A Study on the Elasto-plastical Constitutive Equation for Unsaturated Soil

Makoto Nishigaki *, Yao Sun ** , Iichiro Kohno*

(Received January 7 , 1994)

SYNOPSIS

In this paper, first, an elasto-plastic consitituve equation for unsaturated soil was developed by considering of the basically behavior of unsaturated soil. Second, the results of a number of triaxial test and a set of rigid foundation model tests were simulated by using this constitutive equation, the agreement between observed and computed results was satisfactory and confirms the possibilities of this constitutive equation.

1. INTRODUCTION

Unsaturated soil is often encountered in engineering practice. It is desirable to know how they will behave under various states of stress when considering their use in engineering practice. Much of the early design work for engineering projects using unsaturated soils was based on empirical rules or experience with a particular soil. In many instances this practice is still the basis for current design procedures. There has , however, been a considerable effort extended toward the development of a more reliable means of predicting the behavior of unsaturated soils. In particular, the strength and compressibility of unsaturated soils must be determined in order to make the most efficient use of these materials in practice.

In past decades, there are many researchers to have been strived to establish a reasonable constitution model on the unsaturated soils. Coleman(1962)[1] suggested a set of incremental stress-strain equations by stress variables,(σ_m - Ua), (U_a - U_w) and (σ_1-σ_3), in which ninth coefficients were included. Fredlund (1978)[2] ,Karube (1978)[3]had discussed the physical meaning and determine method for those coefficient by test results. In same stress space as Coleman, Matyas (1968)[4] had studied the volume characteristics of partially saturated soil. For avoiding the difficulty of determining the coefficince about effective stress

* Department of Civil Engineering
** The Graduate School of Natural Science and Technology

equation , Fredlund (1978) modified Mohr-Coulomb failure criteria in (σ_m - Ua), (U_a - U_W), and (σ_1-σ_3) stress condition. In present , Karube(1989)[5] and Alonso (1987),(1990)[6,7] investigated the yield characteristic of unsaturated clay ,sand clay by the experiment in which (U_a - U_W) was controlled, presented a constitutive model for partially saturated soils , respectably. In addition , Kohge (1991)[8] had developed a model to predicate the elasto-plastic deformation behaviors about partially saturated soils. In that research , suction (U_a - U_W) was adopted as a variables to describe the stress-strain behaviors with other stress variables .The presented models almost are the extend ones of Cam-Clay-Model. Because if a stress variable is incremented in model, the constitutive equation will become more complicate. Therefor, one aim in our research is try to make the constitutive equation simple for understanding easily.

In this paper, first, a elasto-plastic constitutive equation for unsaturated soil was developed by considering of the basically behavior of unsaturated soil. Through analysising the relationship among pore-pressure, water content ratio and pore ratio, and interaction effect of the change of pore-pressure and the change of mechanical propertice of unsaturated soil , the reasonableness of selecting degree of saturation as a variable parameter in the constitutive equation of unsaturated soil, was discussed. Second, the results of a number of triaxial test and a set of rigid foundation model tests was simulated by using this constitutive equation.

2. STRESS FRAMEWORK

In the research on the partially saturated soils, first , we must determine the variables that can be easily used for describing the behaviors of mechanics of the partially saturated soils. If we can easily achieve the effective stress equation of unsaturated soils , we are able to make a extension from the result of saturated soil to unsaturated soil , simply. But, as we known , now it is difficulty to apply an equation of effective stress in both strength and deformation calculation. In many effective stress equations ,the Bishop's equation was widely discussed and applied, the reason is considered that is one extension of Terzaghi' effective stress equation for saturated stress

$$\sigma' = \sigma - Ua + \chi(Ua\text{-}Uw) \qquad (1)$$

Where χ is a coefficient, it depends upon the degree of saturation Sr , soil structure, and the cycle of wetting-drying. (Bishop.1960)[8]. Because coefficince value χ is scattered in the different test conditions, it is difficulty to utilize Eq.1.[9]

In addition, Fredlund (1978) had selected σ_{ij} - $\delta_{ij}U_a$,σ_{ij} - $\delta_{ij}U_W$, and (Ua-Uw) as the independent stress variables for describing the behavior of partially saturation soil, in term of those stress fields, stress-strain-strength theory and consolidation equation was presented. But, for formatting the relationships among σ_{ij} - $\delta_{ij}U_a$, σ_{ij} - $\delta_{ij}U_W$, and (U_a-U_W), a many number of tests have to be done. That will need much time and high cost.

In last research, the suction (U_a-U_W) and the excess of total stress over air pressure (σ_{ij} - $\delta_{ij}U_a$

) often has been adopted as the relevant stress variables to establish the constitutive equations of unsaturated soil. Why did the suction (U_a-U_w) was selected as a variable like an external pressure to express the change of mechanical behavior of unsaturated soil ? we think that there exist two reasons almost: 1) the suction can give an clearly mechanical idea for moisture move in unsaturated soil, the moisture move generally produce the internal soil structure change. 2) In triaxial test, Ua can be easily controlled, if their are enough consolidation time, the suction (U_a-U_w) also can be controlled. In practice, the measurement of - Uw in shear test stage is vary difficulty because of influence effect of shear speed and drain condition, so that the suction (U_a-U_w) cannot be controlled easily. In other way, the relationship of water head ψ and Sr can be established by PF test. From PF moisture curve as shown in Fig.1, it can be understand that the ψ has the different values in related to drain and absorb process at same saturated degree point and in Sr = 90～40 % range the value vary of ψ is small, but as we known in this range of Sr the mechanical behavior has the significantly change. According to the analysis as above, we can think that there are the some advantages when degree of saturation is adopted as a parameter. For example, variation of mechanical behaviors can be easily understand from physical properties ; variation of degree of saturation in sample can also be conveniently understand through measurement drain water content ; it can be ease to make a extend of the theory that had been established in saturated soil to unsaturated soil.

In this paper, the excess of total stress over air pressure (σ_{ij} - $\delta_{ij}U_a$) and degree of saturation Sr are adopted as stress variable and variable parameter in the constitutive equation.

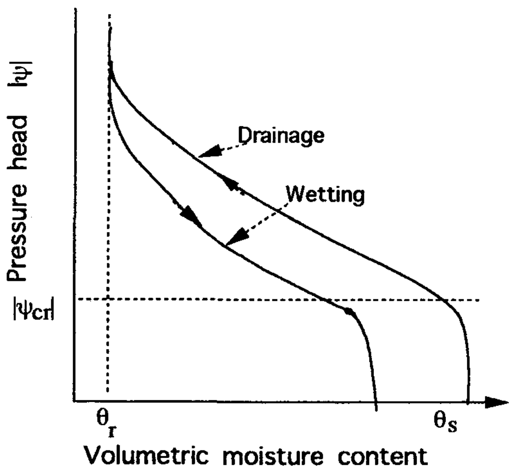


Fig.1 Hysteresis between volumetric moisture content and pressure head.

3. A ELASTO-PLASTIC CONSTITUTIVE EQUATION OF UNSATURATED SOIL

In this section, a elasto-plastic constitutive equation for unsaturated soil is presented in according to the triaxial test results. there are two assumes in the derivation of the constitutive equation 1) the study is limited in isotropy stress state; 2) associated flow rule is satisfied in unsaturated soil.

3.1 Yield Function and Hardening Modulus

Yield Stress Point

By reference to the actuarial manner of determine a yield stress illustrated by Yasfuku (1991,1991)[10,11], a yield point was defined as the state of stress at a marked changes in slope of each stress - strain curve ,we can achieved data of yield stress point from the typical stress - strain curve that was showed in Fig.2.

Comparison of Yield Curve in Different Degree of Saturation

Fig.3 show the yield curves in the $q \sim p(=\sigma_m - Ua)$ space, that was obtained from the different stress path tests . From Fig.3 clearly the shape of yield curves is roughly same in different degree of saturation. Because the triaxial test is restricted in isotropic consolidation range in compressibility side only, the effect of loading-path, stress history cannot be discussed in this paper. Here, we assume that yield curve is symmetrical about both compressive side and extension side.

The slopes of yield curve segments for isotropic unsaturated soil can be considered as an unique function of the stress ratio in different degree of saturation, respectably. From Fig.3, we can achieve $dq/dp \sim \eta$ data in different degree of saturation , directly. And, those are plotted in Fig.4. From Fig.4, we can consider that the $dq/dp \sim \eta$ relationship is notablly same for the different degree of saturation. Therefore, the $dq/dp \sim \eta$ relationships can be represented as a unique function of stress ratio and degree of saturation in the irrespective of the proportional loading path and the stress level. That is

$$dq/dp = G(\eta, Sr) \tag{2}$$

From Fig.4, Eq.2 can be approximately expressed by a hyperbola equation, that is

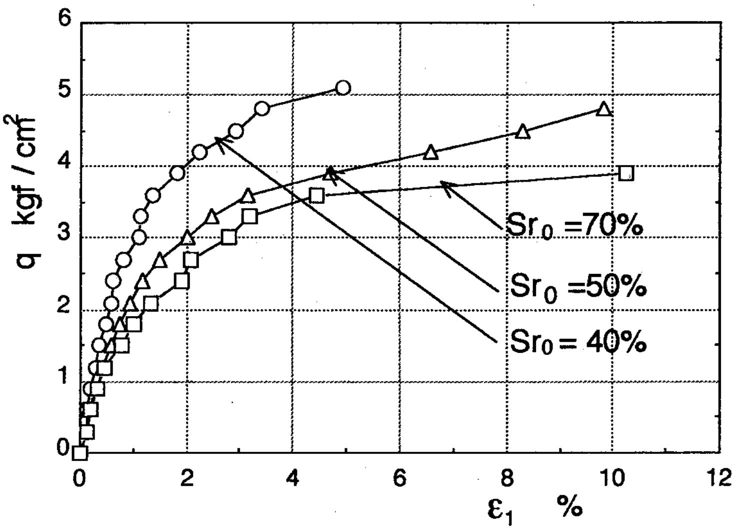
$$\frac{dq}{dp} = \frac{\{\eta - (2-c^y)\}\eta - \{N^y(s_r) - (2-c^y)\}N^y(s_r)}{c^y \eta} \tag{3}$$

Where , $N^y(S_r)$ and C^y are experimental parameters, in which $N^y(S_r)$ is the value of stress ratio when $dq / dp = 0$, and that has a little variation in different degree of saturation; C^y is a parameter of evaluating the shape of a hyperbola. We can take C^y as a constant in the different degree of saturation.

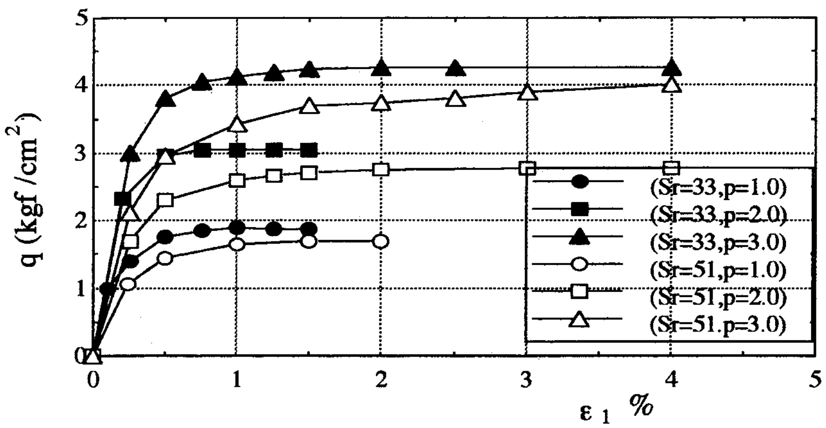
Formulation of Yield Function

Using Eqs.(2),(3) the entire yield curve can be formed as follows . From the equation of $q = \eta p$, we get $dq = \eta dp + pd\eta$. Putting this relationship into Eq.3, we can obtain the following general form :

$$Ln\frac{p}{p_o(s_r)} = \int_0^{\eta} \frac{1}{G(\eta, s_r) - \eta} d\eta \tag{4}$$



Axial Strain versus Deviatoric Stress



Axial Strain versus Deviatoric Stress

Fig.2 The typical stress-strain curve of unsaturated soil ("KAMISAKI" decomposed granite soil)

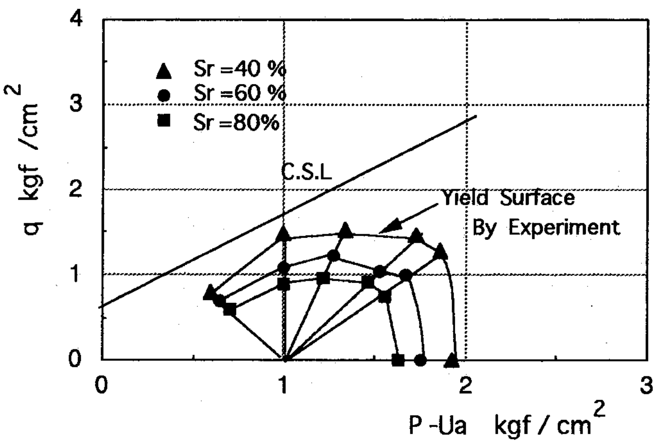


Fig.3 Variation of yield curves with degree of saturation for unsaturated soil ("KAMISAKI" decomposed granite soil)

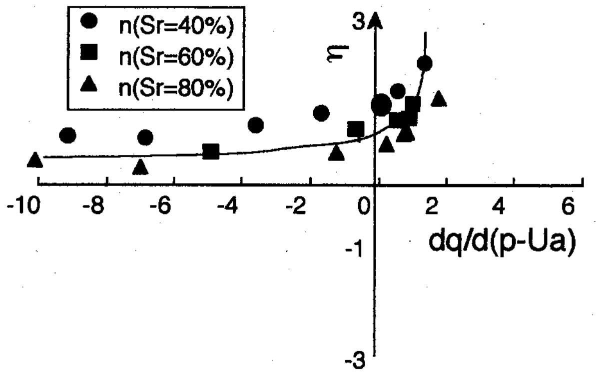


Fig.4 dq/d(p-Ua) ~η relationship (from yield curve of fig.3) ("KAMISAKI" decomposed granite soil)

Where, $P_0(S_r)$ is defined as the yield pressure of consolidation in the different degree of saturation. Substituting eqs.(2,3) into Eq.6 , the following yield function is derived

$$f = Ln\frac{p}{p_0(s_r)}+\frac{c^y}{2(c^y-1)}Ln\{\frac{(N^y)^2-(1-c^y)\eta^2}{(N^y)^2}\}=0 \tag{5}$$

In Eq.5, C^y is not able to equal 1.0. When C^y is equal to 1.0, Eq.5 can be simplified to Eq.(6)

$$f = \eta^2 +2(N^y)^2Ln\{\frac{P}{p_0(s_r)}\}=0 \tag{6}$$

Property of Isotropic Compressive of Unsaturated Soil

In to evaluate relationship of hardening modulus, it is necessary to research isotropic consolidation behaviors. Fig.5 show a set of typical curves of isotropic compressibility of unsaturated soil, in which initial degree of saturation was controlled. The relationship between compressive index λ and degree of saturation could be represented as $\lambda = f(e_0, Sr)$, when e_0 =constant, it can be written as follows,

$$\lambda(Sr) = a + b\,Sr \tag{7}$$

Where, $\lambda(Sr)$ is (a compressive index) stiffness parameter for changes in net mean stress for virgin states of the soil; a,b are parameters of the test for specific elastic stiffness parameter . In addition, we assume that the rebound elasticity modules k, elastic stiffness parameter for changes in net mean stress, is only a function of stress level for unsaturated soil.

For the collapse or swelling deformation problem, first ,we can use Eq.7 to achieve both $\lambda(Sr)_0$ of be-wetting and $\lambda(Sr)$ of pro-wetting in according to Sr values. Second, deformation amounts of be-wetting and pro-wetting can also be calculated by use of constitutive model. Finally, the deformation different between be-wetting and pro-wetting can be considered as collapse or swelling deformation in this stress state.

Evaluation of Hardening Modulus

In to complete the yield function as presented in Eq.5,6, it is necessary to evaluate the hardening modulus, in other words , to formulate the internal variables k^p and $P_0(Sr)$ in the yield function, Eq.5,or 6.

For saturated sand, it is identified that the observed yield curves and contours of equal normalized energy was roughly similar (YASFUKU et al(1991)), in which the normalized energy is a state parameter proposed by Moroto(1976), that is

$$dk^p=d\varepsilon_v^p +\eta d\varepsilon^p \tag{8}$$

In here, k^p is the function of hardening modulus, we assume that Eq.8 is also satisfied in unsaturated soil. On the basis of on the plastic flow rule, the plastic volumetric and shear strain increment is written as follows:

$$d\varepsilon_v^p = \wedge\frac{\partial f}{\partial p} \qquad d\varepsilon^p = \wedge\frac{\partial f}{\partial q} \tag{9}$$

In here,
$$\wedge = \frac{1}{H}\left(\frac{\partial f}{\partial p}dp + \frac{\partial f}{\partial q}dq\right) \tag{10}$$

Where, $\wedge$ is a ratio parameter; H is hardening modulus. Substituting Eq.9,10 into Eq.8, the following Eq.11 is derived.

$$dk^p = \wedge\left(\frac{\partial f}{\partial p}dp + \eta\frac{\partial f}{\partial q}dq\right) = \wedge\overline{k^p} \tag{11}$$

In according to general elasto-plastic theory, yield function can be written as following form, f =(p,q,kp,Sr), in the equation S_r is taken as referent variable and connected in k^p function, that is

$$df = \frac{\partial f}{\partial p}dp + \frac{\partial f}{\partial q}dq + \frac{\partial f}{\partial k^p}d\overline{k^p} = 0 \tag{12}$$

Substituting Eq.10 ,11 into Eq.12 and rearranging Eq12, the Hardening modulus H can be written as follows

$$H = \frac{\partial f}{\partial k^p}\bar{k} \tag{13}$$

and
$$\frac{\partial f}{\partial k^p} = \frac{\partial f}{\partial p_0(s_r)}\frac{\partial p_0(s_r)}{\partial k^p} \tag{14}$$

and based on the yield function presented in Eq.6

$$\frac{\partial f}{\partial p_0(s_r)} = \frac{2(N^y)^2}{p_0(s_r)} \tag{15}$$

In the plan of stress ratio $\eta = 0$, the partial derivative $dp_0(S_r)/dk^p$ in Eq.14 is generally obtained by observing the relationships between k^p and $p_0(Sr)$ during isotropic consolidation . Here is the case of isotropic consolidation . dk^p is assumed to be equivalent to plastic volumetric strain $d\varepsilon^p{}_v$. Now, based on the e ~ p linear relation, the total and elastic volumetric strain increments $d\varepsilon_v$, $d\varepsilon^e{}_v$ under isotropic consolidation can be expressed by :

$$d\varepsilon_v = \frac{de}{1+e_0} = \frac{\lambda(s_r)}{1+e_0}dp_0(s_r) = \frac{\lambda}{1+e_0}dp_0 \tag{16}$$

$$d\varepsilon_v^p = \frac{\lambda(s_r)-\kappa}{1+e_0}dp_0(s_r) = \frac{\lambda-\kappa}{1+e_0}dp_0 \tag{17}$$

If relationship of $P_0(S_r) \sim e$, or $P_0 \sim e$, can be repressed as straight line equation, we can achieve Eq.18, that is

$$p_0(s_r) - \bar{p} = \frac{\lambda}{\lambda(s_r)}(p_0 - \bar{p}) \tag{18}$$

then, Eq.15 can be rewritten as follows :

$$\frac{dp_0(s_r)}{dk^p} = \frac{1+e_0}{\lambda(s_r)-\kappa} \quad ; \quad \frac{dp_0}{dk^p} = \frac{1+e_0}{\lambda-\kappa} \tag{20}$$

where , $\lambda(Sr)$ is determined by Eq.8 ; λ is stiffness coefficient at saturation soil ; $P_0(Sr)$, P_0 are net mean yield pressure in unsaturated and saturated states, respectively. Substituting the above equations into Eq.13, we can explicitly obtain the hardening modulus H .

$$H = \frac{1}{dp_0}\frac{1+e_0}{\lambda(s_r)-\kappa}\left(\frac{\partial f}{\partial p} + \eta\frac{\partial f}{\partial q}\right) \tag{21}$$

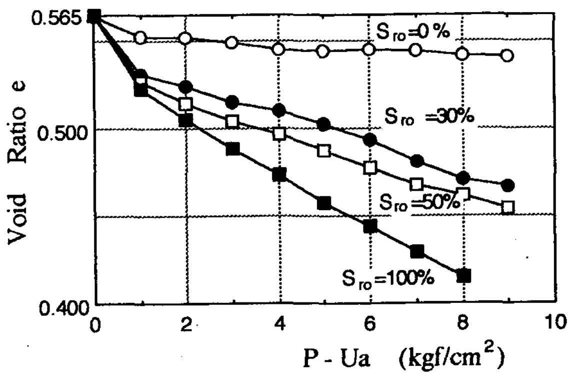


Fig. 5 Void ratio versus (P -Ua) curves ('KAMISAKI" decomposed granite soil)

3.2 Stress - Strain relationship

In according to general elasto-plastic mechanics law , a stress-strain increment form matrix equation can be derived,

$$\begin{vmatrix} d\varepsilon_v \\ d\varepsilon \end{vmatrix} = \begin{bmatrix} \frac{H}{K(s_r)}+(\frac{\partial f}{\partial p})^2 & \frac{\partial f\,\partial f}{\partial p \partial q} \\ \frac{\partial f\,\partial f}{\partial p \partial q} & \frac{H}{3G(s_r)}+(\frac{\partial f}{\partial q})^2 \end{bmatrix} \begin{bmatrix} dp \\ dq \end{bmatrix} \tag{22}$$

In here, K(Sr) and G(Sr) are the elastic incremental bulk and shear module, respectively. They are the functions of compressor and degree of saturation. The hardening coefficient H and yields function **f** can be calculated by using Eqs.21,6.

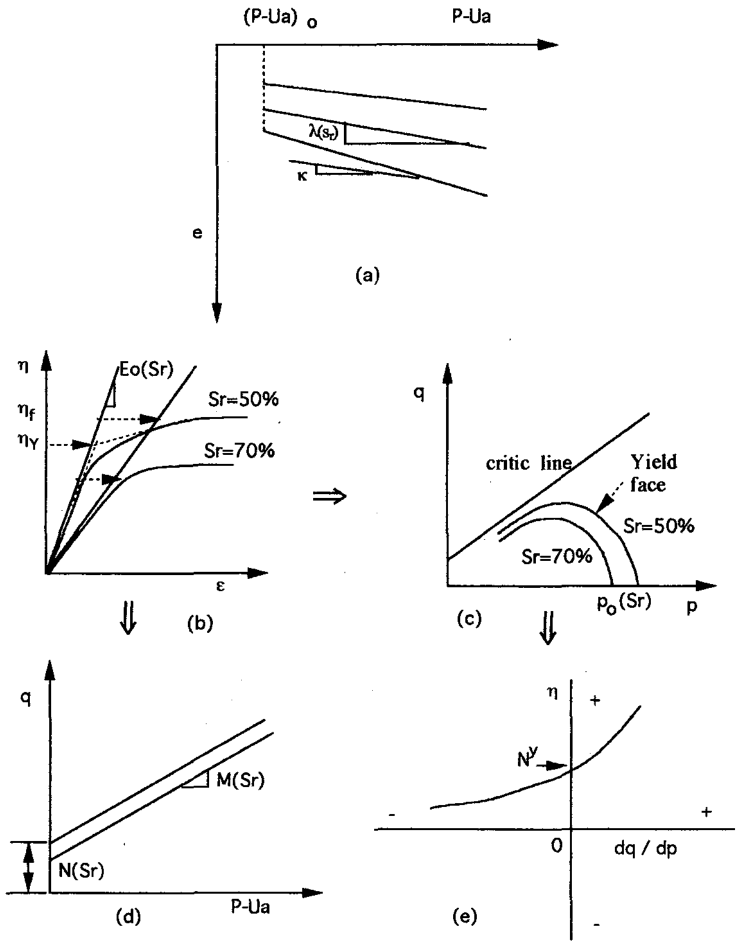


Fig.6 Determination of the parameters for the constitutive equation of unsaturated soil

3.3 Parameters of the constitutive equation and their determination

Parameters of the constitutive

The application of the constitutive equation requires to determine the following stress states and parameters. They are

(a) Initial state: that include initial stresses (p_0, q_0), initial degree of saturation Sro, and initial pore ratio(e_0), defining the initial position of stress state.

(b) Parameters directly associated with the yield curve (isotropic stress state): That include p_0(Sr) (in unsaturated state), and p_0 (in saturated state) of yield stress in isotropic consolidation state of having same plastic volume strain.

λ(Sr) =a + b Sr, compressibility coefficient for unsaturated state along virgin loading;

k, compressibility coefficient along elastic (unloading - reloading) stress paths;

λ, compressibility coefficient for saturated state, along virgin loading.

(c) Parameters directly associated with charges of shape of yield curve: that include C^y (parameter to evaluate the slope of a hyperbola in $\eta \sim$ dp/dq relation curve) and N^y (Sr) (value of η when dq/dp = 0 for different degree of saturation).

(d). Parameter directly associated with changes in elastic modulus and shear strength: that include G(Sr) (shear modulus on the unsaturated soil), K (Sr) (bulk modulus on the unsaturated soil and N(Sr), M(Sr) (strength parameters that include the effect of degree of saturation).

The determination of the parameters

To determine those parameters, require a set of controlling initial saturated degree tests have to be performed as shown in Fig.6.

(1). A set of isotropic drained compression at several initial degrees of saturation will be used to determine λ(Sr), k, λ, P_0(Sr). That is shown in Fig.6(a).

(2). A series of drained shear test under deferent degree of saturation, as shown in Fig.6(b), will be used to determine elastic modulus and yield point.

(3). Using the yield stress data achieved from Fig.6(b), a series of yield curve with different degree of saturation will be formed as shown in Fig.6(c). Parameters C^y, N^y (Sr) can be achieved through Fig.6(e). Generally, C^y, N^y(Sr) is adopted as constant value.

(4), According to critical stress data delivered from Fig.6(b), N(Sr), M(Sr), can be evaluate. This is, of cause, a minimum experimental amount. A more reliable, determination of parameters will certainly require more tests.

4 COMPARISON OF THE CONSTITUTIVE EQUATION PREDICTIONS WITH EXPERIMENTAL RESULTS

4.1 Triaxial Test on Unsaturated Compacted Decomposed Granite Soil

Specimen Preparation and Test Method

"KAMISAKI" decomposed granite soil was studied in the triaxial test. The physical properties of this soil are presented in Table.1. A triaxial setup that developed by Writers was applied, that can accuracy measures the volume deformation, drain water amount, and pore water-pressure, and can easily control pore air-pressure. The two series of stress -path tests performed is as shown in Fig.7. A summary of the parameters of the decomposed granite soil is given in Table.2.

Table.1 The normal physical properties of decomposed granite soil

liquid limit	Plastic limit	Plastic Index	Specific Gravity	Saturated permiablity cm/sec
NP	NP	-	2.66	1.0E-5

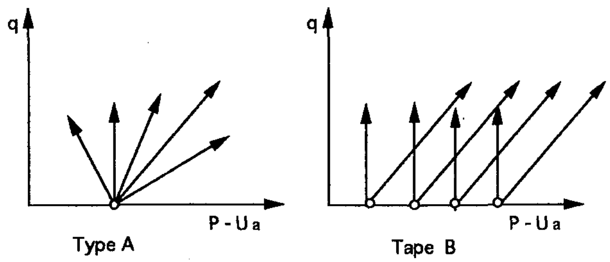


Fig. 7 Stress paths for detmination of behavious of unsaturated decomposed granite soil

Prediction for Triaxial tests results

Figs.8 , 9 show the results of predicted and experimental $q \sim \varepsilon_1$ and $\eta \sim \varepsilon_v$ curves triaxial tests. Fig.8 (a) show the variation of $q \sim \varepsilon_1$ on the triaxial compression side under different stress paths condition ,different initial degree of saturation condition, respectively. Fig.8(b) show the predicted and experimental results for undrained triaxial compression tests. It also indicated that the model is a reasonable representation of the compressive properties of decomposed granite soil under undrained conditions .

Figs.9(a),(b)show the variation of $q \sim \varepsilon_1$ on the triaxial compression side under different stress paths condition ,different initial degree of saturation condition and different stress levels, respectively. From these results ,we can indicate the model presented will be able to describe the tendency for volumetric strains in more complexity condition ,roughly.

From above findings it can be said that the proposed model has a wide applicability in the prediction of triaxial stress -strain behavior in unsaturated state.

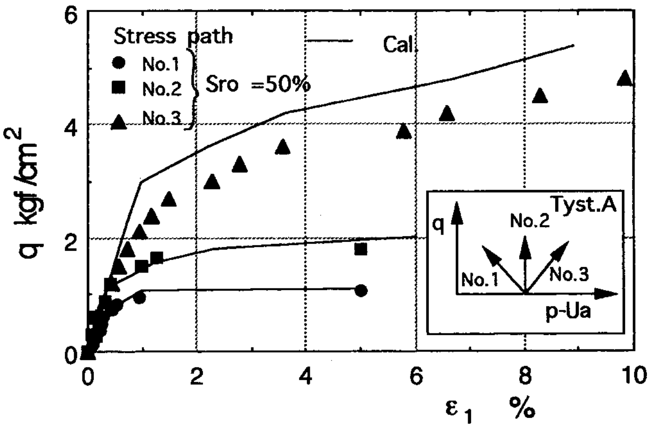


FIg. 8(a) q - ε_1 curve of stress path test of decomposed granit soil under diffrent stress path

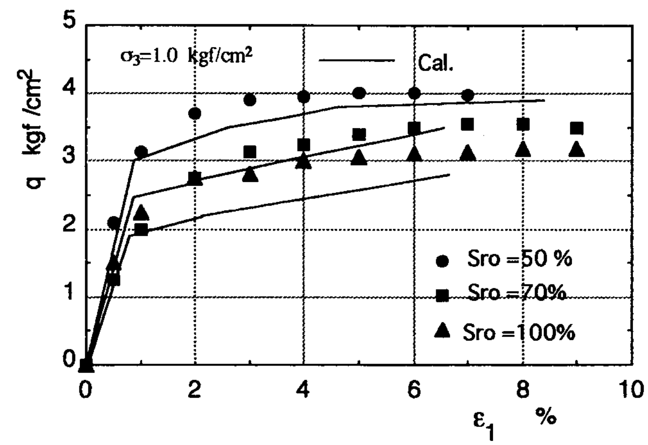


Fig.8(b) q - ε_1 curve of stress path test of decomposed granite soil (Type. B)

Table 2. Model parameters , initial state valus for decomposed granite soil

Initial Values			Elastic moduli		Strength coefficient		Yield function			
e_o	Sro %	P_o	Eo kgf/cm^2	Ko kgf/cm^2	C	ϕ	N^y	C^y	$\lambda(Sr)$ a	b
0.564	40 ~ 100	0.0	500~750	200~600	0.3	38	1.45~ 1.05	2.0	0.35	0.1

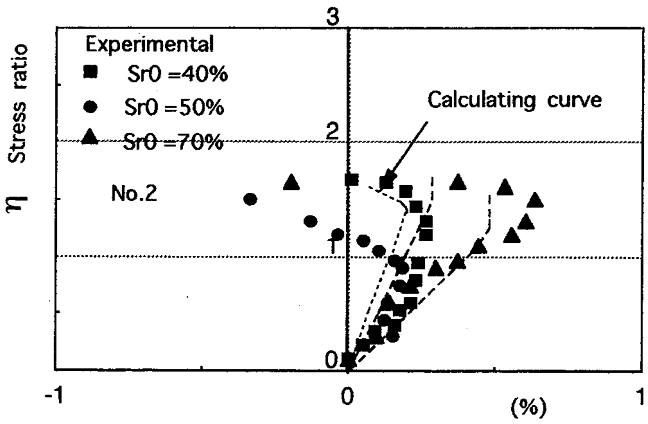


Fig. 9(a) Varition of $\eta \sim \varepsilon v$ curve with degree of saturation in Type.A

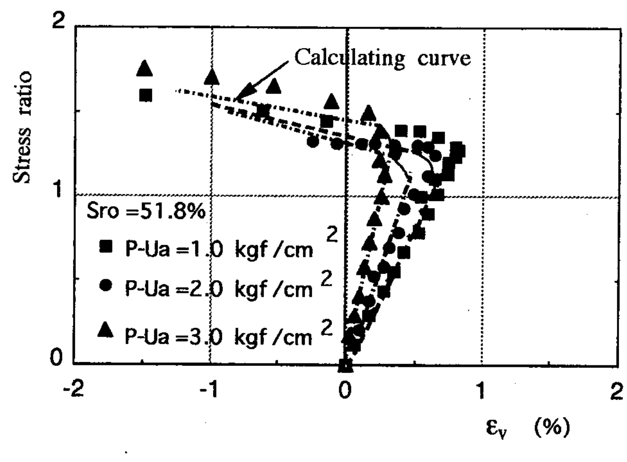


Fig. 9(b) $\eta \sim \varepsilon v$ curve in Test Type B

4.2 Model Test for Mixture Soil of Sand/Bentonit

Physical properties and Model Test

In terms of design requirement in filed, soil used in the model research is mixture materials of TOYOHO sand and Bentonit clay, the mixture ratio of sand and bentonit is 0.85:0.15. The normal physical properties are showed in Table.3. In term of the design requirement 90% optimum's water content and density was selected in the model test and the triaxial test, water content and Dry density of mixture soil are 16% and 1.69 g/cm3 , respectively.

For investigating the deformation and failure behavior of mixture foundation soil of sand/bentonit in unsaturated state and saturated state, a series of model tests was completed in laboratory.

Deformation behavior of foundation soil

By FEM, the deformation behavior of foundation soil in model test can be simulated. In FEM, The elastic-plastic constitution presented in the paper is used. The joint element also was used to simulate the effect of bottom friction of foundation soil.

The mesh model of calculating analysis is showed in Fig.10, the parameters used are summered in Table.4, the detail interpreted is in section 3.3. Fig.11 showed the partly calculating results and test results of settlement of model foundation in different B/H values. By this compare analysis, it can be considered that the FEM analysis is well to simulate settlement variation of model foundation in elastic-plastic deformation stage. The utility of elastic-plastic constitution of unsaturated soil presented in the paper is also shown.

Table.3 The normal physical properties of micture soil of sand/bentonit

liquid limit	Plastic limit	Plastic Index	Specific Gravity	Saturated permiablity cm/sec
25 %	6 %	19	2.626	1.0E-8

Table 4. The parameters , initial state valus for FEM calculation (mixture soil of sand/bentonit)

Initial Values			Elastic moduli		Strength coefficient		Yield function		
e_o	Sro %	P_o	Eo kgf/cm^2	Ko kgf/cm^2	C	ϕ	N^y	C^y	$\lambda(Sr)$ a b
0.545	70~90	0.0	100~450	100~300	0.13	24	0.96	2.0	0.25 0.1

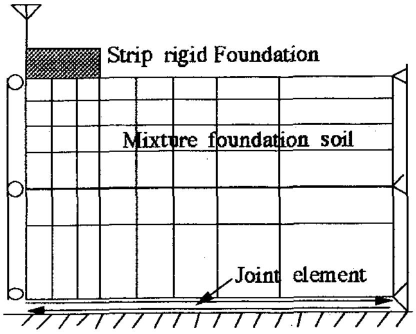


Fig.10 Mesh Model for FEM Calculation

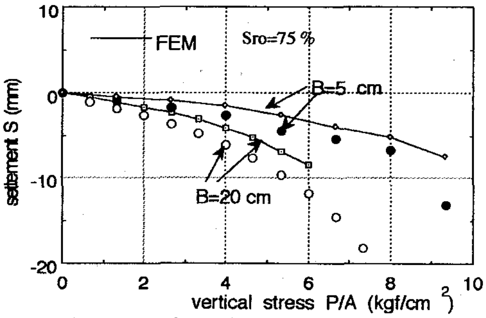


Fig.11 Load-settelment curve of model foundation by FEM and model test

5. CONCLUSION

In this paper, an elasto-plastic constitutive equation for unsaturated soil has been developed and was used to predicate the triaxial test results and model test results for two soil. The conclusions obtained in this paper as follows:

1. Through analysis the behavior of stress variables that has been used in last research for unsaturated soil, we can think that there are the some advantages when saturated degree is adopted as parameter. For example, variation of mechanical behaviors can be easily understand from physical properties; variation of degree of saturation in sample can also be conveniently understand through measurement drain water content; it can be ease to make a extend of the theory that had been established in saturated soil to unsaturated soil.

2. An elasto-plastic constitutive equation presented in the paper can reflect some mechanical behavior of unsaturated soil through increasing the parameter of saturated degree in yield function and hardening modulus. The parameters included in this constitutive equation can be easily determined by a few conventional triaxial tests and those also have the clear physical meaning.

3. The proposed model has a wide applicability in the prediction of triaxial stress-strain behavior in unsaturated state. Through comparison with model test results, the utility of the constitutive equation for predicting deformation of unsaturated soil is shown.

REFERENCES

[1] Coleman,J.D.,(1962) "Stress strain relations for partly saturated soil "Correspondence to Geotechnique . Vol. 12 , No. 4. pp.348-350.

[2] Fredlund,D.G., Morgenstern, N.R. and Widger, R.A.(1978) : Shear strength of unsaturated soil, Canadian Geotechnical Journal. Vol.15, No.3, pp.313-321

[3] Karube,D., Namura,K., Morita,N., Iwasaki,T,(1978): " Fundamental study of the stress-strain behavior of an unsaturated soil ", Geotechnical engineering JSCE .No.269. pp105-119

[4] Matyas,E.L.,Radhakrishna,H.S.,(1968) "Volume change characteristics of partially saturated soils ". Geotechnique .Vol.18 ,No. 4 ,pp.432-448.

[5] Karube,D.,Koto.S.,(1989): "Yield function of unsaturated soil " 12IC-SMFE pp.615-618.

[6] Alonso,E.E. , Gens,A. and Hight,D.W.(1987). Special problem soils . General report. Proc. 9th European Conf. Soil Mech. Fdn Engng, Dublin, 3, pp1087 - 1146.

[7] Alonso,E.E. , Gens,A. and Josa,A. (1991) : " A constitutive model for partially saturated soils," Geotechnique 40, No.3, pp.405 - 430 .

[8] Bishop, A. W. and Blight, G. E.,(1963). Some aspects of effective stress in saturated and partly saturated soils . Geotechnique 13 , No. 3, pp.177 - 197.

[9] Blight, G. E.(11965). A study of effective stress for volume change. In Moisture equilibria and moisture changes in soils beneath covered areas, pp. 259 - 269.

[10] Yasufuku, N., Murata , H. and Hyodo, M.(1991) : " yield characteristics of anisotropically consolidated sand under low and high stresses," Soils and Foundations, Vol. 31, No.1, pp.95- 109.

[11] Yasufuku, N., Murata , H., Hyodo, M.H. and Hyde, A. F. L.,(1991) : " A stress strain relationship for anisotropically consolidated sand over a wide stress regeion,"Soils and Foundations, Vol. 31, No.4, pp.75-92.