A Beam Focusing Antenna for the TEon Mode High-Power Millimeter Wave

Osami Wada † and Masamitsu Nakajima ‡

SYNOPSIS

This paper describes a method to design an antenna to focus millimeter-wave beam generated by a gyrotron. The antenna, which has been proposed by the authors, consists of a stair-cut circular waveguide and two cylindrical reflectors; one is elliptic and the other is parabolic. Its principle is based on the geometrical optics though slightly modified to consider the diffraction effect. Results of low-power experiments agree well with the design on beam direction, beam width and the position of the focal point. At 35.5 GHz using TE_{01} mode, a focused beam with half-power thickness of 13 mm × 10 mm was obtained. This type of antennas find applications to millimeter-wave scattering measurement in fusion plasma research and high-energy-density source for material heating.

1 INTRODUCTION

High-power millimeter wave is used for electron cyclotron resonance heating (ECRH) of magnetically confined fusion plasma, where linearly polarized sharp beam is required for efficient heating. A gyrotron oscillator generates 100 ~ 200 kW of millimeter wave in the range from 28 to 140 GHz. Normally the output from a gyrotron is of a high order mode, especially of an axially symmetric TE_{0n} mode.

Vlasov et al. proposed a quasi-optical reflector antenna which transforms the TE_{0n} mode into a linearly polarized beam. The antenna consists of an oversized circular waveguide having a stair-cut open end and a parabolic cylinder reflector, as shown in Fig. 1. From the geometrical-optics approximation, the width of the beam is to be constant along its axis. But the real beam is broadened with propagation due to diffraction.

†Department of Electrical and Electronic Engineering
‡Department of Electronics, Kyoto University
For localized heating of plasma, it is required to focus the beam and to increase the power density. A popular technique may be to use another ellipsoidal, or paraboloidal, reflector or its modification together with Vlasov's converter for focusing the beam into one point. However, since the size of the Vlasov-type antenna is not sufficiently larger than the wavelength, it is not always successful the design of the reflector being difficult. The curvature of the reflector must be designed with consideration of the diffraction effect which depends on beam widths. Generally the beam widths are different in the E-plane and the H-plane of the antenna, so it is difficult to design a reflector which focuses the beam having different thickness in the two directions.

The authors proposed another type of focusing antenna which consists of a stair-cut circular cylinder waveguide and two cylindrical reflectors; one is an elliptic cylinder and the other is a parabolic cylinder as shown in Fig. 2[2-4]. The two reflectors are cylindrical and control the shape of the beam independently in two planes; the former does in the E-plane and the latter in the H-plane. Therefore the reflectors are designed and shaped more easily than the two dimensionally curved reflectors.

Recently some applications of this type of antennas are being made for millimeter-wave scattering measurement in fusion plasma experiments[5-7] and for a high energy density source for heating various materials[8].

In this paper we will present a detailed method of designing the reflectors on the basis of the geometrical optics. We have taken the diffraction effect into consideration for determining the size of the reflectors. We will also report some results of low-power experiments which were performed at 35.5 GHz using the TE_{01} mode. The measured power distribution agreed well with result of calculation by means of field integration on the equivalent image source based on Huygens-Green's formula[3,4]. The method of numerical calculation was described in the references [3], [4], and [7].

2 PRINCIPLE

The configuration of the original quasi-optical antenna, proposed by Vlasov et al[1], is shown in Fig. 1. It consists of an open-ended circular waveguide with a semicircular cylindrical visor and
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A parabolic cylinder reflector whose focal axis coincides with the waveguide axis. When the TE_{0n} mode is incident, the field radiates from the open end in the lower-half space, and is reflected on the parabolic cylinder to produce a beam of parallel rays.

On the approximation of the geometrical optics, the direction angle of the beam with the waveguide axis \( a \) is determined by

\[
\alpha = \sin^{-1} \left( \frac{k_z}{k} \right) = \sin^{-1} \left( \frac{\chi'_{on}/a}{2\pi/\lambda} \right),
\]

where \( a \) is the radius of the waveguide. \( k = 2\pi/\lambda \) and \( k_z = \chi'_{on}/a \) are the wave number in free space and the cutoff wave number of TE_{0n} mode, respectively. The widths of the beam are

\[
D_E = 4f, \quad D_H = 4a \cos \alpha,
\]

in the E-plane and the H-plane respectively, where \( f \) is the focal distance of the parabolic cylinder. In practice the diffraction effect should be taken into account, since the beam will be broadened with propagation.

Instead of the parabolic cylinder reflector, let us consider an elliptic cylinder reflector which is placed so that one of the focal axes coincides with the waveguide axis as shown in Fig. 2. Then the beam from the waveguide is focused on the other focal axis. The direction of polarization is nearly perpendicular to the symmetrical plane.

In addition, using another parabolic cylinder reflector positioned between the lower and upper focal axes as shown in Fig. 2, the beam can be focused to a point.

The two reflectors are cylindrical and each of them controls the shape of the beam independently in the respective direction; the elliptic reflector focuses the beam in the E-plane and the parabolic one focuses it in the H-plane. Therefore they are easily modified to control the beam as compared with the two dimensionally curved reflectors.

The location of the focusing point can be altered by selecting parameters of the reflectors. The spot size of the beam at the focal point depends on the parameters: an aperture size, focal...
distance, and so on. To be short, the spot size is proportional to the distance between the reflector and the focal point, and is inversely proportional to the width of the aperture of the reflector. The details of the size of reflectors and the thickness of beam are described in section 3.

3 DESIGN OF REFLECTORS BY THE GEOMETRICAL OPTICS

In this section, a procedure for designing the open-ended waveguide and the reflectors used in the focusing antenna will be discussed on the basis of the geometrical optics. Some modifications in size are also mentioned in consideration of the diffraction effect.

3.1 Open end of a circular waveguide

The radiation from an open end of an oversized waveguide is expressed by superposition of localized plane waves which are treated as 'optical' rays in the geometrical optics approximation. Each ray is propagated at the same angle $\phi$ with the waveguide. When the waveguide has a semicircular cylindrical visor at its end, the rays are reflected on the visor and radiated in the opposite direction forming a hollow semiconical horn. If the rays were perfectly 'optical', the length of the visor would be

$$L_0 = 2a \cot \phi.$$  \hfill (3)

In practice, we have to consider the diffraction effect so that the visor should be made longer than $L_o$, as shown in Fig. 3.

The radiation from the waveguide depends upon the length $L$ of the visor. Fig. 4 shows measured radiation field of TE$_{01}$ mode from waveguides with $L = 94.7$ mm and $144.7$ mm; 35.5 GHz, $a = 16$ mm, $\alpha = 18.8$ degree, $L_o = 94.1$ mm. In the case of $L \approx L_o$, compared with the case of longer $L$, the main lobe level is lower and diffraction at the end of the visor causes a higher sidelobe around the waveguide axis, $\theta \approx 0$.

In view of diffraction, the longer the visor is, the better. But the long visor is not convenient
for the reflector system, since a part of the rays reflected from the elliptic cylinder will hit the visor. A suitable length can be estimated by considering the half-power angle of the beam.

The 'optical' beam thickness $D_H$ in parallel with the magnetic field in Fig. 3 is calculated from

$$D_H = 4a \cos \alpha.$$  \hspace{1cm} (4)

Considering diffraction, the half-power angle is estimated as

$$\theta_H \approx \frac{\lambda}{D_H}.$$  \hspace{1cm} (5)

The radiation from the waveguide will occur within the range of

$$\alpha - \frac{\theta_H}{2} \leq \theta \leq \alpha + \frac{\theta_H}{2}.$$  \hspace{1cm} (6)

Thus the length of the visor $L$ is determined to be

$$L \geq 2a \cot (\alpha - \frac{\theta_H}{2}).$$  \hspace{1cm} (7)

The sizes of reflectors should also be determined with consideration of the range of radiation angle, which will be explained in 3.2 and 3.3.

3.2 Elliptic cylinder reflector

3.2.1 Basic design by the geometrical optics

On the approximation of the geometrical optics, the radiation from the open-ended circular waveguide with a visor is equivalent to a group of rays which intersect the waveguide axis within the range of $-L_o \leq z \leq L_o$, where $L_o$ is given in eq. (3). Each ray has the same angle of direction $\alpha$ with the waveguide axis as given by eq. (1).
Let us consider an elliptic cylinder reflector whose lower focal axis coincides with the waveguide axis. We will use a system of coordinates shown in Fig. 5, in which the waveguide axis is along the z-axis and the semicylinder visor is located at \( z = -L_o/2 \) and \( x \geq 0 \). The major axis of the ellipse is contained in the \( zx \)-plane, that is the plane of symmetry of the waveguide with the visor. The elliptic cylinder is defined by the equations

\[
\frac{y^2}{A^2} + \frac{(x - f_e)^2}{B^2} = 1, \tag{8}
\]

\[
f_e = \sqrt{B^2 - A^2}, \tag{9}
\]

where \( A \) and \( B \) are the length of the minor and major radii respectively, and \( f_e \) is one-half of the distance between the two focal axes \( O_1 \) and \( O_2 \) in Fig. 5.

In Fig. 5, the 'optical' width \( D_E \) of the reflector in the \( y \)-direction at \( x = 0 \) is represented as a function of \( A \) and \( B \) as

\[
D_E = 2y_o = \frac{2A^2}{B}. \tag{10}
\]

The \( A \) and \( B \) are solved as functions of \( f_e \) and \( y_o \)

\[
A = \sqrt{\frac{y_o(y_o + \sqrt{y_o^2 + 4f_e^2})}{2}}, \tag{11}
\]

\[
B = \frac{y_o + \sqrt{y_o^2 + 4f_e^2}}{2}. \tag{12}
\]

The group of rays radiated from \( O_1 \) are focused on \( O_2 \) by the elliptic cylinder reflector. Assuming that \( 2f_e \) is larger enough than \( D_E \), the polarization of each ray is nearly parallel with the \( y \)-direction.
The beam width at the focal axis $O_2$ is approximately proportional to the distance $PO'$ along the beam from the aperture of the reflector to the focal axis in Fig. 5, where
\[ PO' = \frac{2f_e}{\sin \alpha}, \]
and inversely proportional to the width of the aperture $D_E$. The half-power width at the focal axis is thus calculated using the equation
\[ w_E \approx \frac{2f_e \lambda}{D_E \sin \alpha}. \]  
This expression gives a good estimation as mentioned in section 4.

### 3.2.2 Size of the reflector in consideration of diffraction

The height of the reflector $h$ in Fig. 5 is determined as follows. If the radiation were perfectly 'optical', the illuminated area would be $x < 0$ and $h$ should be as high as $B - f_e$. But, since we have to consider the diffraction at the edges of the semicircular visor, we will estimate it by means of the Fresnel approximation.

The rays are incident obliquely on the edges at $(x, y) = (0, \pm a)$, the diffraction being three-dimensional. For simplicity, however, we will approximate it by two-dimensional model. Fig. 6 shows the system of coordinates for this problem. Rays radiated cylindrically from the $\eta$-axis are incident perpendicularly to an edge at $(\zeta, \xi) = (a', 0)$, where $a'$ is the radius of the equivalent semicylinder:
\[ a' = \frac{a}{\sin \alpha}. \]  
The polarization is perpendicular to the edge. Radiation occurs from the aperture in the range of $s = s_1$ to $s_2$, where $s$ is the length along the circle from the point M as shown in Fig. 6.
We will calculate the field strength at a point \( P \) on the reflector. If the diffraction angle \( \phi \) is small, the distance between the semicylinder and \( P \) is approximated by \( b \) in Fig. 6, where

\[
b = \frac{y_o - a}{\sin \alpha}.
\]

The strength of the electric field at point \( P \) is expressed using the Fresnel approximation

\[
E = K \sqrt{\frac{a'b\lambda}{2(a' + b)}} E_o R \exp(\omega t - kb - \theta),
\]

where \( E_o \) is the electric field on the cylinder, \( K \) is almost constant, and \( R \) and \( \theta \) is defined as follows.

\[
R = \sqrt{\{f(u_1, u_2)\}^2 + \{g(u_1, u_2)\}^2},
\]

\[
\theta = \tan^{-1}\left(\frac{g(u_1, u_2)}{f(u_1, u_2)}\right),
\]

\[
f(u_1, u_2) = \int_{u_2}^{u_1} \cos\left(\frac{\pi u^2}{2}\right) du = C(u_1) - C(u_2),
\]

\[
g(u_1, u_2) = \int_{u_2}^{u_1} \sin\left(\frac{\pi u^2}{2}\right) du = S(u_1) - S(u_2),
\]

\[
u = \sqrt{\frac{2(a' + b)}{a'b\lambda}}\phi,
\]

where \( C(u) \) and \( S(u) \) are the Fresnel integrals.

The \( R \) without the semicylinder, represented by \( R_o \), is approximately equal to \( \sqrt{2} \), which is calculated using \( s_1 = -a'\pi, \ s_2 = a'\pi, \ -C(u_1) = C(u_2) \approx -S(u_1) = S(u_2) \approx 1/2 \). Therefore the power diffraction coefficient \( D \) at point \( P \) is given by the expression

\[
D = \frac{R^2}{R_o^2} = \frac{1}{2}\left[\left(C(u_1) - C(u_2)\right)^2 + \left(S(u_1) - S(u_2)\right)^2\right]
\]

\[
= \frac{1}{2} \left[\left(C(u_1) - \frac{1}{2}\right)^2 + \left(S(u_1) - \frac{1}{2}\right)^2\right],
\]

\[
u_1 = \sqrt{\frac{2a'(a' + b)}{b\lambda}}\phi,
\]

where \( s_1 = a'\phi \) and \( s_2 = a'(\phi + \pi) \).

The value of \( u_1 \) which gives \(-10\text{dB}\) of \( D \), represented by \( u_{[-10\text{dB}]} \), is approximately 0.5. The \( \phi \) and \( h \) will hence be calculated by use of the equations

\[
\phi_{[-10\text{dB}]} = \sqrt{\frac{b\lambda}{2a'(a' + b)}} u_{[-10\text{dB}]} \approx \sqrt{\frac{b\lambda}{8a'(a' + b)}},
\]

\[
h_{[-10\text{dB}]} = (B - f_e) + \Delta h,
\]

\[
\Delta h = (a' + b) \tan \phi_{[-10\text{dB}]} = \frac{y_o}{\sin \alpha} \tan \sqrt{\frac{(y_o - a)\lambda \sin \alpha}{8ay_o}}.
\]

The next thing we have to do is to determine the length of the elliptic cylinder reflector.
If the radiation from the waveguide were 'optical', rays were radiated within the range of \(\pi/2 \leq \phi \leq 3\pi/2\), where \(\phi\) is the angle measured from the \(x\)-axis as shown in Fig. 5. The minimum distance from the waveguide axis to the reflector is equal to \(B - f_e\) on the plane of \(\phi = \pi\), and the maximum distance is equal to \(y_o\) on the plane of \(\phi = \pi/2\) or \(3\pi/2\). Then the reflector for 'optical' rays may exist in the range of \(Z_{\text{min}} \leq z \leq Z_{\text{max}}\), where

\[
Z_{\text{min}} = (B - f_e - 2a) \cot \alpha,
\]
\[
Z_{\text{max}} = (y_o + 2a) \cot \alpha.
\]
(27)

The real radiation occurs within the range of angle between \(\alpha - \theta_H/2\) and \(\alpha + \theta_H/2\) from the waveguide, as noted in 3.1. The range of the reflector will hence be modified as shown in Fig. 7

\[
Z'_{\text{min}} = (B - f_e - a) \cot \left(\alpha + \frac{\theta_H}{2}\right) - \frac{L_o}{2},
\]
\[
Z'_{\text{max}} = (y_o + 3a) \cot \left(\alpha - \frac{\theta_H}{2}\right) - \frac{L_o}{2}.
\]
(28)

### 3.3 Parabolic cylinder reflector

The rays reflected from the elliptic cylinder are focused in a segment \(S_1S_2\) on the upper focal axis \(O_2\) as shown in Fig. 8. In order to focus them into one point which is denoted by \(F\) in Fig. 8, we will use another reflector.

In the geometrical optics approximation, the rays from the elliptic cylinder reflector are parallel with the \(yz'\)-plane, so that the second reflector should be of parabolic cylinder which is uniform along the \(y\)-direction and the directrix plane of which is perpendicular to the \(z'\)-axis containing \(S'_1, S'_2, O', H\).

Let the coordinates of the focal point \(F\) be \((-x'_f, 0, -2f_p)\) in the coordinate system \((x', y, z')\) in Fig. 8. The symmetric plane of the parabolic cylinder containing \(F, G, H\) is parallel with the
Fig. 8 Beam focusing by a parabolic cylinder reflector.

Fig. 9 Determination of the width of the second reflector: half-power width $w_E$; the width of the parabolic reflector $D'_E$; the width of the elliptic reflector $D_E$. 
yz'-plane. The equation of the parabolic cylinder reflector is given by

$$z' = -\frac{(x' + y')^2}{4f_p} - f_p.$$  

(29)

The size of the reflector can be determined as follows. Since the reflected field from the elliptic cylinder broadens in the xz-plane in the range of angle between $\alpha - \theta_H/2$ and $\alpha + \theta_H/2$ with the z-axis as shown in Fig. 7, the width of the beam at the second reflector is determined to be from $Q_1$ to $Q_2$. With the coordinate system $(x', y, z')$ in Fig. 8, the area of the reflector is hence given by

$$|z'| \leq \frac{D_H'}{2} = \frac{D_H}{2} + \left(\frac{2B}{\sin \alpha} - f_q\right) \tan \frac{\theta_H}{2},$$

(30)

where $D_H'$ is the width of the broadened beam in the H-plane at Q and $f_q$ is the distance $QF$ ($=QO'$) from the second reflector to the focal point shown in Fig. 8.

The beam is focused on the point F by the elliptic and the parabolic cylinder reflectors. The electric field vector is oriented in the y-direction. Let the half-power width of the beam at F in the H-plane be $w_H$, which depends upon the distance $f_q$ ($=QF$) and the beam width $D_H'$ at the reflector. The $w_H$ is approximately given by

$$w_H \approx \frac{f_q}{D_H'}.$$  

(31)

Likewise, using the width of the elliptic reflector $D_E$ and the distance $PO'$ ($= 2f_e/\sin \alpha$), the half-power width in the E-plane $w_E$ is given by

$$w_E \approx \frac{2f_e/\sin \alpha}{D_E}.$$  

(32)

Using the half-power width $w_E$, we can determine the width of the second reflector in the y-direction. Assuming that the field is focused linearly from $D_E$ to $w_E$ in width as shown in Fig. 9, the width $D_E'$ of the reflector placed at $x = x_q$ is calculated to be

$$D_E' = \frac{2f_e - x_q}{2f_e} D_E + \frac{x_q}{2f_e} w_E.$$  

(33)

This estimated width is sufficiently wide.

4 EXPERIMENTAL RESULTS

We designed an experimental antenna on the basis of the geometrical optics as mentioned in the preceding section. Fig. 10 shows the configuration of the antenna; $d = 2a = 16$ mm, $2f_e = 100.0$ mm, $D_E = 97.5$ mm and $f_p = 33.0$ mm. The focal point is located on the waveguide axis in this antenna.

Experiments were performed at 35.5 GHz ($\lambda = 8.45$ mm) using TE_{01} mode. The direction angle $\alpha$ in eq. (1) is 40.1°, and the 'optical' length of the visor $L_o$ is 19.0 mm, $D_H$ 24.5 mm and
\[ \theta = 19.7^\circ. \]

Considering the diffraction effect, the calculated sizes of the elliptic cylinder reflector are: \( z'_{\text{min}} = 9.0 \text{ mm}, \) \( z'_{\text{max}} = 115.4 \text{ mm} \) and \( \Delta h = 21 \text{ mm}. \) The sizes of the experimental antenna were: \( L = 20 \text{ mm}, \) \( z'_\text{min} = 5 \text{ mm}, \) \( z'_\text{max} = 110 \text{ mm} \) and \( \Delta h = 8 \text{ mm}. \) The experimental value of height \( h \) was not large enough.

If the height of the elliptic cylinder reflector \( h + \Delta h \) is not high enough, some of the rays from the waveguide spill over the edges which are parallel with the waveguide axis. But the spilling over causes only power loss, and the effect of diffraction to the power distribution is small, since the electric field vector at the edge is perpendicular to the edges. For this reason the measured distribution agreed well with the estimation described in section 3.

For measurement a quarter-wave monopole antenna made with a semirigid cable was used as a probe. Fig. 11(a) shows the power distribution of the y-polarized field on the \( zz \)-plane without the second reflector. Slant solid line denotes the axis of 'optical' beam, and two broken lines denote the borders of the broadened beam shown by \( PQ_1 \) and \( PQ_2 \) in Fig. 7. The direction and the size of the beam agree well with the estimation in the preceding section. The maximum power density appears at \( x \approx 80 \text{ mm}, \) whereas the 'optical' focal axis is at \( x = 100 \text{ mm}. \) This disagreement is due to the diffraction effect, which decreases as the wavelength is shortened. The authors have calculated the power distribution numerically, which is shown in Fig. 11(b), by means of integration of the equivalent image source on the basis of Huygens-Green's formula. The results of calculation have agreed well with the measured results\(^{[3,4]}\).

The half-power widths at the focal point of this antenna are calculated using equations (31) and (32): \( w_H \approx 8 \text{ mm} \) and \( w_E \approx 13 \text{ mm}. \) Fig. 12 (a) and (b) show the power distribution on the planes parallel with the \( yy \)-plane. The beam width in the \( y \)-direction has its minimum at \( x \approx 80 \sim 90 \text{ mm}. \) The half-power width is about 13 mm at \( x = 90 \text{ mm}, \) which agrees well with the calculated \( w_E \). Fig. 13 shows the power distribution at the focal point of the antenna consisting of two reflectors. The measured half-power widths agree well with the calculated \( w_H \) and \( w_E \).

![Fig. 10 Dimensions of the experimental antenna.](image_url)
Fig. 11 Measured and calculated power distributions of $y$-polarized field focused by an elliptic cylinder reflector only; the dimensions of the reflector is shown in Fig. 10; $\text{TE}_{01}$ mode; waveguide diameter $d = 2a = 16 \text{ mm}; f = 35.5 \text{ GHz}; \alpha = 40.1^\circ; L = 20 \text{ mm}$. 

(a) Experimental results.

(b) Theoretical results.
Fig. 12 Measured power distribution on the planes parallel with the $yz$-plane focused by an elliptic cylinder reflector only; equipower contours at levels of 0.75, 0.5 and 0.25.
(a) around the position of the second reflector.
(b) around the upper focal axis of the elliptic cylinder.

Fig. 13 Measured power distributions around the focal point focused by two cylindrical reflectors; equipower contours at levels of 0.75, 0.5 and 0.25.
5 CONCLUSIONS

We have described the method of designing the beam-focusing antenna using two cylindrical reflectors. The sizes of the reflectors were determined on the basis of the geometrical optics with some modifications by considering the diffraction effect.

We have also reported some experimental results at 35.5 GHz using TE_{01} mode, which agreed well with the estimation by the approximate design method and also with the results of calculation based on Huygens-Green's formula. The half-power thickness of the measured beam at the focal point was 13 x 10 mm², which agrees well with quasi-optical estimation.

As a result of the experiments, it was proved that the quasi-optical treatment and the simple estimation of diffraction are effective enough for practical design of the two-dimensional focusing antenna described in this paper. More detailed characteristics of the antenna can be calculated by the method presented in the references [3], [4], and [7]. The optimization of the shape of reflectors is the subject for a future study.

ACKNOWLEDGMENTS

We wish to thank Prof. T. Idehara, Fukui University, Japan, and Prof. S. Tanaka, Kyoto University, Japan, for their kind help and support on this work. The authors also wish to thank Dr S. Miyake, Osaka University, Japan, for his interest and eagerness in application of the antenna to a gyrotron as an high-energy-density heat-source. This research was supported by the Grant-in-Aid for Fusion Research, from the Ministry of Education, Japan.

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