On the Friction Coefficient for Turbulent Flow Through Sectionally Roughened Square Ducts*

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The friction coefficient of sectionally rough pipes has not been studied yet. The friction coefficient for turbulent flow through sectionally roughened square ducts is experimentally studied. Four arrangements of rough surfaces are used to obtain the sectionally roughened square ducts. It is attempted to predict the friction coefficient for sectionally roughened square ducts from the friction coefficients for alloverly roughened and smooth square ducts.

§ 1. Introduction

The friction coefficient for alloverly roughened circular pipes is analytically and experimentally studied by Nikuradse and the others. In recent years the interest has been developed in friction coefficient for sectionally roughened ducts because of problems encountered in nuclear reactor technology and superheater.

In this study, as an example of sectionally rough pipes the duct of square cross section is used.

§ 2. Experimental Studies

Fig. 1 shows the schematic diagram of the over-all flow circuit. Air from the room was drawn into the blower and passed through a flow straightener, suitable foreflow section and test section before exhausting into the atmosphere. The average velocities in the ducts were obtained using the following method. The square cross section of a duct was equally divided into eighty-one sections, and the velocity \( v \) at the center of each section was measured by a pitot tube. The average velocity \( V \) in the duct was calculated from the arithmetical mean value of \( v \). At the same time, the pressure difference between the pressure tap I and II in the Fig. 1 was measured and related with the average velocity \( V \). Using this relation, the average velocities at the measuring of the pressure drops along the ducts were determined.

The pressure drop was measured for eleven square ducts. All ducts were 3.6 meters long and the cross-sectional area of each duct was \( 90 \times 90 \text{mm}^2 \). Their rough walls were coated over with closely packed uniform sand grains selected with the suitable sieves. The mean diameter \( k \) of the sand grain and the relative roughness \( k/4m \) are shown in Table 1, where \( m \) is the hydraulic radius and \( 4m \) is called the hydraulic diameter. Fig. 2 shows the arrangements of rough surfaces. Fourteen pressure taps were located at intervals of 0.25 meters along the length of each duct allowing a precise determination of the local pressure.

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Table 1. Values $k$ and $k/4m$

<table>
<thead>
<tr>
<th>$k$</th>
<th>$k/4m$</th>
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<tr>
<td>0.905</td>
<td>0.01</td>
</tr>
<tr>
<td>1.840</td>
<td>0.02</td>
</tr>
</tbody>
</table>

§ 3. Results and Discussion

Foreflow-section of Square Ducts.

Fig. 3 shows representative pressure-gradient measurements taken at successive positions down the length of the square ducts. The pressure drop between two taps divided by the distance of two taps is taken to be the pressure gradient existing at a point halfway between the taps. It is plotted against the distance from the duct entrance made dimensionless with the hydraulic diameter. The foreflow-section length required for the pressure drop per unit length to reach a constant value unchanging with distance, has been determined experimentally. From Fig. 3 the length of approximately twenty times hydraulic diameters were required to arrive at a fully established flow.

Friction Coefficients for Turbulent Flow Through Square Ducts.

It is customary to express the dimensionless coefficient $\lambda$ frequently called a friction coefficient in the form

$$\lambda = \frac{\Delta P}{\tau l} \cdot \frac{4m}{l} \cdot \frac{2g}{V^2}$$  \hspace{1cm} (1)

where $\Delta P$ is the pressure drop in square duct length $l$, $\tau$ is the specific weight of the air and $g$ is the gravitational acceleration. The friction coefficient is a function of the Reynolds Number $Re = V(4m)/v$ ($v$ is the kinematic viscosity of air.) and the relative roughness $k/4m$.

Fig. 4 shows the obtained friction coefficients for alloverly roughened and smooth square ducts versus Reynolds Number. Assuming these values are represented by the function $\lambda = aRe^b$, Table 2 shows the values of $a$ and $b$. 

The smaller pressure differences were measured by a Göttingen manometer, and larger pressure differences were measured by means of a simple U-tube manometer. The alcohol was used in the one and the water was used in the other.

The temperature of the air at the test section was measured with a alcohol-in-glass thermometer, and the room pressure level was measured by a Fortin's barometer.

For each duct the flow rate was varied to obtain Reynolds Numbers in the range of $9 \times 10^4$ to $3.5 \times 10^5$. At a given flow rate, the pressures at the taps starting at the duct entrance and proceeding toward the duct exit were measured. The foreflow-section length necessary to establish fully developed flow could be directly determined from the knowledge of the pressure gradient along the duct. The particular combination of pressure-tap connections for each individual run was determined by the magnitude of the pressure drop per unit length. At the lower flow rates, the measurements were made over a larger distance to obtain readings of sufficient magnitude to insure accuracy. These measurements were made for all eleven ducts.
In Fig. 4, the solid lines show these functions, the chain line shows the Nikuradse formula \( \lambda = 0.0032 + 0.221/R_e^{0.237} \) for a circular smooth pipe and the dotted lines show the Kármán-Nikuradse formula \( \lambda = 1/(2 \log d/k + 1.138) \) for the circular rough pipe of diameter \( d \). In the figure, the experimental values obtained with the rough square duct is higher than the Kármán-Nikuradse formula for each same relative roughness value, and the experimental data for smooth square duct is lower than the Nikuradse formula. The maximum difference is about 19 per cent. Perhaps these disagreements will be due to the secondary flows in the square duct and a little difference of artificially roughened surfaces.

The major objective of this study is to predict the pressure drop for the flow through sectionally roughened square ducts. Let \( \lambda_n, \lambda_r \) and \( \lambda_s \) represent the friction coefficients for sectionally roughened, all-overly roughened and smooth square ducts respectively. It is assumed to express \( \lambda_n \) in the form

\[
\lambda_n = \frac{n \lambda_r + (4-n) \lambda_s}{4}
\]

(\( n = 1, 2, 3 \) )

where \( n \) is the number of rough walls.

Figs. 5 and 6 show the obtained \( \lambda_n \) versus \( R_e \). In the figures the solid lines show the predicted value from Equation (2). The predicted values are seen to lie above the experimental results for each case, and the maximum difference is about 15 per cent. These differences will be due to using the simple Equation (2) obtained by the method of arithmetical mean.

<table>
<thead>
<tr>
<th>Table 2. Values ( a ) and ( b )</th>
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<tbody>
<tr>
<td>( k/4m )</td>
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<tr>
<td>0.01</td>
</tr>
<tr>
<td>0.02</td>
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<td>0</td>
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</tbody>
</table>

Fig. 4 Friction coefficients for alloverly rough and smooth square ducts.

Fig. 5 Comparison of experimental and predicted friction coefficients for sectionally rough square ducts. \( (k/4m = 0.01) \).
§ 4. Conclusion

In the light of this investigation the following conclusions may be drawn in practice:

If the error of 20 per cent. is accepted,

1. The friction coefficient for flow through alloverly rough and smooth square ducts can be predicted from the one for the circular pipe respectively.

2. The friction coefficient for the sectionally roughened square ducts can be predicted from the Equation (2).

![Graph Comparison of experimental and predicted friction coefficients for sectionally rough square ducts (k/4m = 0.02)](image-url)