

Transient Displacement Response to Pulse Excitations on Periodontal Tissues

Hisao OKA*, Tatsuma YAMAMOTO* and Yoshiharu ISAYAMA*

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SYNOPSIS

In the field of dental study it is most fundamental and necessary to estimate the condition of periodontium. In order to examine a mechanical characteristics of periodontium, the theoretical displacement response to periodontal mechanical model (three elements model) are strictly solved in case of some pulse excitations. Impact excitations (rectangular, triangular and half-cycle sine pulse) are given in physical and mathematical definitions and complete solutions to the impact excitations are provided. The triangular pulse excitation which is obtained by means of a fracture of pencil-lead is most suitable. The mechanical parameters of periodontium are given using this input excitation. This is experimentally confirmed by artificial tooth model. The obtained mechanical characteristic of the periodontal tissues can be applied to clinical diagnosis.

* Department of Electrical and Electronic Engineering

1 INTRODUCTION

For dental diagnostic purpose, the tooth mobility test is the act of moving a tooth with fingers or tongue blades in order to determine whether it is loosely or tightly attached to the alveolus. Teeth normally have a certain range of mobility. Pathologic tooth mobility is recognized as an alarming sign of periodontal disturbance. Several variables contribute to pathologic mobility. The most common are loss of alveolar bone, alteration of the width of the periodontal ligament space or a combination of these factors⁽¹⁾. Therefore the mobility test should be used only as a supplementary form of diagnosis.

Measurements of tooth mobility have already been fully studied and some static⁽²⁾ and sinusoidal dynamic⁽³⁾ measurements have been proposed. The forces applied on the tooth, in occlusion or mastication, are of course basically different from static or sinusoidal loads. They should be considered as impulsive loads. In recent dental research there are many reports, which deal with the spectra of physical properties caused by impulsive loads⁽⁴⁾.

An observation of the gingiva surface appearance, a measurement of the periodontal pocket depth, a radiograph examination, a histopathological examination *etc.* are useful for dental diagnostic purposes. But their examinations demand skill and experience in the determination of mobility in general. In recent dental research, there are many reports which deal with the displacement response to impulsive loads applied on the tooth⁽⁵⁾. This study gives full consideration to the relationship between impulsive loads and physical properties based on an analysis of the mechanical elements. The hitherto tooth mechanical model is examined thoroughly and the displacement response is obtained theoretically. They are experimentally confirmed by artificial tooth model.

2 PHYSICAL PROPERTIES OF PERIODONTAL TISSUES

The supporting tissues of the teeth are referred to collectively as the periodontium. These tissues consist of the gingiva, periodontal ligament, cementum, alveolus, and supporting bone. Among them, the periodontal ligament has direct effects upon the tooth mobility. This chapter explains a structure of periodontium and its dynamic mechanical model.

2.1 Structure of periodontium

Fig.1 shows a structure of periodontium. Periodontium consists of the gingiva, periodontal ligament, cementum, alveolus, and supporting bone. The cementum covers a surface of tooth root. The mature periodontal ligament (0.1~0.3 mm) is organized into the principal and secondary fiber bundles. The principal bundles traverse the periodontal space obliquely and insert into the cementum and alveolar bone as Sharpey's fibers. The secondary bundles are randomly oriented between the principal bundles. The periodontal ligament and space contain a rich network of blood vessels, lymphatics, and nerves⁽⁶⁾. The gingiva is the mucous membrane covering the alveolar ridge and forming a cuff around the neck of each tooth.

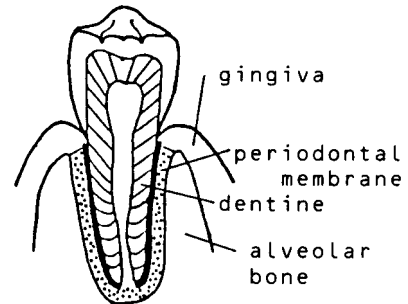


Fig.1 Structure of periodontium⁽⁶⁾.

Teeth normally have a certain range of mobility. The mobility is principally in a horizontal direction. It also occurs axially, but to a much lesser degree. Tooth mobility occurs in two stages: (1) The initial or intrasocket stages, in which the tooth moves within the confines of the periodontal ligament. this is associated with viscoelastic distortion of the ligament and redistribution of the periodontal fluids, interbundle content, and fibers. (2) The secondary stage, which occurs gradually and entails elastic deformation of the alveolar bone in response to increased horizontal forces. The tooth itself is also deformed by the impact of a force applied to the crown, but not to a clinically significant degree. When a force such as that normally applied to teeth in occlusion is discontinued, the teeth return to their original positions in two stages: the first is an immediate, spring-like elastic recoil; the second is a slow, asymptomatic recovery movement. The recovery movement is pulsating and is apparently associated with the normal pulsation of the periodontal vessels, which occurs in synchrony with the cardiac cycle. Pathologic mobility is caused by one or more of the following factors⁽⁷⁾:

(1) Loss of tooth support (bone loss). The amount of tooth movement

depends upon the severity and distribution of the tissue loss on individual root surface, the length and shape of the roots, and root size compared to that of the crown.

(2) The extension of inflammation from the gingiva into the periodontal ligament results in degenerative changes which increase mobility. The changes usually occur in periodontal disease which has advanced beyond the early stages, but pathologic mobility is sometimes observed in severe gingivitis.

2.2 Mechanical model of periodontal tissues

The movement which results in a human incisor tooth to the crown of which a horizontal force is applied may be divided into three characteristic phases: the range of initial tooth mobility, of intermediate tooth mobility, and of terminal tooth mobility⁽²⁾. Within the initial tooth mobility the resistance which the tooth puts up against being moved in its socket is very small. If the force is increased regularly with a dynamometer, it will be noted that with a certain amount of force (around 100 gf) this resistance suddenly increases. Far greater forces are now necessary, in order to push the crown over a distance similar to the phase of initial tooth mobility. Within the range of approximately 100 to 1500 gf force, the increase in motion stays in linear relation to the increase in force. With an increase in force over 1500 gf (maximal force), pain is registered. When a quasistatic horizontal or axial force is removed abruptly from the crown, the tooth will in both cases return towards its original position in two phases.

The first linear phase is an immediate spring-like elastic recoil movement. The second phase is a slow asymptomatic recovery movement (creep, elastic after-effect). Under normal conditions, the distance of the recoil movement is proportional (1) to the degree of elastic deformation of the socket and the surrounding bone and (2) to the elastic tooth distortion produced by tooth loading. If the tooth is loaded and unloaded with single horizontal forces different in magnitude, it will in both cases quickly return in a linear form to the same position at some distance from the original rest position. After rapid recoil the tooth gradually moves towards its original rest position. The slow recovery phase can be observed after horizontal or axial recoil movements and is similar. It is advantageously studied by measuring probes having no contact with the tooth. In teeth with

normal mobility and loaded horizontally complete recovery may last up to 90 seconds.

Though a number of mechanical models for periodontal tissues are feasible, three elements model shown in Fig.2 is suitable for the expression of the physical characteristics of periodontium. When the periodontium is necessarily diseased, the values of three elements (two springs and dashpot) change, and then the physiological changes of periodontium would be clear. A displacement response of the 3 elements model to impact load is similar to an actual case of periodontium. Then the three elements model is proper for a mechanical model of displacement response to impact load. When the applied excitation force on three elements model is $f(t)$, the displacement is $x(t)$ and the forces and displacements of the mechanical elements as shown in Fig.2 are the following respectively.

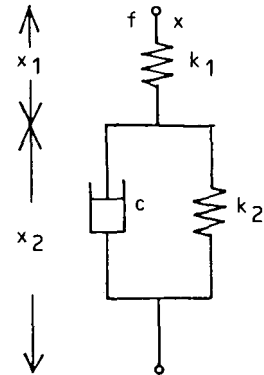


Fig.2 Three elements model (two springs and dashpot).

$$\left. \begin{aligned} x(t) &= x_1(t) + x_2(t) \\ f(t) &= c \frac{dx_2}{dt} + k_2 x_2, \quad f(t) = k_1 x_1 \end{aligned} \right\} \quad (1)$$

If the assumption is that the initial conditions of x , x_1 , x_2 and dx/dt are zero at the time $t=0$, the following equation in $x(t)$ is obtained.

$$X(s) = \{F(s)/k_1\} \{s + (k_1 + k_2)/c\} / (s + k_2/c) \quad (2)$$

Where, $X(s)$ and $F(s)$ are the Laplace transforms of $x(t)$ and $f(t)$ respectively.

3 DISPLACEMENT RESPONSES TO SOME PULSE EXCITATIONS

The input excitation $f(t)$ applied on the three elements model is examined in cases of rectangular, triangular and half sine pulse excitation.

3.1 Rectangular pulse excitation

When the input excitation is a rectangular pulse $f(t)$ with the magnitude F_0 , $f(t)$ is a superposition $f_1 + f_2$ of two step functions f_1 and f_2 . Where $f_1(t)$ is $F_0 u(t)$, $f_2(t)$ is $-f_1(t-d)$, $u(t)$ is a unit step

function and d is a weighting time. The displacement response is

$$\left. \begin{aligned} x(t) &= x_1(t) + x_2(t) \\ x_1(t) &= (F_0/k_1) \{ (k_1+k_2)/k_2 - (k_1/k_2) e^{-(k_2/c)t} \} u(t) \\ x_2(t) &= -x_1(t-d) \end{aligned} \right\} \quad (3)$$

$x_1(t)$ and $x_2(t)$ are step responses to f_1 and f_2 respectively.

3.2 Triangular pulse excitation

When the input excitation is a triangular pulse $f(t)$, $f(t)$ is a superposition $f_3+f_4+f_2$ of two ramp functions $f_3(t)=tF_0u(t)/d$, $f_4(t)=-f_3(t-d)$ and a step function f_2 as shown in Fig.3. When $f(t)$ in eqn.1 is a triangular pulse, a displacement response $x(t)$ is

$$\left. \begin{aligned} x(t) &= x_3(t) + x_4(t) + x_2(t) \\ x_3(t) &= (F_0/k_1 d) \{ (-ck_1/k_2^2) + (k_1+k_2)/k_2 t + (ck_1/k_2^2) e^{-(k_2/c)t} \} u(t) \\ x_4(t) &= -x_3(t-d) \end{aligned} \right\} \quad (4)$$

x_2 , x_3 and x_4 are responses to f_2 , f_3 and f_4 respectively.

3.3 Half sine pulse excitation

When the input excitation is a half sine pulse excitation $f(t)$, $f(t)$ is a superposition f_5-f_6 of two sine waves $f_5(t)=F_0u(t) \sin \gamma t$ and $f_6(t)=f_5(t-d)$. When $f(t)$ in eqn.1 is a half sine pulse, a displacement response $x(t)$ is

$$\left. \begin{aligned} x(t) &= x_5(t) + x_6(t) \\ x_5(t) &= (F_0/k_1) \{ \gamma Q e^{-(k_2/c)t} + M \sin(\gamma t - \psi) \} u(t) \\ x_6(t) &= -x_5(t-d) \end{aligned} \right\} \quad (5)$$

where $\gamma = \pi/d$

$$Q = k_1 c / (k_2^2 + \gamma^2 c^2)$$

$$M = \{ 1 / (k_2^2 + c^2 \gamma^2) \} \sqrt{ k_2^2 (k_1 + k_2)^2 + c^2 \gamma^2 (k_2^2 + 2k_1 k_2 + 2k_2^2) + c^4 \gamma^4 }$$

$$\psi = \tan^{-1} [\{ k_2 (k_1 + k_2) + c^2 \gamma^2 \} / c \gamma k_1]$$

3.4 Determination of the mechanical model parameter

The recovery of a tooth on removal of a horizontal load is in two phases. Initially, there is a fast return of the tooth towards its starting position (in a linear manner with time). As the tooth approached the starting position, a second and logarithmic phase developed. Fig.4 illustrates the pattern for human maxillary central incisor following the sudden application and sudden removal of a

load⁽⁸⁾. Thus, on applying the load there is an initial rapid displacement away from the force, followed by a phase of a more gradual displacement, and also the bi-phasic response on removal of the load. The resistance of the periodontal tissues to horizontal loading cannot be explained in terms of a simple linear mechanical model. The pattern shown in Fig.4 might suggest a viscoelastic system composed of Maxwell or Voigt elements (springs and dashpots in series or in parallel).

Fig.5 shows a theoretical displacement response in case of rectangular pulse excitation. Fig.6

shows in case of triangular pulse excitation. R_i is an elastic recoil movement. R_e is a slow recovery movement. Fig.7 shows a theoretical response in case of half sine pulse excitation and also has a slow recovery movement at sudden removal of load.

When a maximum of displacement is X_0 , a damping ratio is σ and a slope of tangent is D ,

$$\left. \begin{aligned} X_0 &= (F_0/k_1 k_2 d) e^{-(k_2/c)} \{ (ck_1/k_2) (1 - e^{(k_2 d/c)}) + k_1 d e^{(k_2/c)} \} \\ \sigma &= 1/\tau = k_2/c \\ D &= F_0(k_1 + k_2)/(k_1 k_2 d) \end{aligned} \right\} \quad (6)$$

These mechanical parameters of the damped curve can be obtained experimentally. The values of the mechanical elements can be obtained from three parameters of eqn.6.

$$\left. \begin{aligned} c &= (F_0/X_0) \{ e^{-\sigma d} + \sigma d - 1 \} / (\sigma^2 d) \\ k_1 &= \{ F_0 (e^{-\sigma d} + \sigma d - 1) \} / \{ (e^{-\sigma d} + \sigma d - 1) d D - X_0 \sigma d \} \end{aligned} \right\} \quad (7)$$

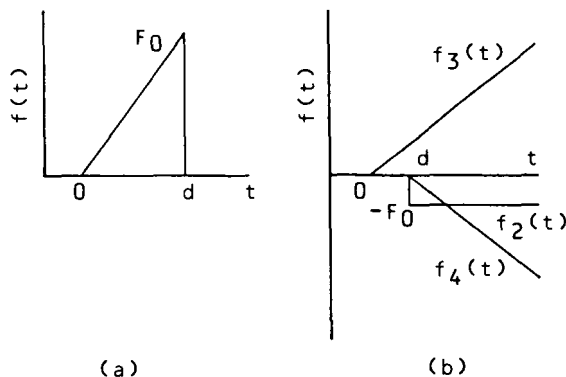


Fig.3 Triangular pulse excitation.

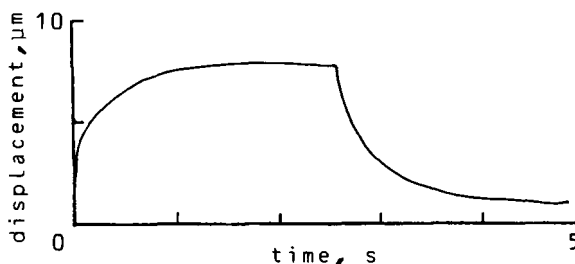


Fig.4 Relationship between horizontal tooth displacement and time following the sudden application and removal⁽⁸⁾.

$$k_2 = (F_0/X_0) \{e^{-\sigma d} + \sigma d - 1\} / (\sigma^2 d)$$

The input excitation, however, must be measured and F_0 and d must be known.

4 MEASURING SYSTEM FOR IMPACT RESPONSE

In a dental study, the small-sized piezoelectric accelerometer which is responsive to mechanical acceleration, is used and adheres to the labial surface of tooth. A mechanical excitation is applied by means of an impact hammer. In this study, an optical displacement transducer (PT-A165, KEYENCE) which is out of contact with tooth and a fragment of a pencil-lead which is used in the field of acoustic emission⁽⁹⁾ are used.

Fig.8 shows a block diagram of a measuring system. The displacement transducer is held 20mm away from the surface of tooth. The optical axis of transducer is perpendicular to tooth surface and the range of detection is ± 2 mm. A load-cell (strain gauge) of pencil-lead is placed on to pencil tube and the input excitation force can be

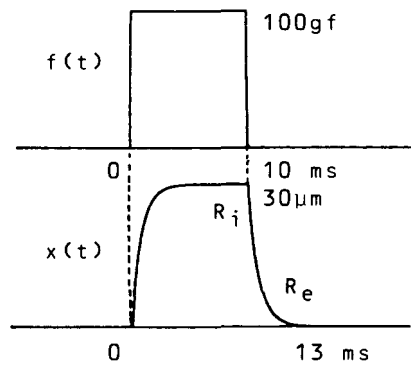


Fig.5 Theoretical displacement and time following in case of rectangular pulse excitation.

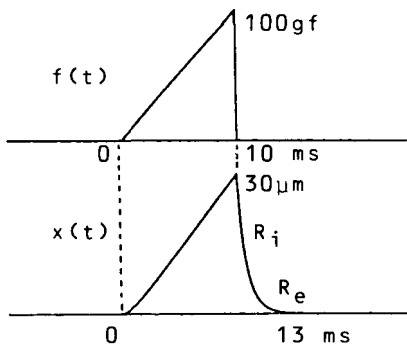


Fig.6 Theoretical displacement and time following in case of triangular pulse excitation.

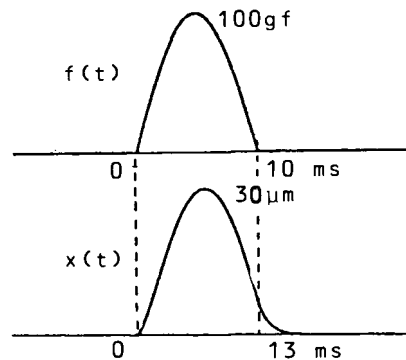


Fig.7 Theoretical displacement and time following in case of half sine pulse excitation.

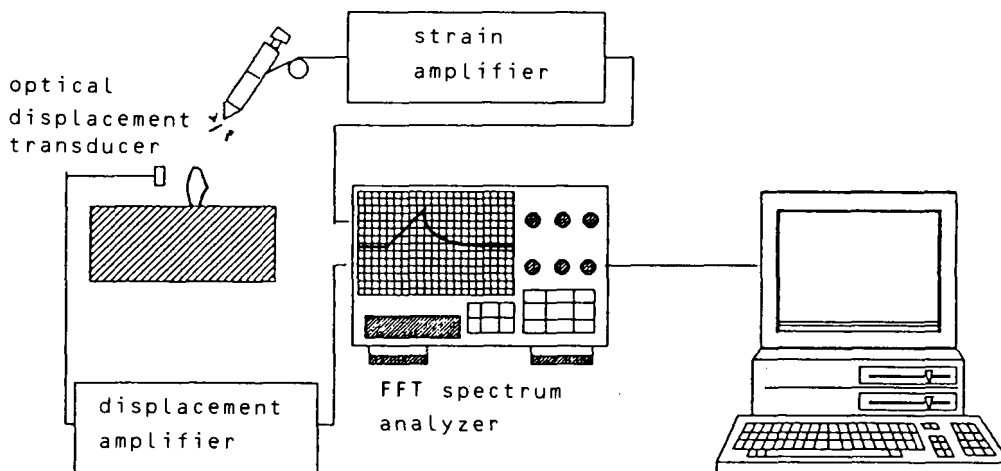


Fig.8 Block diagram of measuring system.

measured. The digital processing of the excitation force and displacement response is performed on a FFT spectrum analyzer (SM2701, IWATSU) and then a data analysis is performed on a general-purpose computer (PC9801VX2, NEC), or the data time-courses are inscribed on a recorder. The experimental results are easily shown on the monitor display of the computer, copied and stored on a floppy disk.

A triangular pulse excitation is obtained by means of a pencil-lead fragment⁽¹⁰⁾. The pencil-lead is snapped at once on the incisal edge and it is possible to obtain a relative stable triangular pulse. The weighting interval d can be sufficiently varied and unloading interval is about $1 \mu\text{s}$.

In the measurement of human teeth *in vivo*, the subject must sit on the dentist-chair without a voluntary immobilization of his head, and open his upper and lower lips using an angle-wider. The weighting interval of the input excitation using a pencil-lead is so short that it is possible to measure the input excitation in the maxillary and mandibular periodontal tissues.

5 IMPACT RESPONSE OF ARTIFICIAL TOOTH MODEL

Some tooth models illustrated in Fig.9 are made. The thickness of silicone impression material (Flexicon Regular Type) simulating periodontal ligament are changed $0.28 \sim 0.84\text{mm}$. The weight of epoxy

resin tooth is 0.95g and the length of tooth root is 15mm. The pencil-lead is fractured in the lingual direction from labial. Table 1 shows the good linearity of mechanical model parameters to magnitude of excitation. F_0 is 56.9 ~ 80.7gf and is determined by the type and protruded length of the pencil-lead (length: 5mm; diameter: 0.5mm, degree of optical density on paper:HB).

The response to triangular pulse excitation (70gf) is shown in Fig.10(a) ~ (c). Table 2 shows their mechanical model parameters (average values of 6 measurements). (b) and (c) assume pathologic teeth and are characterized by a slope of tangent D and damping ratio σ bigger than those of a healthy tooth (a). It is adequate that the mechanical model parameters of the pathological teeth are smaller than those of the healthy tooth.

6 CONCLUSION

A new impact response method is proposed, which adopts a relatively long triangular pulse at the fracture of pencil-lead which is easily obtained experimentally and is easily expressed mathematically. In the usual shock

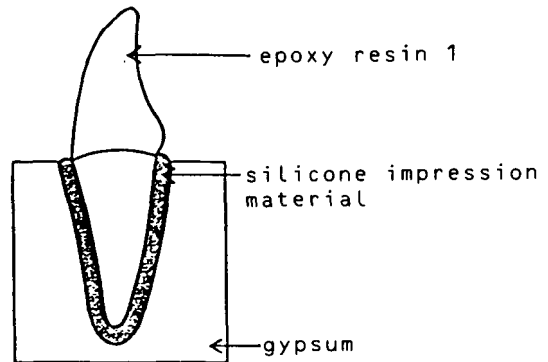


Fig.9 Artificial tooth model with silicone impression material.

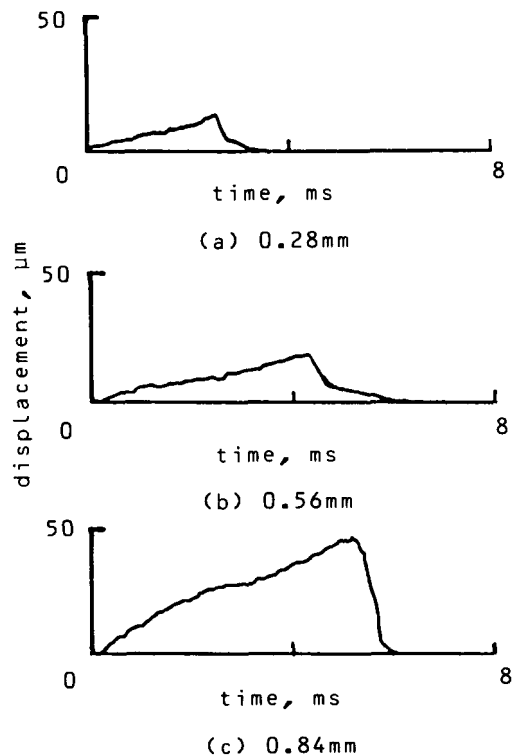


Fig.10: Experimental responses to triangular pulse excitation (70gf); (a)0.28mm, (b)0.56mm and (c)0.84mm.

Table 1 The linearity of mechanical model parameters to input magnitude.

input magnitude gf	c Ns/m	k_1 $\times 10^4$ N/m	k_2 $\times 10^4$ N/m
56.9	13.4	21.7	3.51
67.4	12.6	21.7	3.47
80.7	12.7	22.5	4.00

Table 2 The mechanical parameters in case of some thickness of silicone impression material; (a)0.28mm, (b)0.56mm and (c)0.84mm.

thickness mm	c Ns/m	k_1 $\times 10^4$ N/m	k_2 $\times 10^4$ N/m
(a) 0.28	35.1	101	8.41
(b) 0.56	14.1	24.1	4.04
(c) 0.84	4.76	6.76	1.61

measurement of dentistry, a frequency spectrum of the acceleration response after input excitation is measured. In that analysis, the input excitation is not considered and only the acceleration response is investigated. In this study, the input excitation is strictly defined physically and mathematically and complete solutions to some of the input excitations are provided for the first time. This method requires both the measurements of displacement response and input excitation. The model parameters of which the mechanical characteristic of periodontal tissues is expressed, can be obtained and applied to clinical diagnosis.

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