Bubble Formation at Single Circular Hole

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In this report the formation of bubbles at a single circular hole is considered theoretically, as a fundamental study on contacting devices for the purpose of mass transfer operation in chemical engineering.

From many previous experimental data, it is found that the mechanism of bubble formation is classified into steady bubble growth system and potential bubble formation system.

§ 1. Introduction

Contacting a gas with a liquid for the purpose of mass transfer is very important operation in chemical engineering. Therefore the investigation for the behavior of bubble formed at a single circular hole submerged beneath various quiescent liquids has been developed by many workers. 1-13

But the theoretical analysis is not adequate because the mechanism of bubble formation is complex. Hitherto, the principal factors which might be expected to affect the bubble size have been made clear to some extent by the experimental approach.

However the results and conclusions of different workers do not always agree. This fact suggests that not only the physical property of liquids and the diameter of holes but also the geometrical dimensions of experimental apparatus effect the bubble formation.

Authors have re-examined many previous experimental data 1-13 in this view point. From the above results it was found that the capillary length is a significant factor. Then authors have divided the mechanism of bubble formation semi-theoretically into two cases, i.e., the formation of bubble at the orifice shapes of thin plate thickness and the capillary shapes of long length compared with its hole diameter.

Consequently, the theoretical equations which are suitable to many previous experimental data over wide operating conditions were obtained.

§ 2. Summary of the previous works.

It has been generally known that the principal factors which might be expected to affect the size of bubble formed at a single circular hole submerged beneath various quiescent liquid are following terms. 14-15

1) the orifice diameter
2) the volumetric flow rate of gas through the orifice
3) the gas density
4) the gas viscosity
5) the liquid density
6) the liquid viscosity
7) the static surface tension of the liquid
8) the dynamic surface tension of the liquid
9) the surface elasticity of the liquid
10) the surface viscosity of the liquid
11) the contact angle of the liquid with the orifice material
12) the wetting property of the material of the orifice
13) the pressure drops across the orifice
14) the volume of gas chamber below the orifice
15) the velocity of the sound in the gas
16) the submergence of the orifice below the liquid surface
17) the shape of the orifice
18) the angle of inclination of the orifice
19) the liquid flow rate past, and its turbulence near the orifice

However, the previous works are different in the experimental conditions and the evaluations on the effective factors, so it may be difficult to compare these results directly. The correlation of the bubble volume against volumetric gas flow rate \( G \) obtained in
the previous works is shown in Fig. 1. It is found from this figure that the previous works are short of the caution against the experimental conditions and the results are not always available. The summary of previous works is given in Table 1.

§ 3. Theoretical consideration

Considering the interaction of the plate thickness and the gas chamber volume as the principal factor of the discrepancy of the previous experimental data, next two limiting states exist.

A) Plate thickness is adequately long compared with its hole diameter, or gas chamber volume is negligible small.

B) Plate thickness is adequately thin, or gas chamber volume is limited.

In both cases, a certain pressure difference is required to form the bubble at the tip of the hole. However, if the bubble is formed once, in case A, the pressure fluctuation at the tip of a capillary is decreased by the friction loss of gas flow through that, so it is not conveyed to the lower end of the capillary and steady gas flow is obtained at the position of bubble formation. On the other hand, in case B, after excess gas volume accumulated in the gas chamber is released into liquid as a bubble, the pressure of the gas chamber drops suddenly and the bubble is not formed until the pressure is restored by continuous gas flow.

As mentioned above, there are different states of the bubble formation, and the bubble size has different values depending upon the two states. Therefore states of bubble formation may be classified into the steady bubble growth system and the potential bubble forma-
Table 1. Summary of previous works

<table>
<thead>
<tr>
<th>worker</th>
<th>hole shape</th>
<th>hole diameter</th>
<th>plate thickness</th>
<th>chamber volume</th>
<th>liquid</th>
<th>experimental correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>c</td>
<td>0.014 ~ 0.034</td>
<td>1.1</td>
<td>small</td>
<td>alcohol water</td>
<td>$V_B = 0.33 \rho_0$ (a)</td>
</tr>
<tr>
<td>2)</td>
<td>c</td>
<td>0.0118 ~ 0.63</td>
<td></td>
<td></td>
<td>ethanol water</td>
<td>$d_B/\rho_0 = 1.82 \left(\rho/\rho_0\right)^{1/4}$ (b)</td>
</tr>
<tr>
<td>3)</td>
<td>c</td>
<td>0.055 ~ 0.95</td>
<td></td>
<td></td>
<td>glycerol water...</td>
<td>turbulent region $d_B = (72G^2/\pi^2g)^{1/5}$ (c)</td>
</tr>
<tr>
<td>4)</td>
<td>c</td>
<td>0.061 ~ 0.293</td>
<td></td>
<td></td>
<td>alcohol water</td>
<td>$V_B = C_d\rho/\rho$ (d)</td>
</tr>
<tr>
<td>5)</td>
<td>o **</td>
<td>0.2 ~ 0.48</td>
<td>thin</td>
<td>small</td>
<td>ethanol water</td>
<td>$d_B/\rho_0 = 1.737 \rho_0^{0.33} \rho_0^{0.125} \rho_0^{0.02}$ (e)</td>
</tr>
<tr>
<td>6)</td>
<td>c</td>
<td>0.038 ~ 0.11</td>
<td>5</td>
<td>1.64 ~ 90</td>
<td>glycerol acetone</td>
<td>$V_B = 1.378 \left(G_2/\rho g\right)^{3/5}$ (f)</td>
</tr>
<tr>
<td>7)</td>
<td>o</td>
<td>0.101 ~ 0.317</td>
<td>thin</td>
<td>large</td>
<td>glycerol water...</td>
<td>$d_B &lt; 2100 \rho_0 = 0.219 \rho_0^{1.16}(Re)^{1/3}$ (g)</td>
</tr>
<tr>
<td>8)</td>
<td>o</td>
<td>0.034 ~ 1.58</td>
<td>0.32</td>
<td>4 ~ 4000</td>
<td>water oil...</td>
<td>$V_B = \pi d_B \sigma/\rho g$ (h)</td>
</tr>
<tr>
<td>9)</td>
<td>o</td>
<td>0.0419 ~ 0.321</td>
<td>$l/d_B \leq 0.125$</td>
<td>large</td>
<td>butanol water</td>
<td>$d_B/\rho_0 = 6 + 2.5 \left(Wr/Fr\right)^{0.5}$ (i)</td>
</tr>
<tr>
<td>10)</td>
<td>o</td>
<td>0.0794 ~ 0.397</td>
<td>0.318</td>
<td>1322</td>
<td>alcohol water...</td>
<td>$d_B/\rho_0 = 6 + 2.5 \left(Wr/Fr\right)^{0.5}$ (k)</td>
</tr>
<tr>
<td>11)</td>
<td>o</td>
<td>0.0393 ~ 0.165</td>
<td>0.04</td>
<td>0 ~ 1940</td>
<td>alcohol water...</td>
<td>$d_B/\rho_0 = 6 + 2.5 \left(Wr/Fr\right)^{0.5}$ (l)</td>
</tr>
</tbody>
</table>

* capillary  ** orifice

A) Steady bubble growth system

Consider that the gas flow into bubble is steady and that the bubble grows as sphere shapes at any instant. And when its buoyancy conquers the viscous drag or inertia, the bubble is released into liquid. Conveniently it is considered on next two gas flow rate regions.

i) Low gas flow rate region

The buoyancy of growing bubble at the hole balances with Stokes' viscous drag.

$$\pi d^3 \rho g = 3 \pi \mu d v$$

Gas which flows into bubble during $t$ seconds after bubble formation started is

$$Gl = \pi d^3 / 6$$

From Eqs. (1) and (3),

$$v = \frac{dz}{dt} = \left(11 \rho V / 16 \right) \left(6 Gl / \pi^2 \right)^{1/3}$$

Integrating Eq. (3) with $t$, on the assumption that bubble detaches off at $z = d_B/2$

$$V_B = \pi d_B^3 / 6 = 6.48 \left(\pi G / g\right)^{1/4}$$

ii) High gas flow rate region

The buoyancy of bubble balances the inertia of liquid accompanied with bubble,

$$V \rho g = d \left(11 \rho V / 16 \right) d z / d t / d t$$

The initial conditions are

$$d z / d t = 0 \text{ for } t = 0$$

$$z = 0 \text{ for } t = 0$$

The final condition is

$$V = V_B \text{ for } d_B / 2 = z$$

Integrating Eq. (5) under the condition of Eq. (6),

$$V_B = 1.378 \left(G^2 / g\right)^{3/5}$$

where authors obtained 7.44 and 1.722 respectively on the coefficient of Eqs. (4) and (7) by rewriting one of previous experimental equation. 3)

Then, it is considered that both viscous drag and inertia influence bubble size in the intermediate region, adding Eq. (4) to Eq. (7),

$$V_B = 6.48 \left(G^2 / g\right)^{3/4} + 1.378 \left(G^2 / g\right)^{3/5}$$

Eq. (8) indicates generally the bubble size
at detachment time. In Fig. 2 the experimental results\(^3,12\) and the theoretical values calculated by Eq. (8) are compared for steady bubble growth system. From this figure it is recognized that the theoretical values agree well with the experimental one over wide operating conditions from low to high gas flow rate region. But as it is difficult for experimental conditions to satisfy limiting condition \(A\), the states of potential bubble formation may appear in very low gas flow rate region.

Consequently the most important factors are gas flow rate \(G\) and kinetic viscosity of liquid \(\nu\) in chainlike bubble formation. And bubble volume is able to be approximated by Eq. (8).

**B) Potential bubble formation system**

In this case the bubble is formed by the pressure difference between gas chamber and liquid head of the liquid depth above the orifice. And gas flow rate into bubble changes as bubble grows. Authors consider an empirical correction from a static force balance because a simple differential equation cannot be derived.

i) Low gas flow rate region

The bubble will grow until its buoyancy exceeds the surface tension forces tending to hold it on the hole, when it will detach itself. That is

\[ V_{bg} \rho g = C \pi d_{or} \]  

(9)

Where correction coefficient \(C=0.796\) is obtained by the theoretical consideration on the growth of gas-liquid interface\(^13\). And authors obtained Fig. 3 by resettling the previous experimental data, and from this figure \(C\) is
0.835.

However it has been confirmed that Eq. (9) is applicable only when gas chamber volume is sufficiently small. This fact can be explained as follows.

As the gas flows into the gas chamber, the pressure raises gradually and excess gas volume is released into liquid when the pressure reaches to \((P_0 + \Delta P)\).

The volume of gas to be released is

\[
V_o = \Delta PV_o / \rho = 4\sigma V_c / \rho
\]  (10)

If \(V_o\) is smaller than \(V_B\) defined by Eq. (9), \(V_o\) increases with the continuous flow and arrives at \(V_B\) finally. But if \(V_c\) is sufficiently large and \(V_o\) is larger than \(V_B\) defined by Eq. (9), \(V_o\) itself indicates the bubble volume at detachment time. The critical value of \(V_c\) is given by equalizing \(V_B\) and \(V_o\).

\[
V_c = \pi d^2 P / 4 \rho g
\]  (11)

So

\[
V_c \geq V_c^c: \quad V_B = \pi C \cdot d_o \sigma / \rho g
\]  (12)

\[
V_c \leq V_c^c: \quad V_B = 4\sigma V_c / d_o P
\]  (13)

ii) High gas flow rate region

Considering the balance of the bubble buoyancy and the drag force toward the growing bubble,

\[
V_B \rho g = \epsilon (\rho \nu^2 / 2) \pi d^2 / 4
\]  (14)

On the other side, a continuous equation on the gas flow is

\[
G = (\sigma / 4) d_f \nu
\]  (15)

And introducing the empirical relation on \(d_f\) and \(d_B\),

\[
d_f / d_B = \epsilon (d_B / d_o)^{1/2}
\]  (16)

The following equation is obtained from Eqs. (14) (15) and (16).

\[
V_B = KG (d_o / g)^{1/2}
\]  (17)

Where \(K\) is not made clear theoretically, but it has been reported by L. Davidson \(^3\) and Tadaki and Maeda \(^3\) that experimentaly \(K = 1.62\) and \(K = 1.67\) respectively.

Assuming that the bubble size is given by Eqs. (12) (13) and (17) as an additional quality, the following equations are obtained.

\[
V \leq V_c^c: \quad V_B = C (\pi d_o \sigma / \rho g) = KG (d_o / g)^{1/2}
\]  (18)

\[
V \geq V_c^c: \quad V_B = 4\sigma V_c / d_o P = KG (d_o / g)^{1/2}
\]  (19)

Generally Eqs. (18) and (19) represent the bubble volume at detachment time. The comparisons between the experimental values \(^\text{6,8}\) and the calculated values obtained from Eqs. (18) and (19) are shown in Fig. 4. From this figure it is recognized that the calculated values agree with the experimental one over wide gas flow rate region.

Consequently the bubble volume is able to be approximated to Eqs. (18) and (19), and is little influenced with liquid viscosity \(\mu\) in this type bubble formation.

§ 4. Conclusions

As a fundamental study on contacting a gas with a liquid, the behaviors of bubble formation were studied theoretically.

The results are as follows;

1) The mechanism of bubble formation is classified into steady bubble growth system and potential bubble formation system by the interaction of the gas chamber and the plate thickness.

2) For cases of steady bubble growth system,
the bubble volume is calculated from Eq. (8).

3) For cases of potential bubble formation system, the bubble volume is calculated from Eqs. (18) and (19).

**Nomenclature**

- $c$: sound velocity in the gas \([ \text{cm/s} ]\)
- $C_D$: drag coefficient \([-\] ]
- $C$: correction coefficient defined by Eq. (19) \([-\] ]
- $d$: equivalent bubble diameter at any instant \([ \text{cm} ]\)
- $d_B$: equivalent bubble diameter at detachment time \([ \text{cm} ]\)
- $d_h$: hole diameter \([ \text{cm} ]\)
- $g$: gravitational acceleration \([ \text{cm/s}^2 ]\)
- $G$: mean volumetric gas flow rate \([ \text{cm}^3/\text{s} ]\)
- $G'$: critical volumetric gas flow rate \([ \text{cm}^3/\text{s} ]\)
- $K$: empirical coefficient defined by Eq. (17) \([-\] ]
- $l$: plate thickness \([ \text{cm} ]\)
- $P$: atmospheric pressure+liquid static head above the orifice \([ \text{dyne/cm}^2 ]\)
- $\Delta P$: pressure difference by the surface tension \([ \text{dyne/cm}^2 ]\)
- $t$: time \([ \text{s} ]\)
- $u_o$: gas velocity through the hole \([ \text{cm/s} ]\)
- $v$: bubble growth velocity \([ \text{cm/s} ]\)
- $V$: bubble volume at any instant \([ \text{cm}^3 ]\)
- $V_B$: bubble volume at detachment time \([ \text{cm}^3 ]\)
- $V_G$: gas chamber volume \([ \text{cm}^3 ]\)
- $z$: vertical distance from the center to the upper surface of the bubble \([ \text{cm} ]\)
- $\varepsilon$: empirical coefficient defined by Eq. (16) \([-\] ]
- $\sigma$: surface tension \([ \text{dyne/cm} ]\)
- $\rho_g$: gas density \([ \text{g/cm}^3 ]\)
- $\rho$: liquid density \([ \text{g/cm}^3 ]\)
- $\mu$: liquid viscosity \([ \text{g/cm-s} \text{ or } [\text{c.P.}] ]\)
- $\nu$: kinetic viscosity of liquid, \((\mu/\rho)\) \([ \text{cm}^2/\text{s} ]\)
- $Re$: Reynolds number, \((u_o d_p/\nu)\) \([-\] ]
- $We$: Weber number, \((d u_o^2/2\rho/\sigma)\) \([-\] ]
- $Fr$: Froud number, \((u_o^2/2g d_o)\) \([-\] ]

**Literature Cited**