteristics, as shown in Fig. 3(a). First, in the half-cycle in which the a-c source voltage applies across the SCR in forward direction, until the SCR is fired by the gate input, the full voltage of reactor will be moved toward the positive saturation level. In this period, the load current is negligible. Once, the SCR is turned on, the load current is flowed through the SCR and the source voltage e_s appears across the load, and the flux level of reactor stays still since the reactor voltage is negligible.

Second, in the succeeding half cycle, the source voltage e_s in turn, appears across the SCR in the backward direction and the load...
current is blocked. In turn, the source voltage across the reactor cause the flux level to move toward the negative saturation. When the flux level of the reactor reaches to the negative saturation level, the load current begins to flow through the reactor in the reverse direction.

Since, in steady state, the flux change of the reactor during one half cycle, must be equal that during the succeeding one half cycle, the following equation will be valid.

\[
\frac{1}{N_0} \int_0^{\pi} e_s d(\alpha t) + \frac{1}{N_0} \int_\pi^{2\pi} e_s d(\alpha t) = 0
\]  

(1)

where \( N_0 \) : number of turns of reactor.
\( \omega t \) : angular velocity of source voltage.
\( \alpha_t \) : firing angle of SCR
\( \alpha_r \) : firing angle of reactor.
\( e_s \) : source voltage in instantaneous value.

From Eq. 1, the following relation between \( \alpha_t \) and \( \alpha_r \) may be obtained.

\[
\alpha_t + \pi = \alpha_r
\]  

(2)

The phase angle of source voltage \( \omega t = \beta \) when the operating point comes to the point (1) is given by

\[
\beta_1 = \sin^{-1} \frac{I_{0} R}{E_m}  
\]  

(4)

Mode A of operation; At \( \omega t = \beta \), the mode A of operation starts, because the reactor becomes unsaturated and the SCR is blocked in forward direction. The load current \( i_R \) is expressed as follows.

\[
i_R = -\frac{E_m}{\sqrt{R^2 + \omega^2 L^2}} \left\{ \sin(\alpha t - \theta) + I_{0} e^{-\frac{R}{L} \left( t - \frac{\beta}{\omega} \right)} \sin(\beta_1 - \theta) \right\} + I_{0} e^{-\frac{R}{L} \left( t - \frac{\beta}{\omega} \right)} 
\]  

(5)

And also the reactor voltage \( e_{SR} \) is given by

\[
e_{SR} = L \frac{di_R}{dt} = L \left\{ \frac{E_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t - \beta_1) - j \right\} + \frac{R}{L} e^{-\frac{R}{L} \left( t - \frac{\beta_1}{\omega} \right)} \sin(\beta_1 - \theta) \right\} + \frac{R}{L} I_{0} e^{-\frac{R}{L} \left( t - \frac{\beta_1}{\omega} \right)} 
\]  

(6)

where \( L \): inductance of reactor.
\( \theta \): arc tan \( \omega L/R \).

Mode A of operation continues until the
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Fig. 3 Models of approximated B-H loops, reactor and load voltage waveforms.

point (2), \( \alpha_1 \), when the SCR turns on by the gate input. The flux change \( \Phi_i \) (the flux resetting value) from the point (1) to (2) is

\[
\Phi_i = \frac{1}{N_0} \int E_{scr} d(\omega t)
\]

The correlation of the current and flux gives the flux resetting value as follows.

\[
\Phi_i = \frac{2}{L_1} \left( i_{SR1} - I_0 \right)
\]

where \( i_{SR1} \): Reactor current at \( \alpha_1 \).

Mode B of operation; After the SCR turns on, the circuit condition is mode B. The load voltage \( e_R \), the reactor voltage \( e_{SR} \) and the reactor current \( i_{SR} \) are given respectively.

\[
e_R = E_{scr}
\]

\[
e_{SR} = E_{scr}
\]

\[
i_{SR} = \frac{E_{scr}}{L} \left( t - \frac{\alpha_1}{\omega} \right) + i_{SR1}
\]

The flux resetting value \( \Phi_i' \) in the duration of mode B of operation is

\[
\Phi_i' = \frac{E_{scr}}{N_0} (\beta_2 - \alpha_1)
\]

Mode A1 of operation; After \( \beta_2 \), the circuit condition is mode A1 of operation similar to mode A of operation. The load current \( i_R \) and the reactor voltage \( e_{SR} \) are given in Eqs. (5) and (6) by substituting \( \beta_2 \) for \( \beta_1 \) and also \( i_{SR2} \) at \( \beta_2 \) for \( I_0 \). The boundary point \( \beta_3 \) between this mode and the succeeding, is determined by \( e_{SR} = 0 \). The operating point of reactor is then at (4) in Fig. 3(d).

Mode D2 and D, of operation; After \( \beta_3 \), the operating point moves into the inner of the B-H loop. At \( \omega t = \pi \), the operating point is on the vertical axis and at \( \omega t = \beta_4 \), reaches to the left side of the B-H loop. The duration from \( \beta_3 \) to \( \pi \) corresponds to mode D2, and the duration from \( \pi \) to \( \beta_4 \) does to mode D. \( \beta_1 \) is as follows.

\[
\beta_4 = \sin^{-1} \left( \frac{R}{L} (i_{SR3} - I_3) \right)
\]

where \( i_{SR3} \): reactor current at \( \beta_3 \).

Mode C of operation; After \( \beta_1 \), the circuit condition is mode C of operation. The load current \( i_R \) and the reactor voltage are expressed in Eq. (5) and (6) by substituting \( \beta_1 \) for \( \beta_2 \) and \( i_{SR1} \) for \( I_0 \).

At the phase angle \( \alpha_2 \) when the reactor is saturated, the relation \( i_R = -I_2 \) is valid. Further more, since the total flux changes over one cycle in steady state is zero, \( \alpha_2 \) is given by

\[
\Phi_i + \Phi_i'' + \Phi_i'' + \Phi_i'' = 0
\]
where \( \phi_i' \): Flux resetting value in mode \( A_1 \) of operation.

\( \phi_i \): Flux gating value in mode \( C \) of operation, i.e., "flux gating value" means the flux change in the direction from \( +\phi_i \) to \( -\phi_i \).

Mode \( D \) of operation; After \( \alpha_2 \), the circuit condition is mode \( D \) of operation. This mode continues until \( \omega t = 2\pi \). The load current \( i_R \) is \( \epsilon/\omega \).

3.4 Control characteristics. As above mentioned, since boundary conditions and behaviors of modes of operation over one cycle are determined for any firing angle \( \alpha_i \), the wave forms of load voltage and therefore the control characteristics are determined for \( \alpha_i \).

The above analysis of the circuit operation will be valid to other B-H loops in Fig. 3 by using the following expressions, that is, \( I_{\lambda 1} = I_{\lambda 2} = 0 \) in Fig. 3(a), \( I_0 = I_{\lambda 2} = I_0 \) in Fig. 3(b), and \( I_{\lambda 1} = -I_{\lambda 2} = -I_0 \) in Fig. 3(c).

3.5 Calculated. According to the analysis, an example has been calculated numerically. Circuit constants and boundary conditions, wave forms and control characteristics are shown in Table 2, Fig. 4 and Fig. 5 respectively.

<table>
<thead>
<tr>
<th>Table 2 Circuit constants and boundary conditions.</th>
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<tbody>
<tr>
<td>( I_{\lambda 1} )</td>
</tr>
<tr>
<td>( I_{\lambda 2} )</td>
</tr>
<tr>
<td>( L )</td>
</tr>
<tr>
<td>( R )</td>
</tr>
<tr>
<td>( E_{m} )</td>
</tr>
<tr>
<td>( f )</td>
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<td>( E_{S/R} )</td>
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<tr>
<td>( \beta_1 )</td>
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<tr>
<td>( \alpha_1 )</td>
</tr>
<tr>
<td>( \beta_2 )</td>
</tr>
<tr>
<td>( \beta_3 )</td>
</tr>
<tr>
<td>( \beta_4 )</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
</tr>
</tbody>
</table>

The calculated in the case of idealized B-H loop, is also shown with dotted line in Fig. 5 for comparison.

§ 4. Experiments results

In order to investigate experimentally the dependence of the control characteristics upon the magnetizing characteristics of reactors, the measurement are carried out used with four reactors, that is, Fe 50\%—Ni 50\% oriented
core, oriented silicon steel core, oriented silicon steel cut-core with air gaps and silicon steel stack core for power transformer.

The B-H loops, the minor loops at $\alpha_i = \pi/2$ and the wave forms of the reactor and voltages at $\alpha_i = \pi/2$ are shown in Fig. 6(a), (b) and (c) respectively. The measured of voltage control characteristics with resistive load are shown in Fig. 7, where (a) is in effective value and (b) is in average.

§ 5. Discussion

Though it seems to be the good identification between the calculated and the measured, the discussion on the foregoing analysis and measurements more over reveals some features of the control characteristics as follows.

5.1 No d-c component in load current. In every modes of operation, the following equation is always true.

$$ e_s = e_{SR} + e_R \tag{16} $$

But, in steady state, since the total flux changes over one cycle is zero, the following equation may be obtained.

$$ \int_0^{2\pi} e_{SR} d(\omega t) = 0 \tag{17} $$

From Eq.s (16) and (17),
\[
\int_{\alpha_1}^{\alpha_2} d (\omega t) = 0
\]  
\tag{18}

Eq. (18) tells that the d-c component is not included in the load current. This fact is good for the case in which the resistance of reactor windings is negligible.

Otherwise, it is not and the following equation have to be taken into account in mode B of operation.

\[
e_{SR} = E_{SCR} - R_{SR} i_{SR}
\]  
\tag{19}

Where, if \( i_{SR} \) is negative, the flux resetting value \( \Phi' \) is increased. \( \Phi' \) affects to increase \( a_2 \) in mode D of operation.

And also, in mode D of operation, the load voltage \( e_R \) is

\[
e_R = - \frac{R}{R_{SR}} e_s
\]  
\tag{20}

The increment of \( a_2 \) and the decrement of \( e_R \) in mode D of operation result in the existence of d-c component in the load current.

5.2 Dependence of control characteristics upon the magnetizing characteristics of reactors. As \( \alpha_1 \) approaches to \( \beta_2 \), \( i_R \) decreases and minimized at \( \alpha_1 = \beta_2 \). Then, in the case of the magnetizing characteristics in Fig. 3 (a), the exciting current of reactor does not flow through the load, but, it does in the cases of Fig. 3 (b), (c) and (d).

As \( \alpha_1 \) decreases and leaves away from \( \beta_2 \), the load current flowing through the \( SCR \) increases, but, in the forward load current, that is, the load current in the direction of the \( SCR \) current, the component of the exciting current decreases, and then, the proportion of the exciting current to the load one decreases extremely. Therefore, the forward current in effective value and also in average value, is independent on the magnetizing characteristics of reactor except in the vicinity of \( \alpha_1 = \beta_2 \). And more, the backward load current in average value is true, since the load current has not d-c component. However, the backward load current in effective value is not, because the backward current waveform is not the same to the forward one.

5.3 Effect of \( SCR \) forward voltage drop \( E_{SCR} \). As the load voltage \( e_R \) when the \( SCR \) is conducting is indicated in Eq. (9), and decreases by \( E_{SCR} \). Further more, \( E_{SCR} \) resets the flux by \( \Phi' \) in Eq. (13), which results in the increased \( a_2 \). In other word, \( E_{SCR} \) results in the reduced not only \( SCR \)’s but also reactor’s current.

§ 6. Conclusion

The a-c voltage control circuit with a resistive load has been analyzed. The results of analysis and measurement have revealed the fact as follows.

1) The control characteristics of the a-c output voltage in average value is independent on the magnetizing characteristics of reactors except in the vicinity of minimum output point. But, in effective value, it is not valid.

2) A \( SCR \)’s forward voltage drop results in the reduction of not only \( SCR \)’s current but also reactor’s current.

3) The a-c output current does not include the d-c component, if the winding resistance of the reactor is negligible.